Chapter 19
Resonant Conversion

Introduction to Resonant Conversion

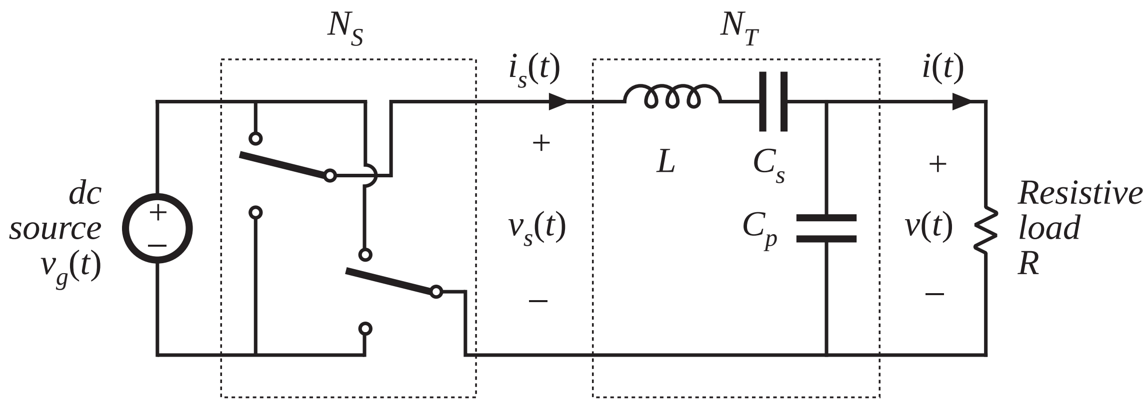
Resonant power converters contain resonant L-C networks whose voltage and current waveforms vary sinusoidally during one or more subintervals of each switching period. These sinusoidal variations are large in magnitude, and the small ripple approximation does not apply.

Some types of resonant converters:

- Dc-to-high-frequency-ac inverters
- Resonant dc-dc converters
- Resonant inverters or rectifiers producing line-frequency ac

A basic class of resonant inverters

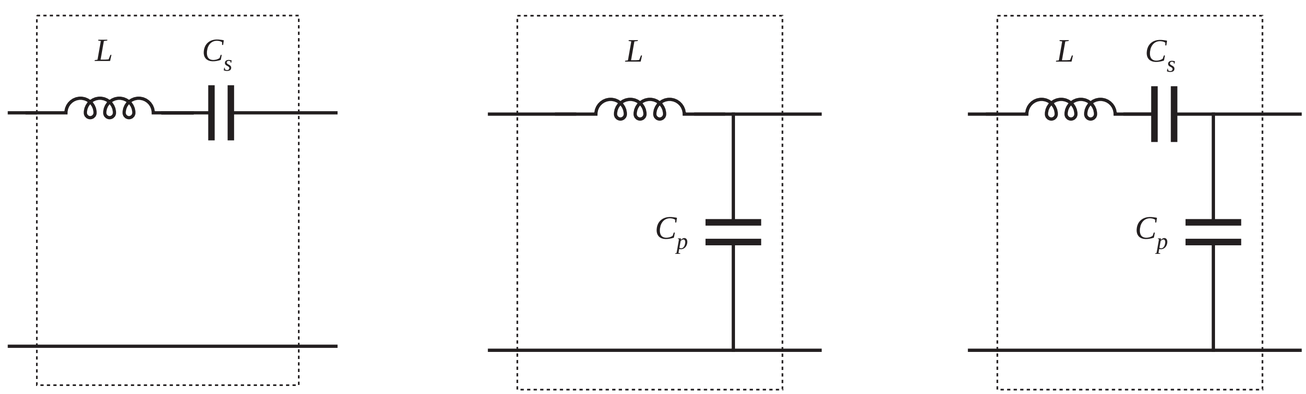
Basic circuit



Switch network

Resonant tank network

Several resonant tank networks

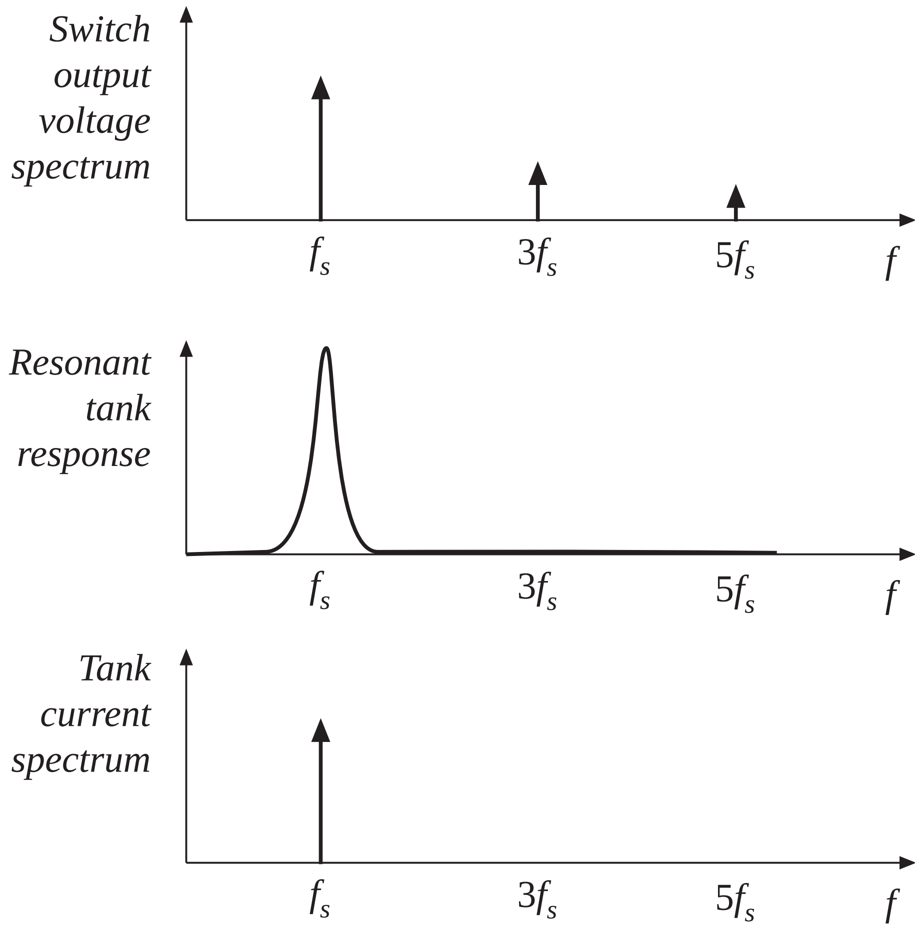


Series tank network

Parallel tank network

LCC tank network

Tank network responds only to fundamental component of switched waveforms

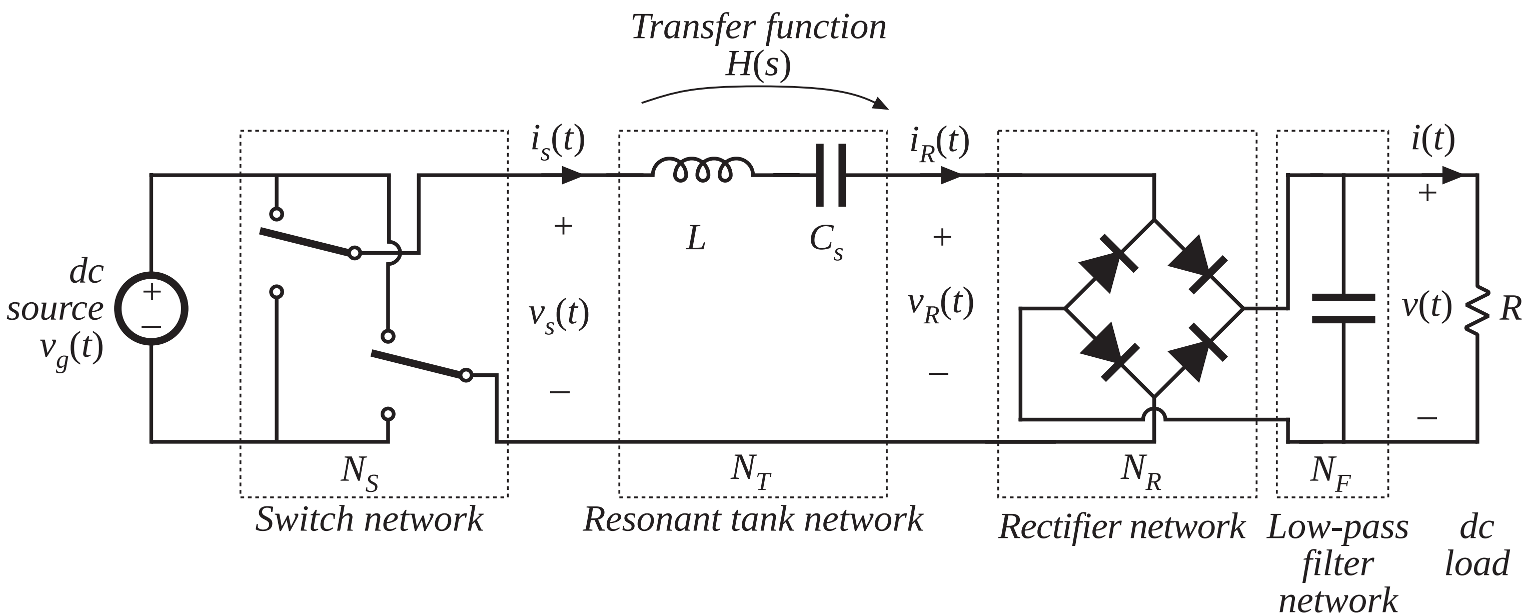


Tank current and output voltage are essentially sinusoids at the switching frequency f_s.

Output can be controlled by variation of switching frequency, closer to or away from the tank resonant frequency

Derivation of a resonant dc-dc converter

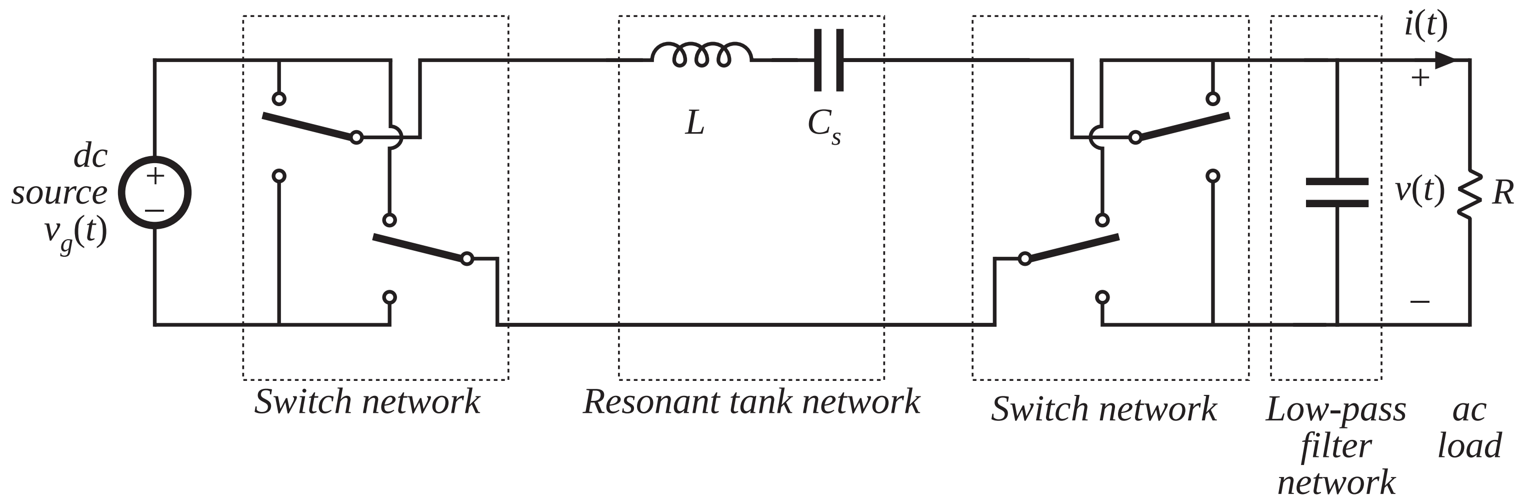
Rectify and filter the output of a dc-high-frequency-ac inverter



The series resonant dc-dc converter

A series resonant link inverter

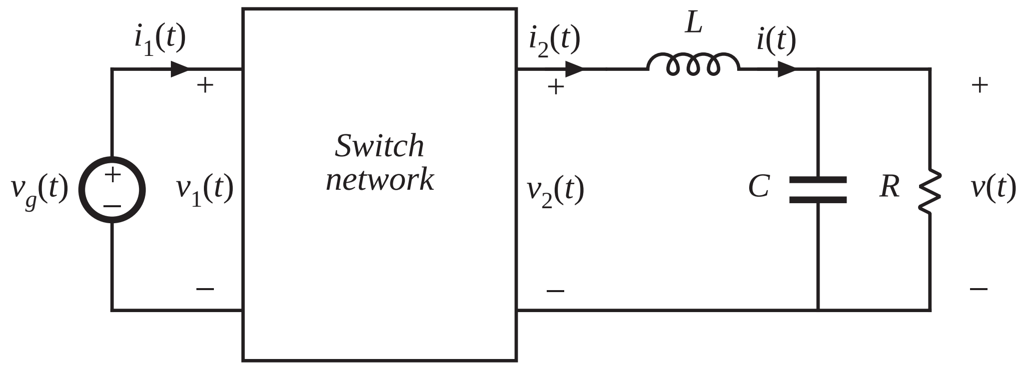
Same as dc-dc series resonant converter, except output rectifiers are replaced with four-quadrant switches:



Quasi-resonant converters

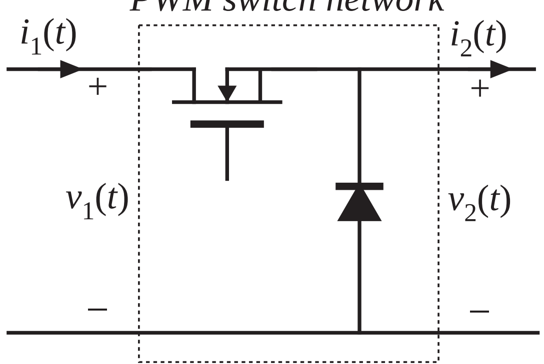
In a conventional PWM converter, replace the PWM switch network with a switch network containing resonant elements.

Buck converter example

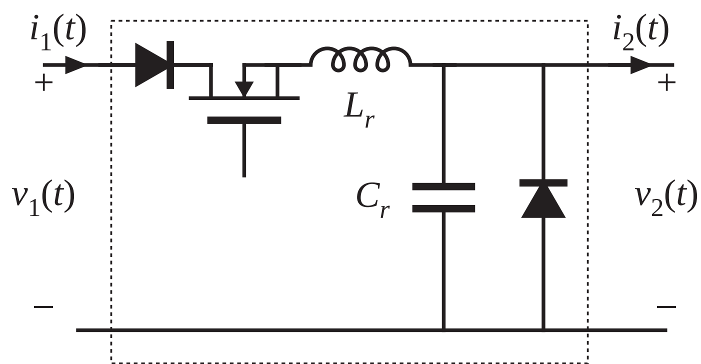


Two switch networks:

PWM switch network



ZCS quasi-resonant switch network



Resonant conversion: advantages

The chief advantage of resonant converters: reduced switching loss

Zero-current switching

Zero-voltage switching

Turn-on or turn-off transitions of semiconductor devices can occur at zero crossings of tank voltage or current waveforms, thereby reducing or eliminating some of the switching loss mechanisms. Hence resonant converters can operate at higher switching frequencies than comparable PWM converters

Zero-voltage switching also reduces converter-generated EMI

Zero-current switching can be used to commutate SCRs

In specialized applications, resonant networks may be unavoidable

High voltage converters: significant transformer leakage inductance and winding capacitance leads to resonant network

Resonant conversion: disadvantages

Can optimize performance at one operating point, but not with wide range of input voltage and load power variations

Significant currents may circulate through the tank elements, even when the load is disconnected, leading to poor efficiency at light load

Quasi-sinusoidal waveforms exhibit higher peak values than equivalent rectangular waveforms

These considerations lead to increased conduction losses, which can offset the reduction in switching loss

Resonant converters are usually controlled by variation of switching frequency. In some schemes, the range of switching frequencies can be very large

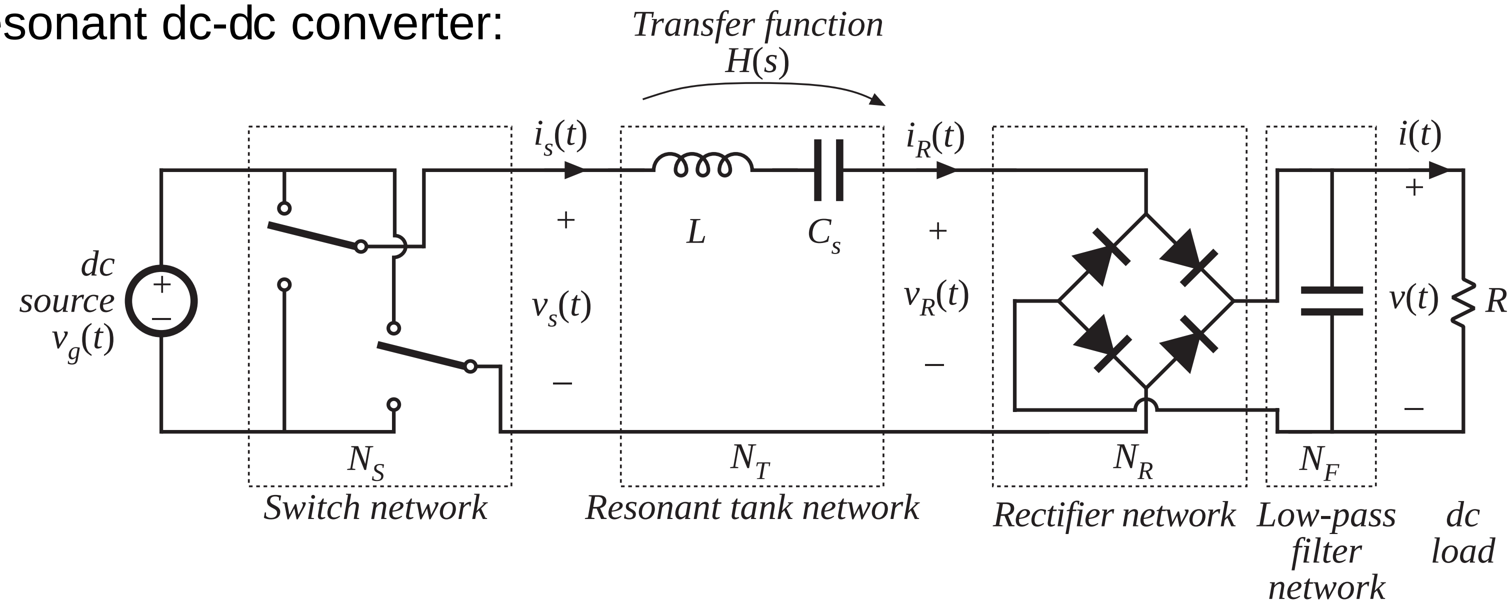
Complexity of analysis

Resonant conversion: Outline of discussion

- Simple steady-state analysis via sinusoidal approximation
- Simple and exact results for the series and parallel resonant converters
- Mechanisms of soft switching
- Circulating currents, and the dependence (or lack thereof) of conduction loss on load power
- Quasi-resonant converter topologies
- Steady-state analysis of quasi-resonant converters
- Ac modeling of quasi-resonant converters via averaged switch modeling

19.1 Sinusoidal analysis of resonant converters

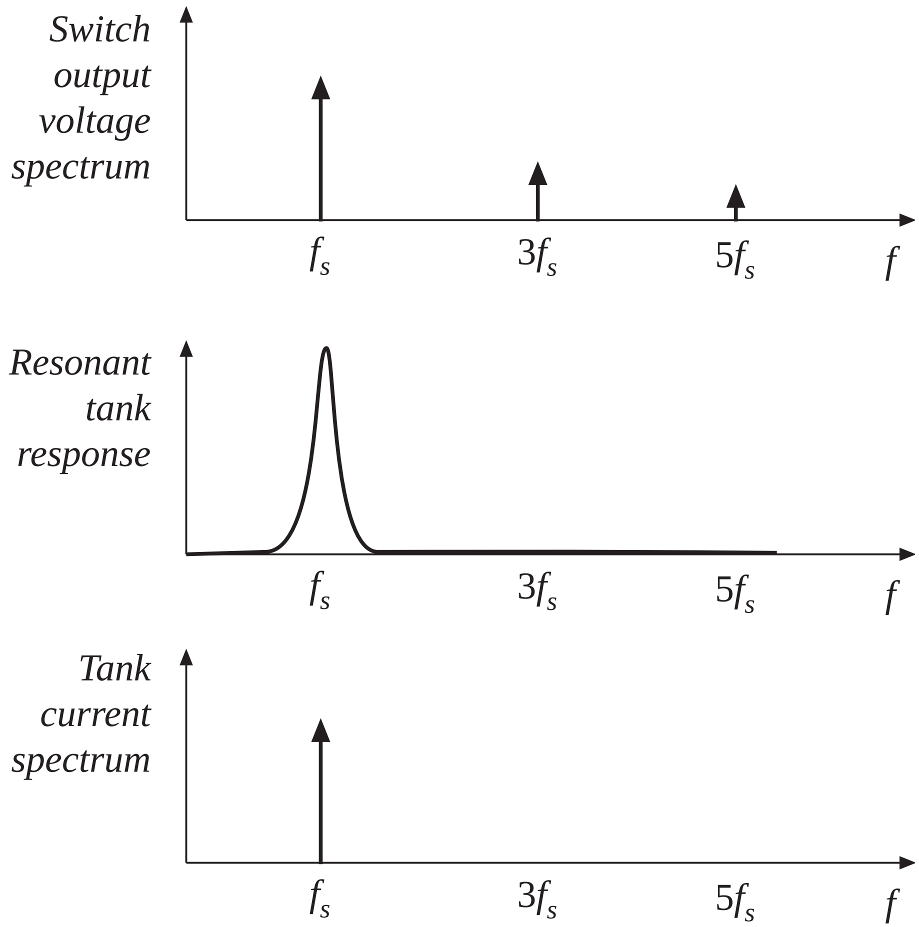
A resonant dc-dc converter:



If tank responds primarily to fundamental component of switch network output voltage waveform, then harmonics can be neglected.

Let us model all ac waveforms by their fundamental components.

The sinusoidal approximation

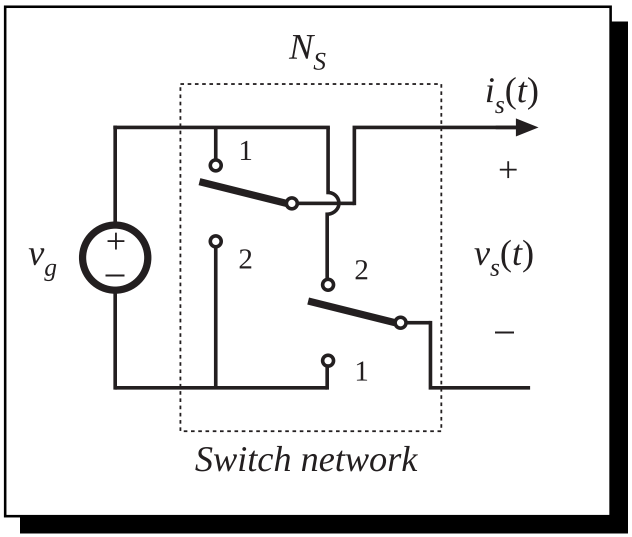


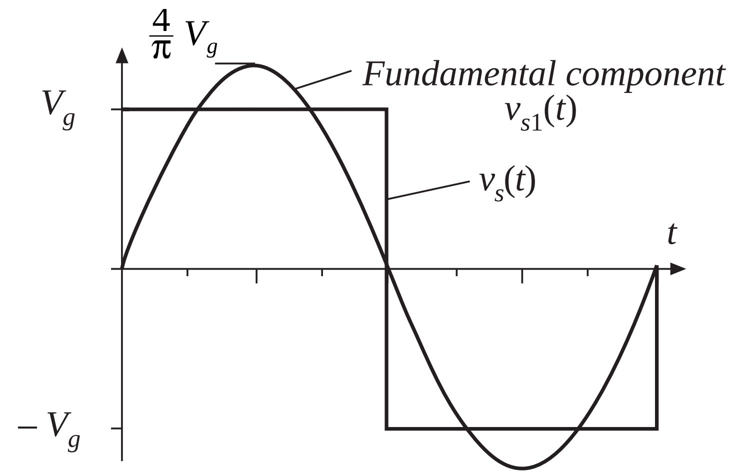
Tank current and output voltage are essentially sinusoids at the switching frequency f_s.

Neglect harmonics of switch output voltage waveform, and model only the fundamental component.

Remaining ac waveforms can be found via phasor analysis.

19.1.1 Controlled switch network model





If the switch network produces a square wave, then its output voltage has the following Fourier series:

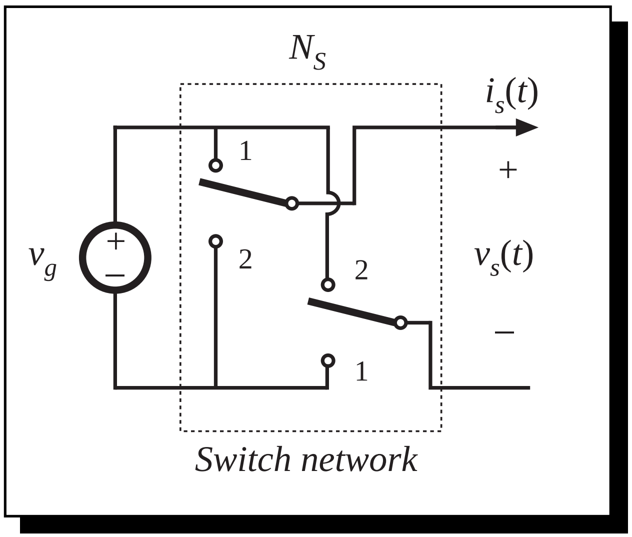
$$v_s(t) = \frac{4V_g}{\pi} \sum_{n = 1, 3, 5, \ldots} \frac{1}{n} \sin (n\omega_s t)$$

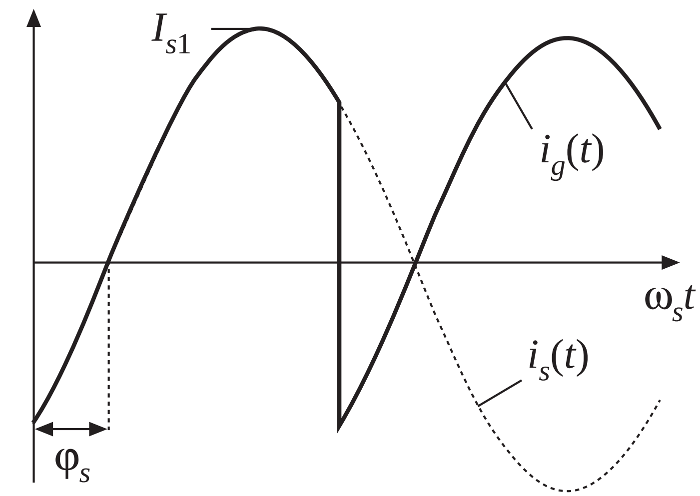
The fundamental component is

$$v_{s1}(t) = \frac{4V_g}{\pi} \sin (\omega_s t) = V_{s1} \sin (\omega_s t)$$

So model switch network output port with voltage source of value $v_{s1}(t)$

Model of switch network input port





Assume that switch network output current is

$$i_s(t) \approx I_{s1} \sin(\omega_s t - \varphi_s)$$

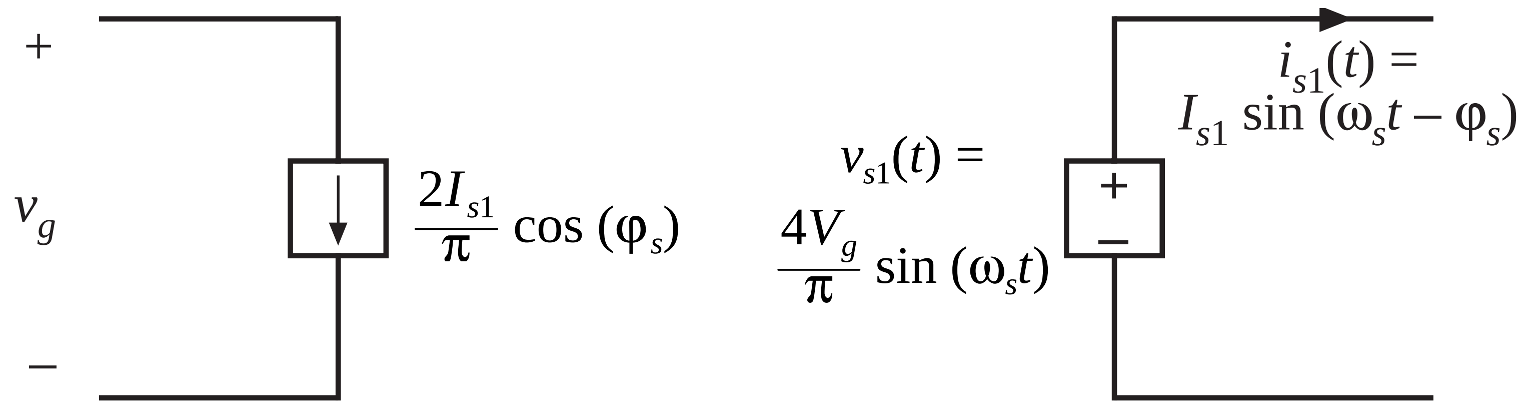
It is desired to model the dc component (average value) of the switch network input current.

$$\left\langle i_g(t) \right\rangle_{T_s} = \frac{2}{T_s} \int_0^{T_s/2} i_g(\tau) d\tau$$

$$\approx \frac{2}{T_s} \int_0^{T_s/2} I_{s1} \sin(\omega_s \tau - \varphi_s) d\tau$$

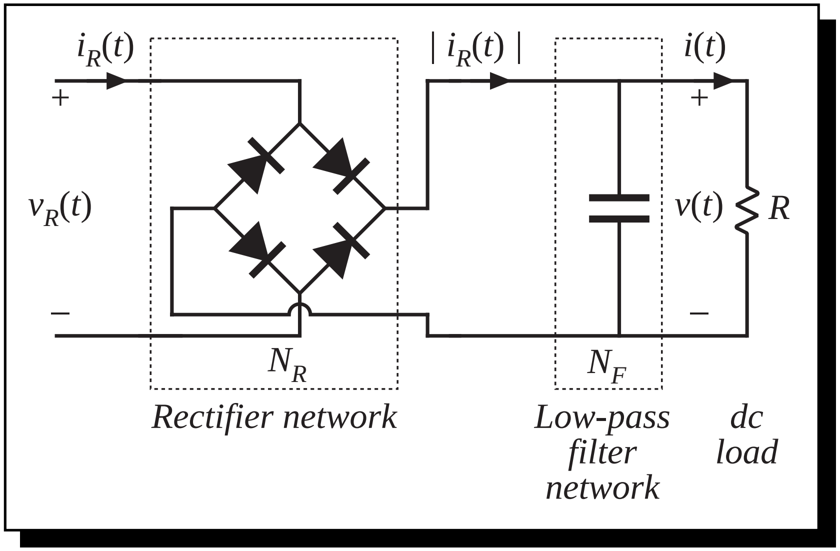
$$= \frac{2}{\pi} I_{s1} \cos(\varphi_s)$$

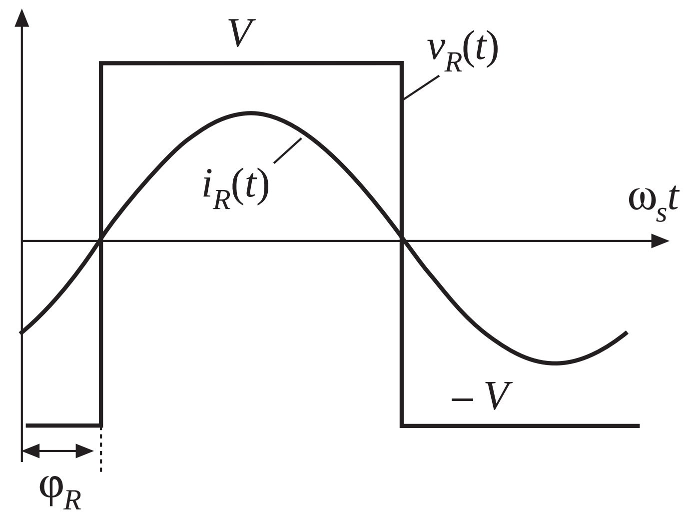
Switch network: equivalent circuit



- Switch network converts dc to ac
- Dc components of input port waveforms are modeled
- Fundamental ac components of output port waveforms are modeled
- Model is power conservative: predicted average input and output powers are equal

19.1.2 Modeling the rectifier and capacitive filter networks





Assume large output filter capacitor, having small ripple.

$v_R(t)$ is a square wave, having zero crossings in phase with tank output current $i_R(t)$.

If $i_R(t)$ is a sinusoid:

$$i_R(t) = I_{R1} \sin(\omega_s t - \varphi_R)$$

Then $v_R(t)$ has the following Fourier series:

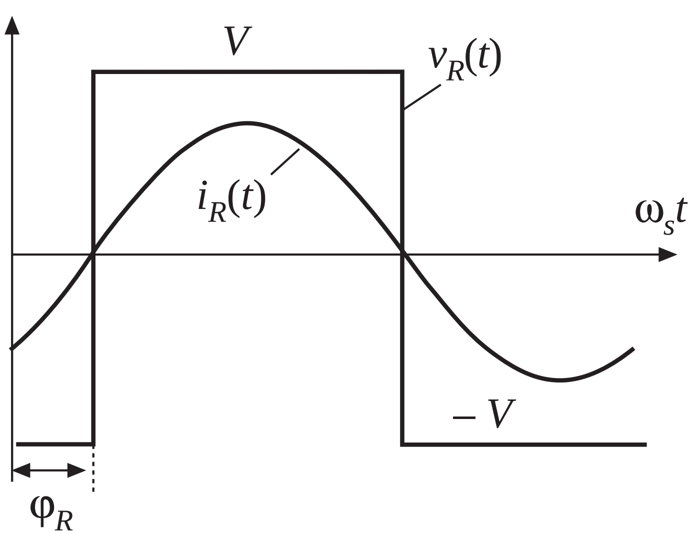
$$v_R(t) = \frac{4V}{\pi} \sum_{n=1,3,5,\cdots}^{\infty} \frac{1}{n} \sin(n\omega_s t - \varphi_R)$$

Sinusoidal approximation: rectifier

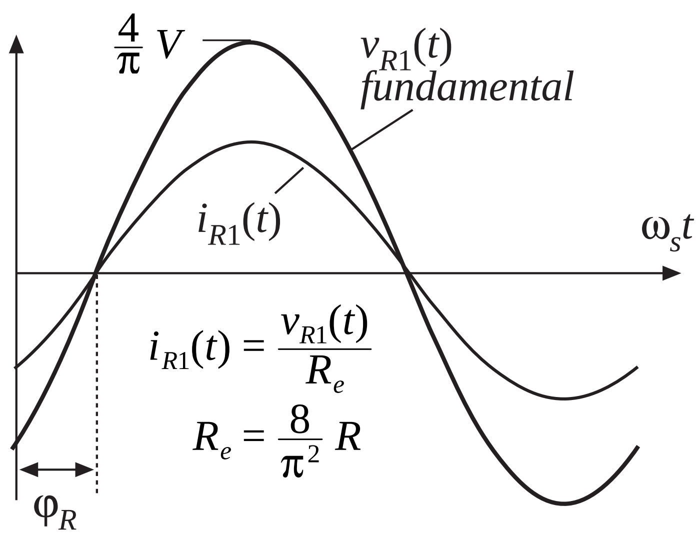
Again, since tank responds only to fundamental components of applied waveforms, harmonics in $v_R(t)$ can be neglected. $v_R(t)$ becomes

$$v_{R1}(t) = \frac{4V}{\pi} \sin\left(\omega_s t - \varphi_R\right) = V_{R1} \sin\left(\omega_s t - \varphi_R\right)$$

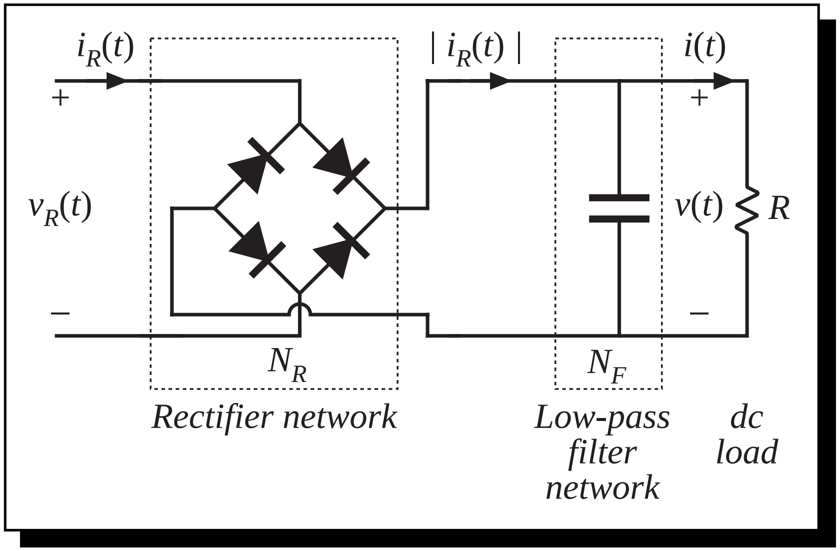
Actual waveforms

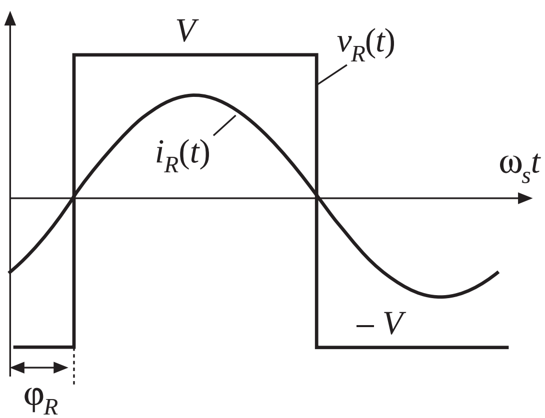


with harmonics ignored



Rectifier dc output port model





Output capacitor charge balance: dc load current is equal to average rectified tank output current

$$\left\langle \left| i_R(t) \right| \right\rangle_{T_s} = I$$

Hence

$$I = \frac{2}{T_S} \int_0^{T_s/2} I_{R1} \left| \sin(\omega_s t - \varphi_R) \right| dt$$

$$= \frac{2}{\pi} I_{R1}$$

Equivalent circuit of rectifier

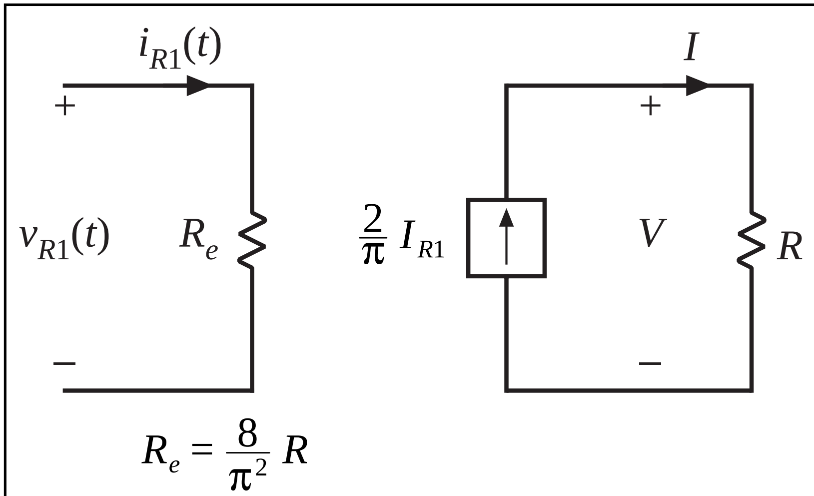
Rectifier input port:

Fundamental components of current and voltage are sinusoids that are in phase

Hence rectifier presents a resistive load to tank network

Effective resistance R_e is

$$R_e = \frac{v_{R1}(t)}{i_R(t)} = \frac{8}{\pi^2}\frac{V}{I}$$

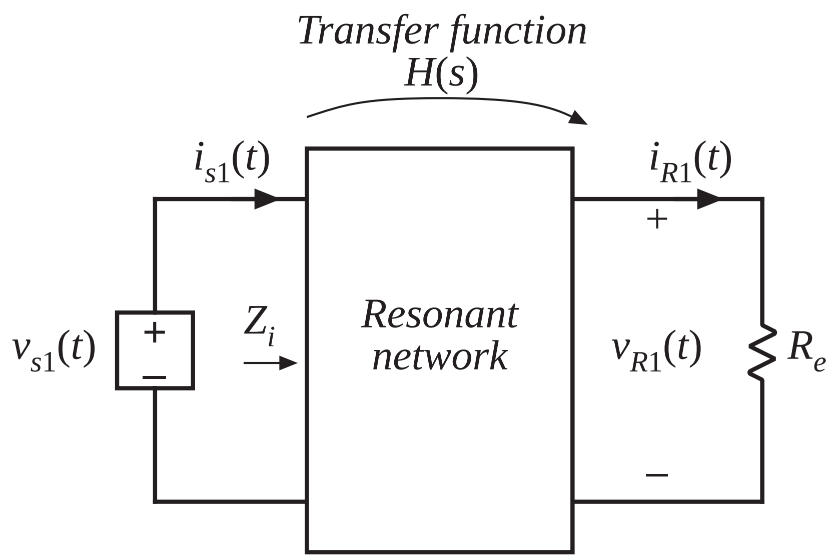


Rectifier equivalent circuit

With a resistive load R, this becomes

$$R_e = \frac{8}{\pi^2} R = 0.8106R$$

19.1.3 Resonant tank network



Model of ac waveforms is now reduced to a linear circuit. Tank network is excited by effective sinusoidal voltage (switch network output port), and is load by effective resistive load (rectifier input port).

Can solve for transfer function via conventional linear circuit analysis.

Solution of tank network waveforms

Transfer function:

$$\frac{v_{R1}(s)}{v_{s1}(s)} = H(s)$$

Ratio of peak values of input and output voltages:

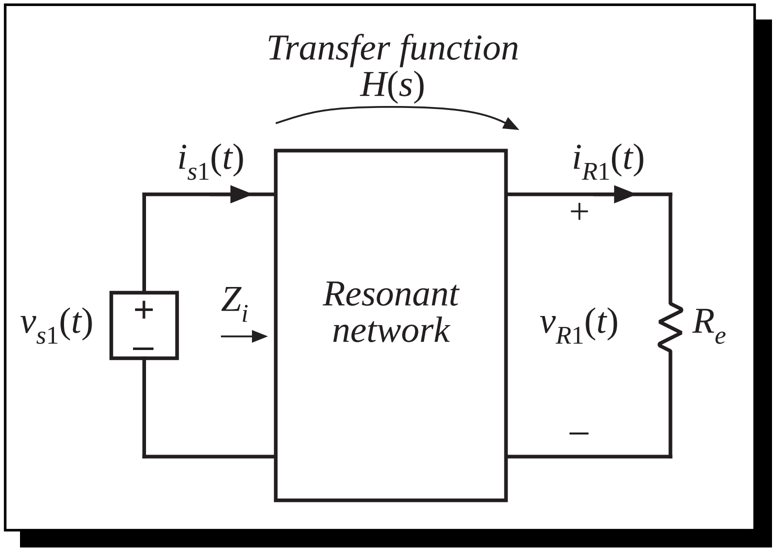
$$\frac{V_{R1}}{V_{s1}} = \left\| H(s) \right\|_{s = j\omega_s}$$

Solution for tank output current:

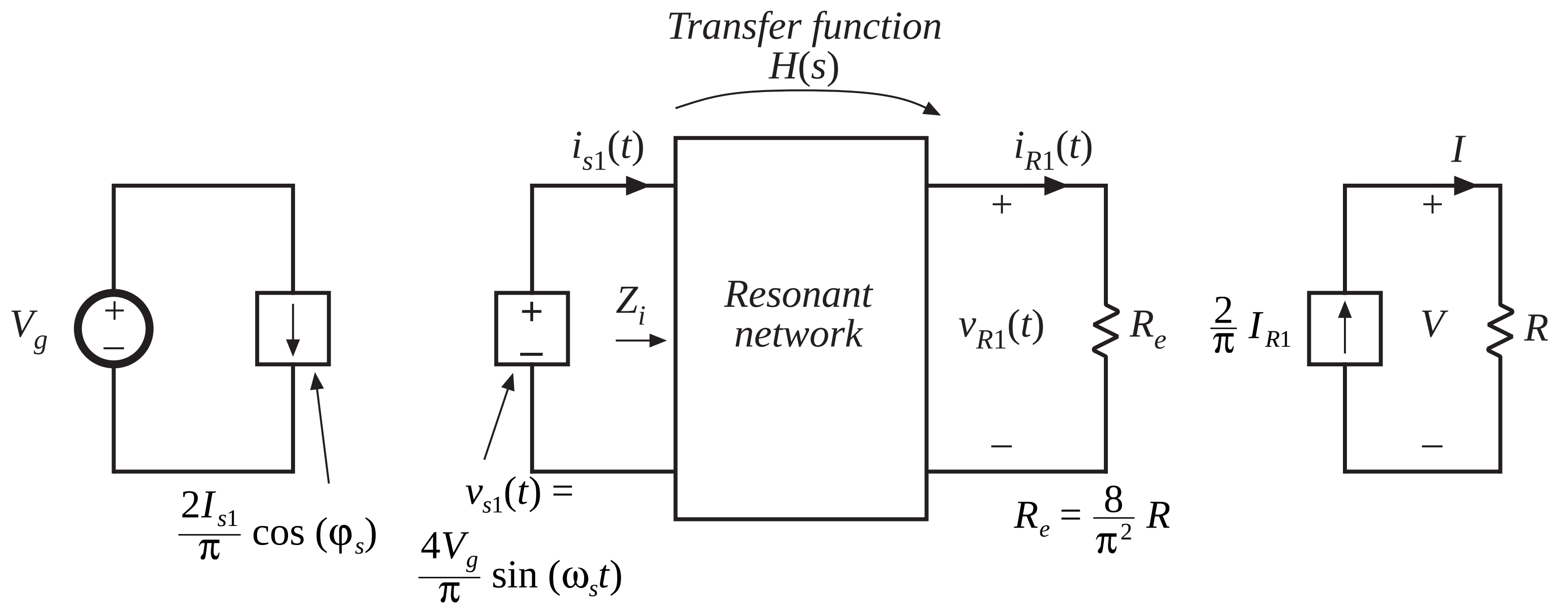
$$i_R(s) = \frac{v_{R1}(s)}{R_e} = \frac{H(s)}{R_e}\, v_{s1}(s)$$

which has peak magnitude

$$I_{R1} = \frac{\left\| H(s) \right\|_{s = j\omega_s}}{R_e}\, V_{s1}$$



19.1.4 Solution of converter voltage conversion ratio $M = V / V_g$



$$M = \frac{V}{V_g} = \underbrace{\left(R\right)}_{\left(\frac{V}{I}\right)} \underbrace{\left(\frac{2}{\pi}\right)}_{\left(\frac{I}{I_{R1}}\right)} \underbrace{\left(\frac{1}{R_e}\right)}_{\left(\frac{I_{R1}}{V_{R1}}\right)} \underbrace{\left(\left\| H(s) \right\|_{s = j\omega_s}\right)}_{\left(\frac{V_{R1}}{V_{s1}}\right)} \underbrace{\left(\frac{4}{\pi}\right)}_{\left(\frac{V_{s1}}{V_g}\right)}$$

Eliminate R_e:

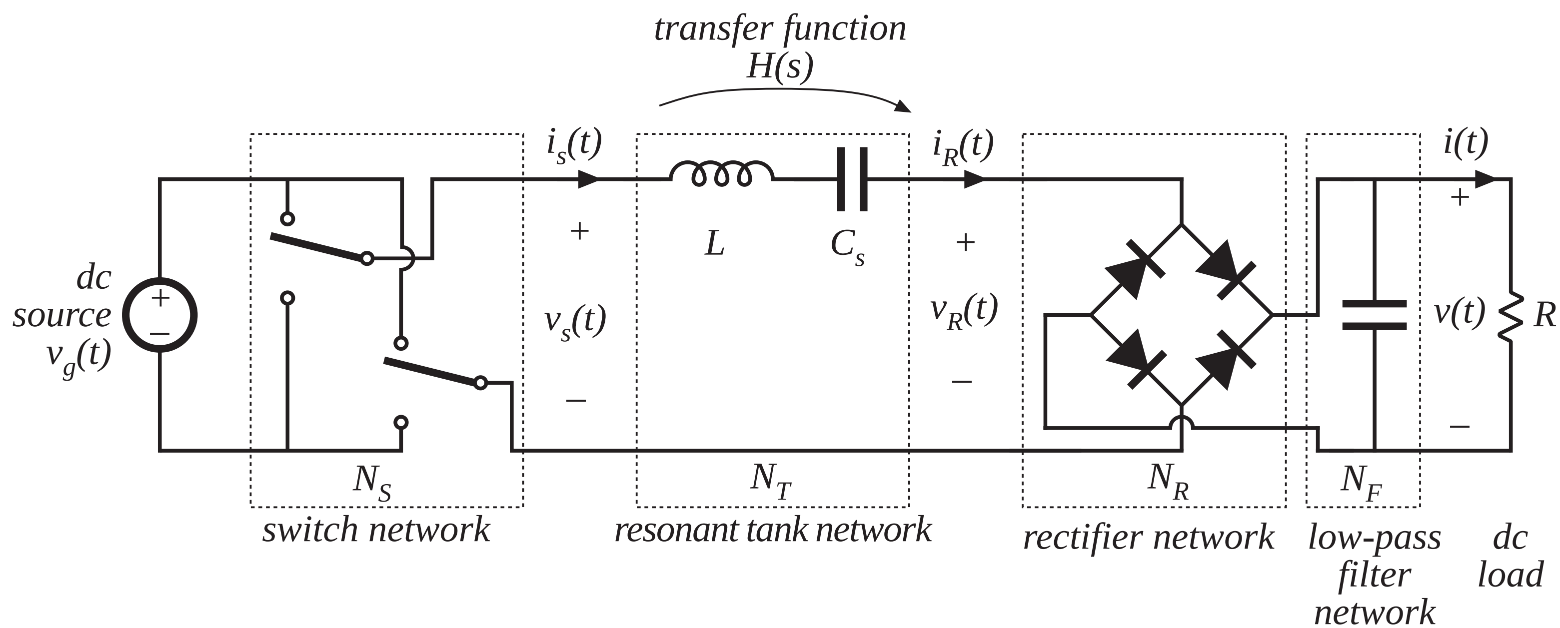
$$\frac{V}{V_g} = \left\| H(s) \right\|_{s = j\omega_s}$$

Conversion ratio M

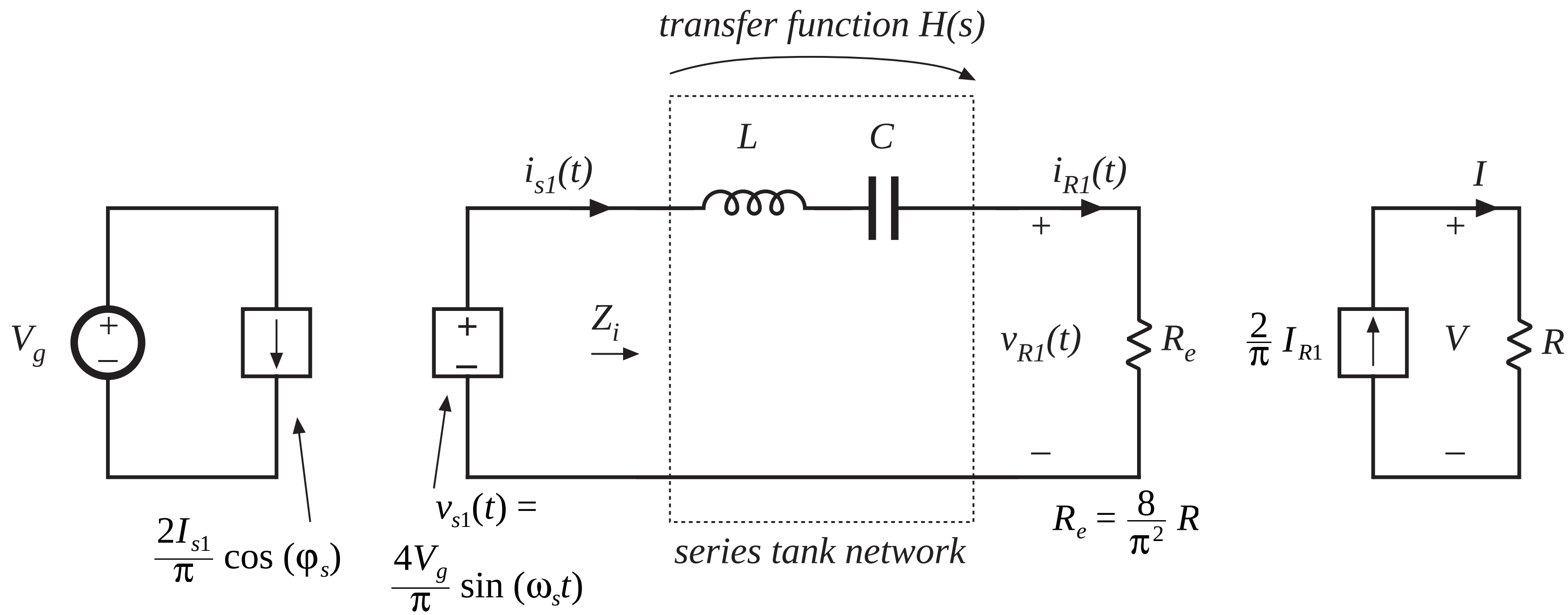
$$\frac{V}{V_g} = \left\| H(s) \right\|_{s = j\omega_s}$$

So we have shown that the conversion ratio of a resonant converter, having switch and rectifier networks as in previous slides, is equal to the magnitude of the tank network transfer function. This transfer function is evaluated with the tank loaded by the effective rectifier input resistance R_e.

19.2 Examples
19.2.1 Series resonant converter



Model: series resonant converter



$$H(s) = \frac{R_e}{Z_i(s)} = \frac{R_e}{R_e + sL + \frac{1}{sC}}$$

$$= \frac{\left(\frac{s}{Q_e\omega_0}\right)}{1 + \left(\frac{s}{Q_e\omega_0}\right) + \left(\frac{s}{\omega_0}\right)^2}$$

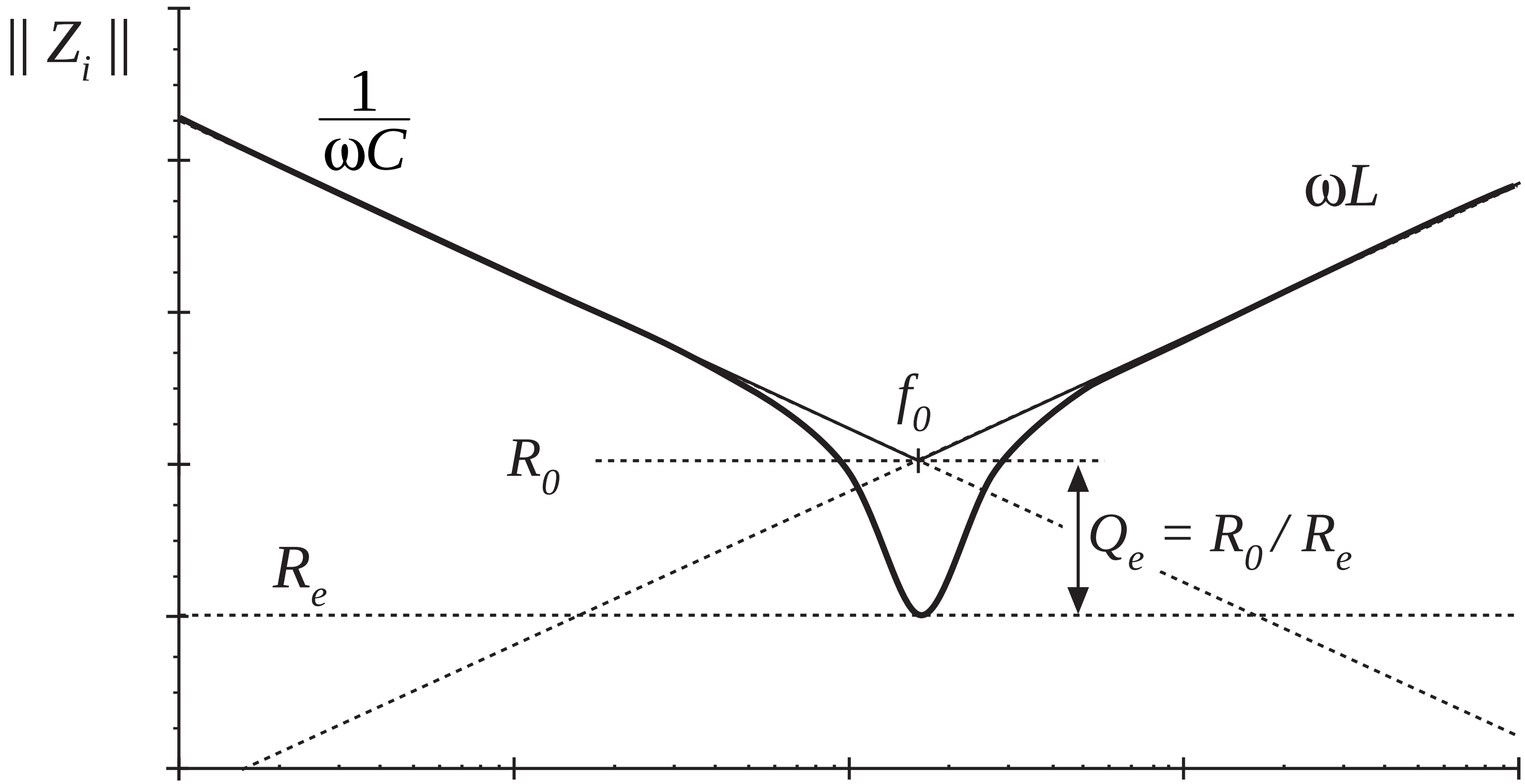
$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0$$

$$R_0 = \sqrt{\frac{L}{C}}$$

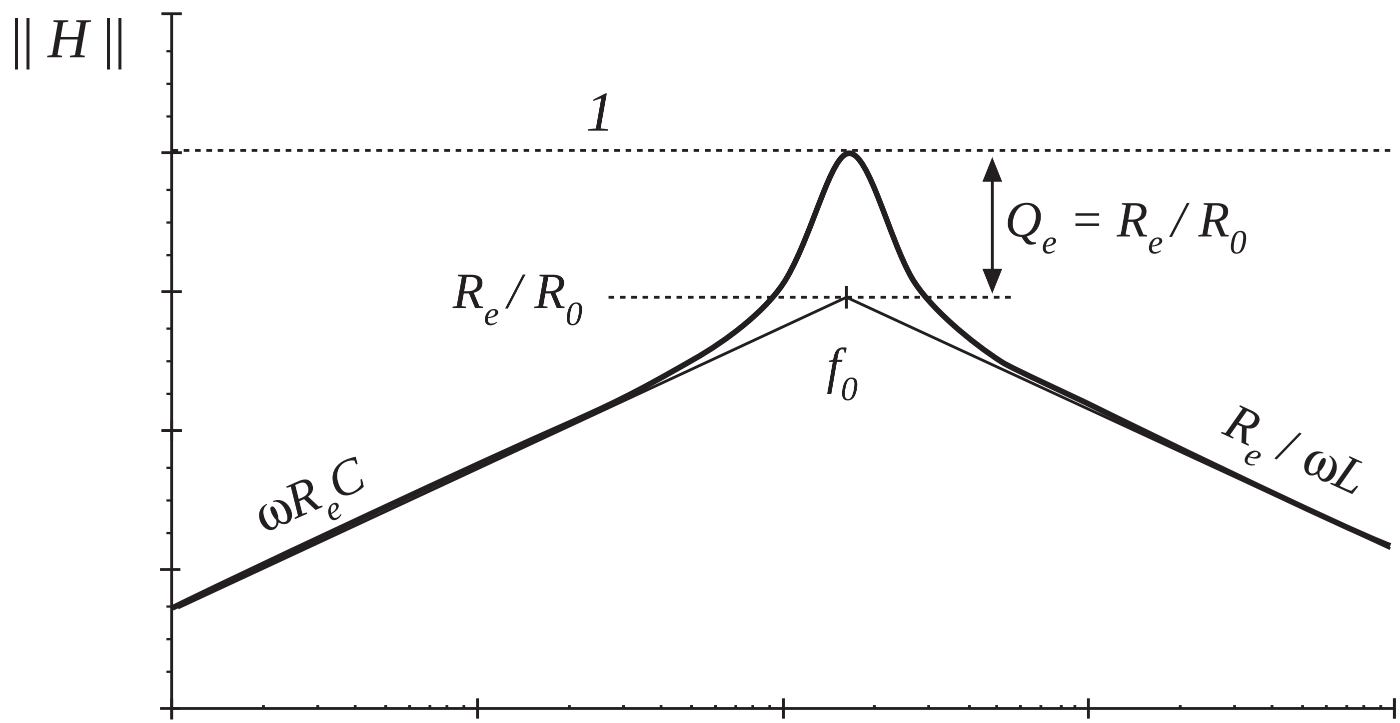
$$Q_e = \frac{R_0}{R_e}$$

$$M = \| H(j\omega_s) \| = \frac{1}{\sqrt{1 + Q_e^2\left(\frac{1}{F} - F\right)^2}}$$

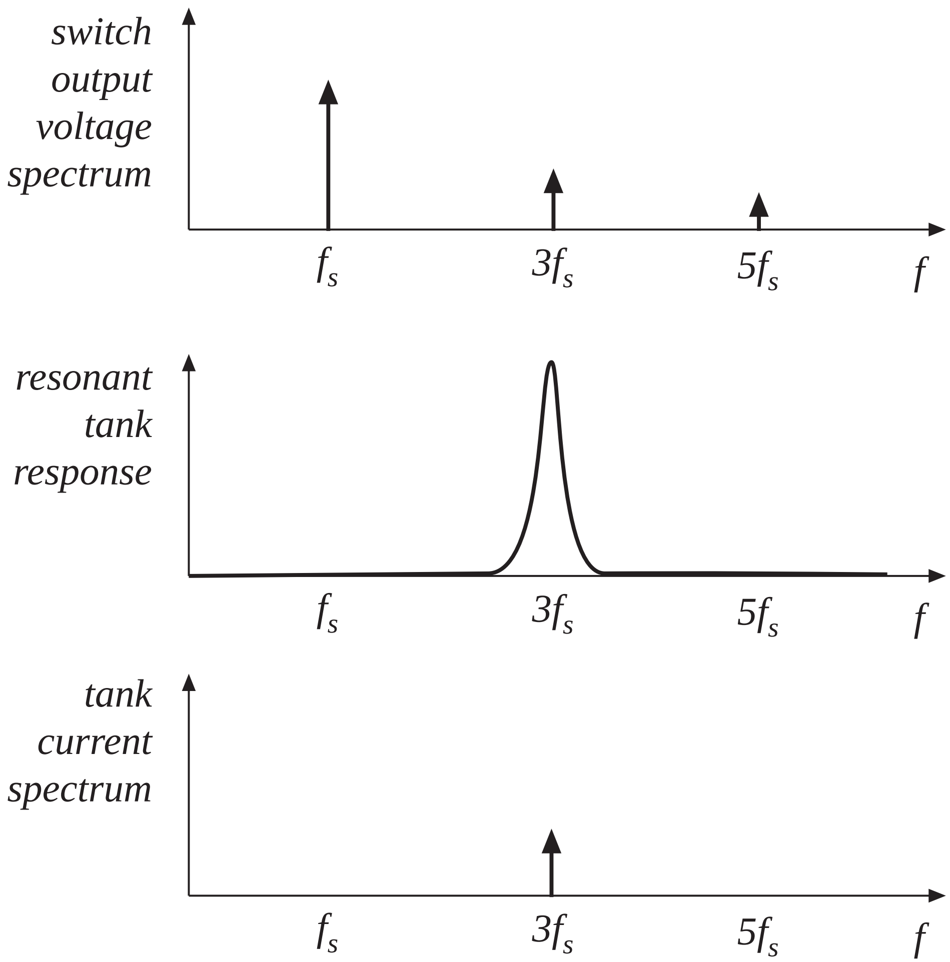
Construction of Z_i



Construction of *H*



19.2.2 Subharmonic modes of the SRC



Example: excitation of tank by third harmonic of switching frequency

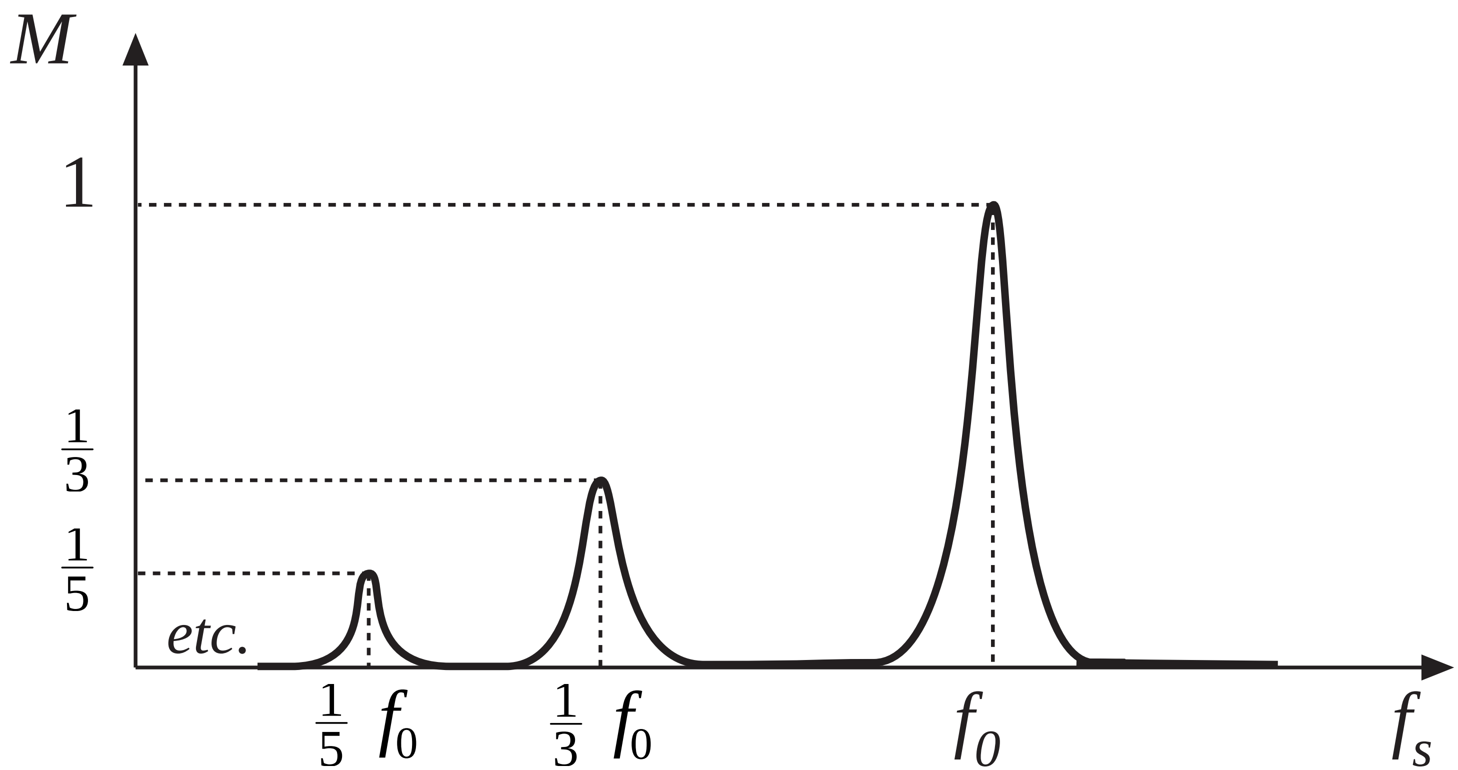
Can now approximate $v_s(t)$ by its third harmonic:

$$v_s(t) \approx v_{sn}(t) = \frac{4V_g}{n\pi} \sin(n\omega_s t)$$

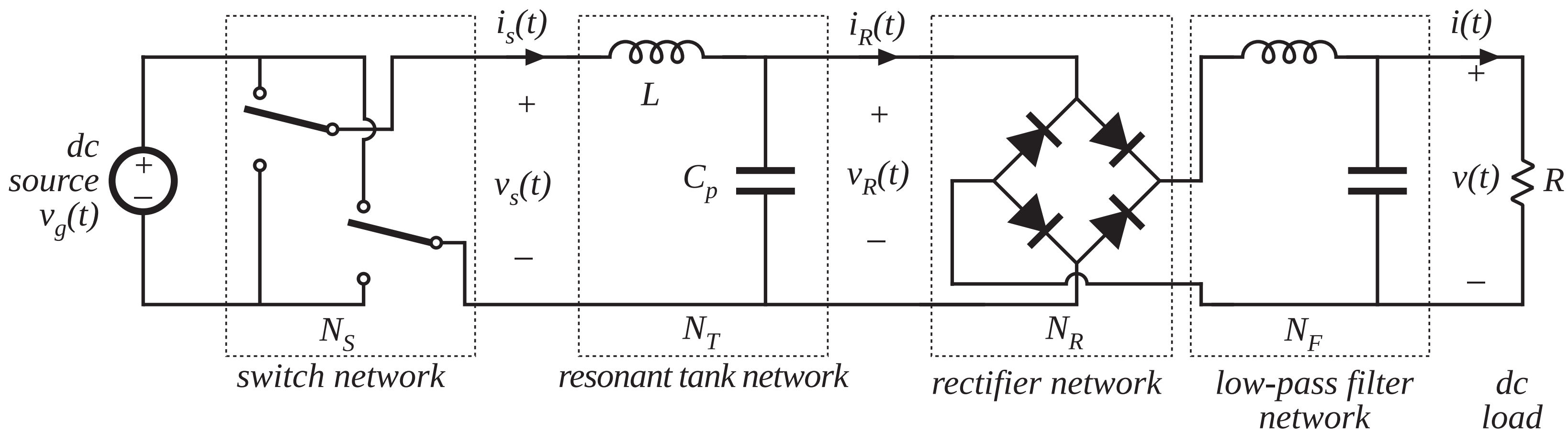
Result of analysis:

$$M = \frac{V}{V_g} = \frac{\| H(jn\omega_s) \|}{n}$$

Subharmonic modes of SRC



19.2.3 Parallel resonant dc-dc converter



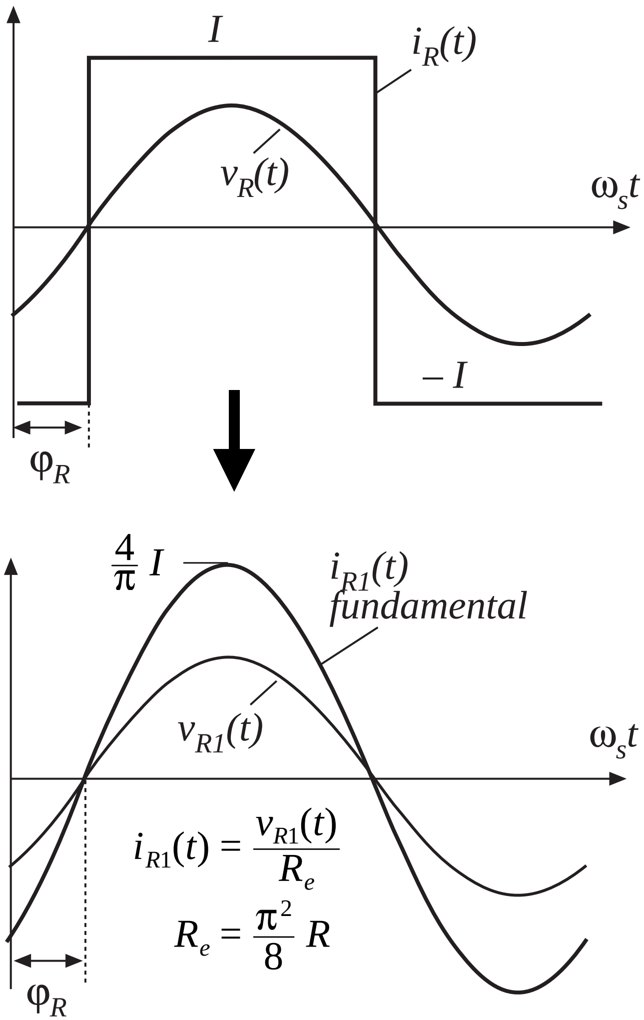
Differs from series resonant converter as follows:

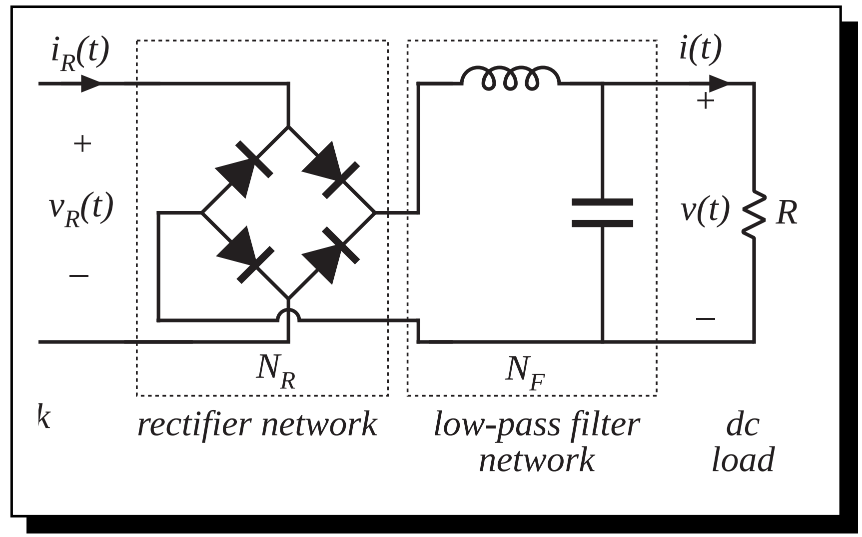
Different tank network

Rectifier is driven by sinusoidal voltage, and is connected to inductive-input low-pass filter

Need a new model for rectifier and filter networks

Model of uncontrolled rectifier with inductive filter network





Fundamental component of $i_R(t)$:

$$i_{R1}(t) = \frac{4I}{\pi}\sin\left(\omega_s t - \varphi_R\right)$$

Effective resistance R_e

Again define

$$R_e = \frac{v_{R1}(t)}{i_{R1}(t)} = \frac{\pi V_{R1}}{4I}$$

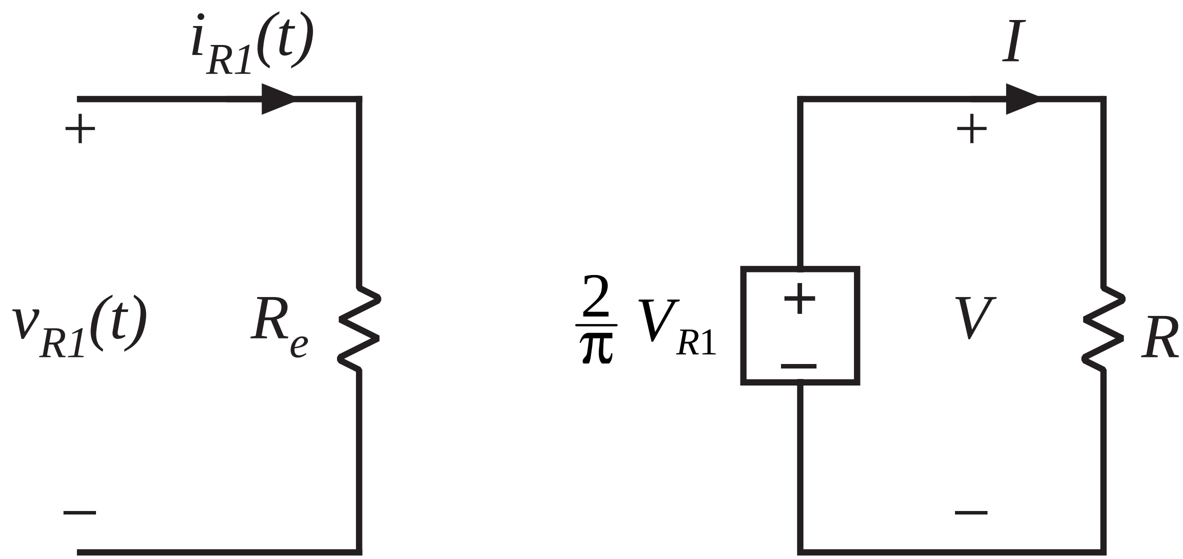
In steady state, the dc output voltage V is equal to the average value of $| v_R |$:

$$V = \frac{2}{T_S} \int_0^{T_S/2} V_{R1} \left| \sin\left(\omega_s t - \varphi_R\right) \right| dt = \frac{2}{\pi} V_{R1}$$

For a resistive load, $V = IR$. The effective resistance R_e can then be expressed

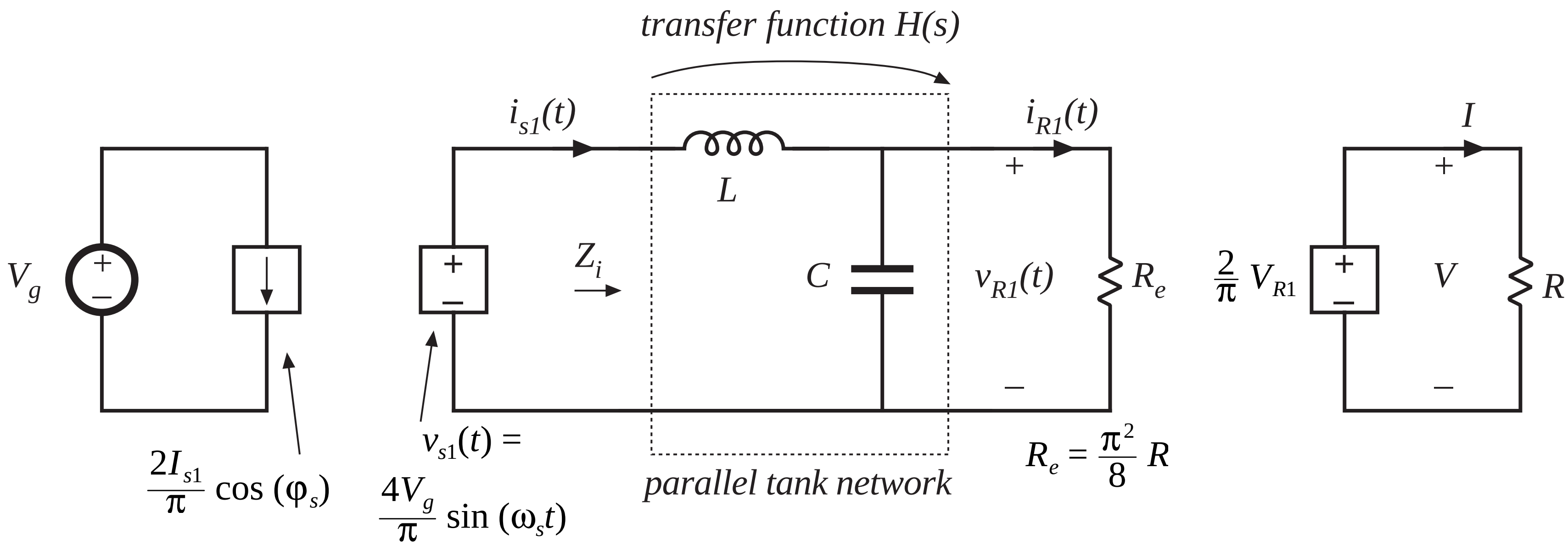
$$R_e = \frac{\pi^2}{8} R = 1.2337R$$

Equivalent circuit model of uncontrolled rectifier with inductive filter network



$$R_e = \frac{\pi^2}{8} R$$

Equivalent circuit model
Parallel resonant dc-dc converter

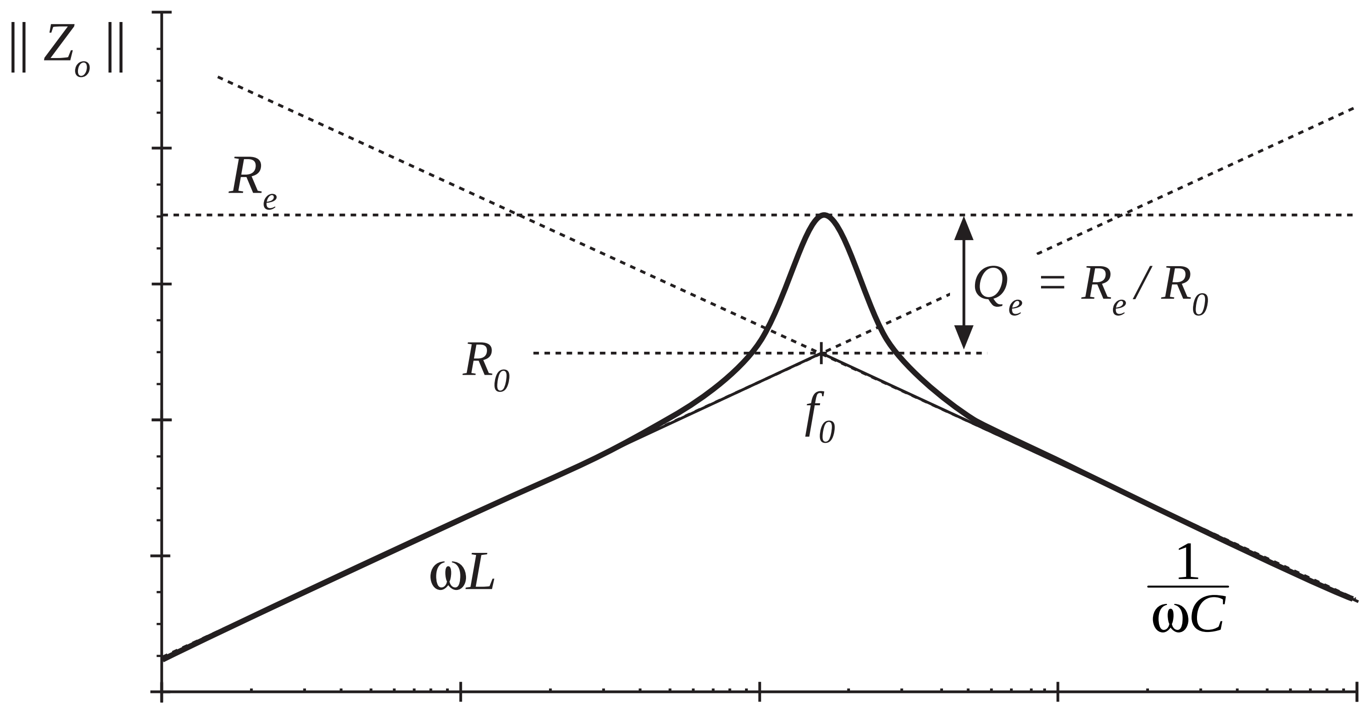


$$M = \frac{V}{V_g} = \frac{8}{\pi^2} \left\| H(s) \right\|_{s = j\omega_s}$$

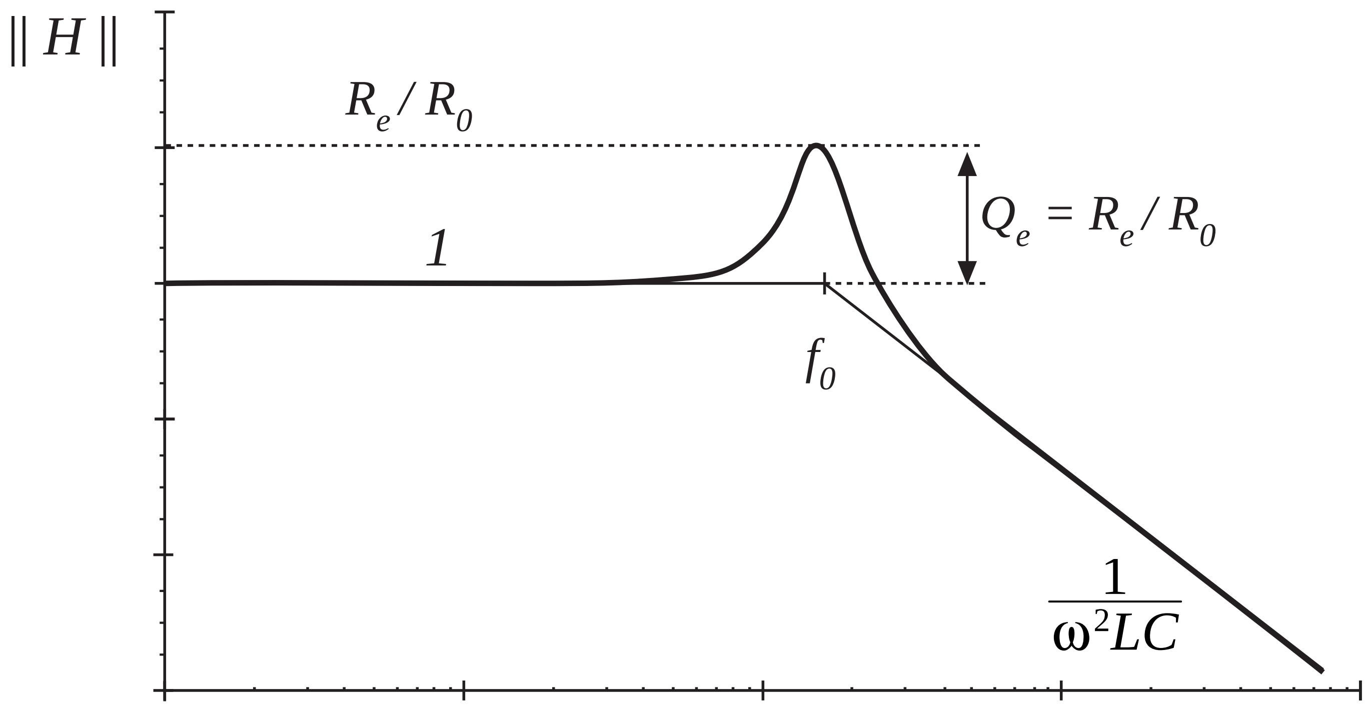
$$H(s) = \frac{Z_o(s)}{sL}$$

$$Z_o(s) = sL \parallel \frac{1}{sC} \parallel R_e$$

Construction of Z_o



Construction of H



Dc conversion ratio of the PRC

$$M = \frac{8}{\pi^2} \left\| \frac{Z_o(s)}{sL} \right\|_{s = j\omega_s} = \frac{8}{\pi^2} \left\| \frac{1}{1 + \frac{s}{Q_e \omega_0} + \left(\frac{s}{\omega_0} \right)^2} \right\|_{s = j\omega_s}$$

$$= \frac{8}{\pi^2} \frac{1}{\sqrt{\left(1 - F^2\right)^2 + \left(\frac{F}{Q_e}\right)^2}}$$

At resonance, this becomes

$$M = \frac{8}{\pi^2} \frac{R_e}{R_0} = \frac{R}{R_0}$$

- PRC can step up the voltage, provided $R > R_0$
- PRC can produce M approaching infinity, provided output current is limited to value less than V_g / R_0

19.3 Exact characteristics of the series and parallel resonant dc-dc converters

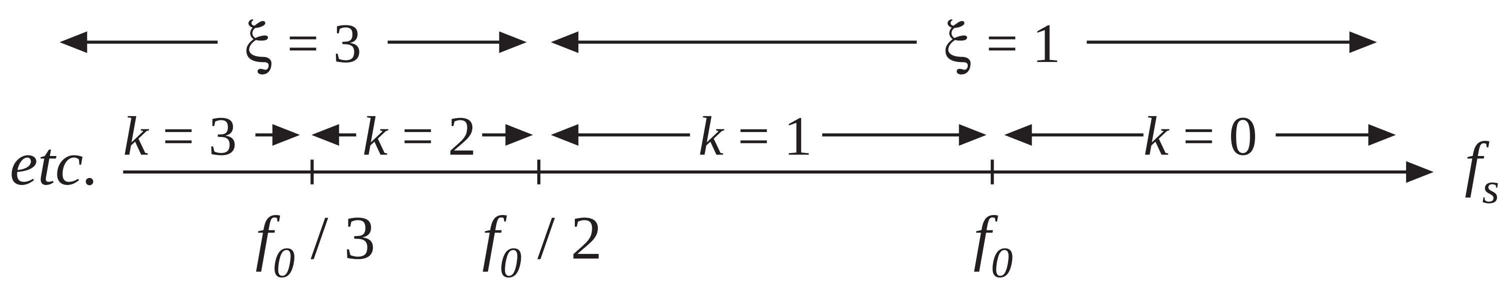
Define

$$\frac{f_0}{k+1} < f_s < \frac{f_0}{k} \qquad \text{or} \qquad \frac{1}{k+1} < F < \frac{1}{k}$$

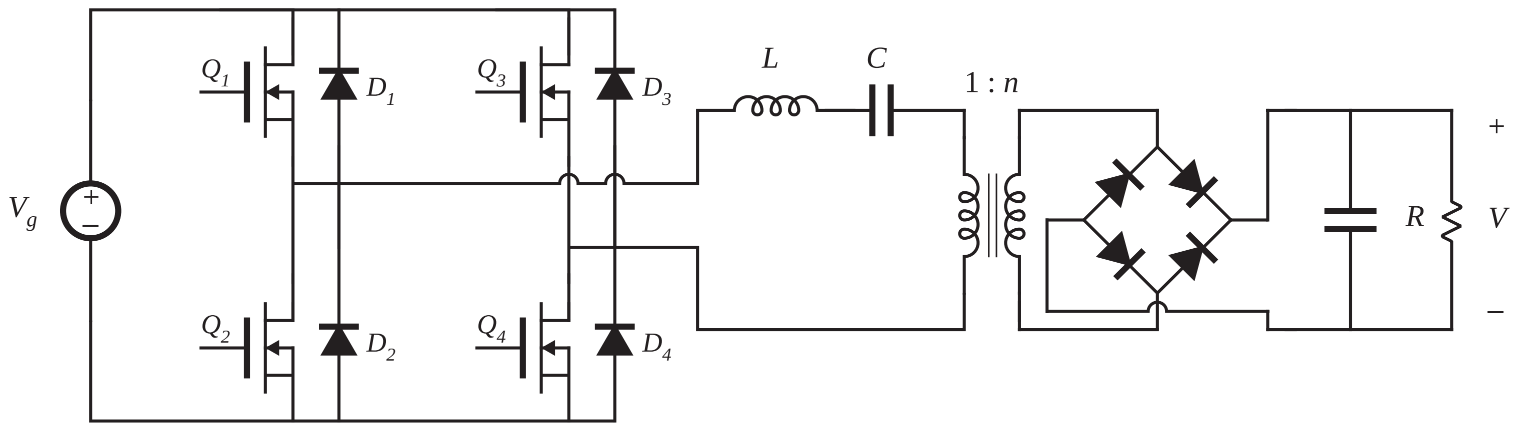
mode index k

$$\xi = k + \frac{1 + (-1)^k}{2}$$

subharmonic index ξ



19.3.1 Exact characteristics of the series resonant converter



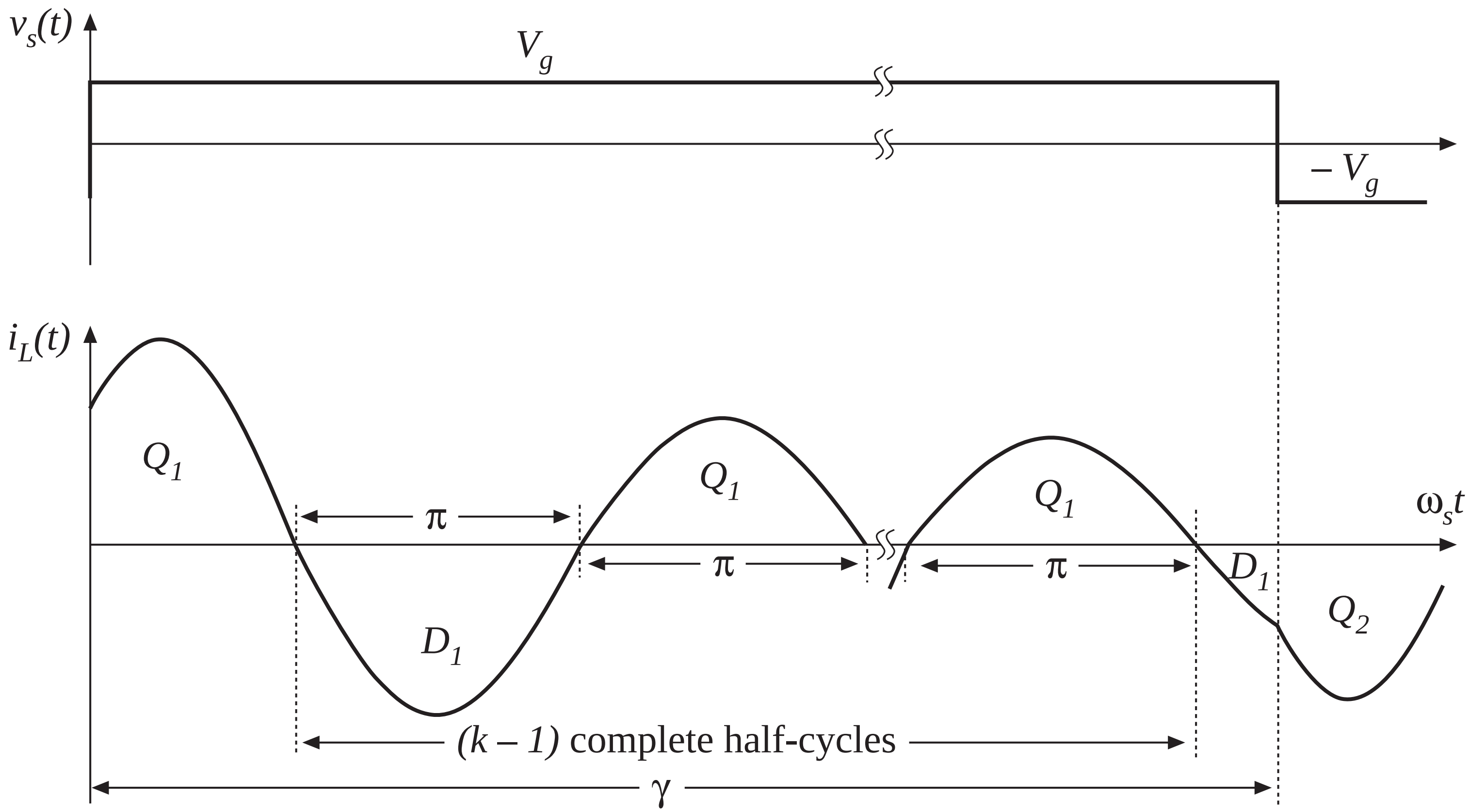
Normalized load voltage and current:

$$M = \frac{V}{nV_g} \qquad J = \frac{InR_0}{V_g}$$

Continuous conduction mode, SRC

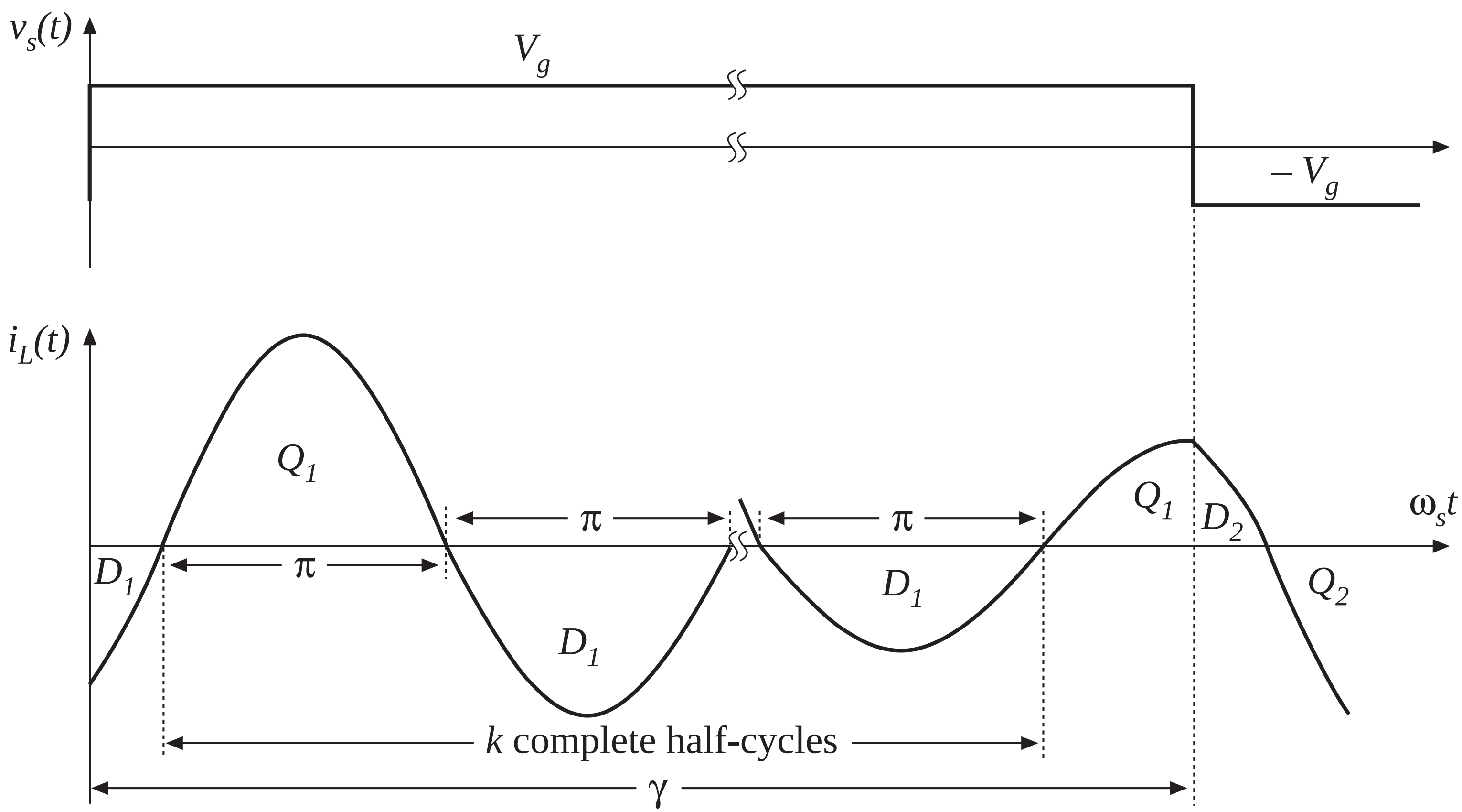
Tank current rings continuously for entire length of switching period

Waveforms for type k CCM, odd k :



Series resonant converter

Waveforms for type k CCM, even k :



Exact steady-state solution, CCM

Series resonant converter

$$M^2\xi^2 \sin^2\left(\frac{\gamma}{2}\right) + \frac{1}{\xi^2}\left(\frac{J\gamma}{2} + (-1)^k\right)^2 \cos^2\left(\frac{\gamma}{2}\right) = 1$$

where

$$M = \frac{V}{nV_g} \qquad J = \frac{InR_0}{V_g}$$

$$\gamma = \frac{\omega_0 T_s}{2} = \frac{\pi}{F}$$

- Output characteristic, i.e., the relation between M and J, is elliptical
- M is restricted to the range

$$0 \le M \le \frac{1}{\xi}$$

Control-plane characteristics

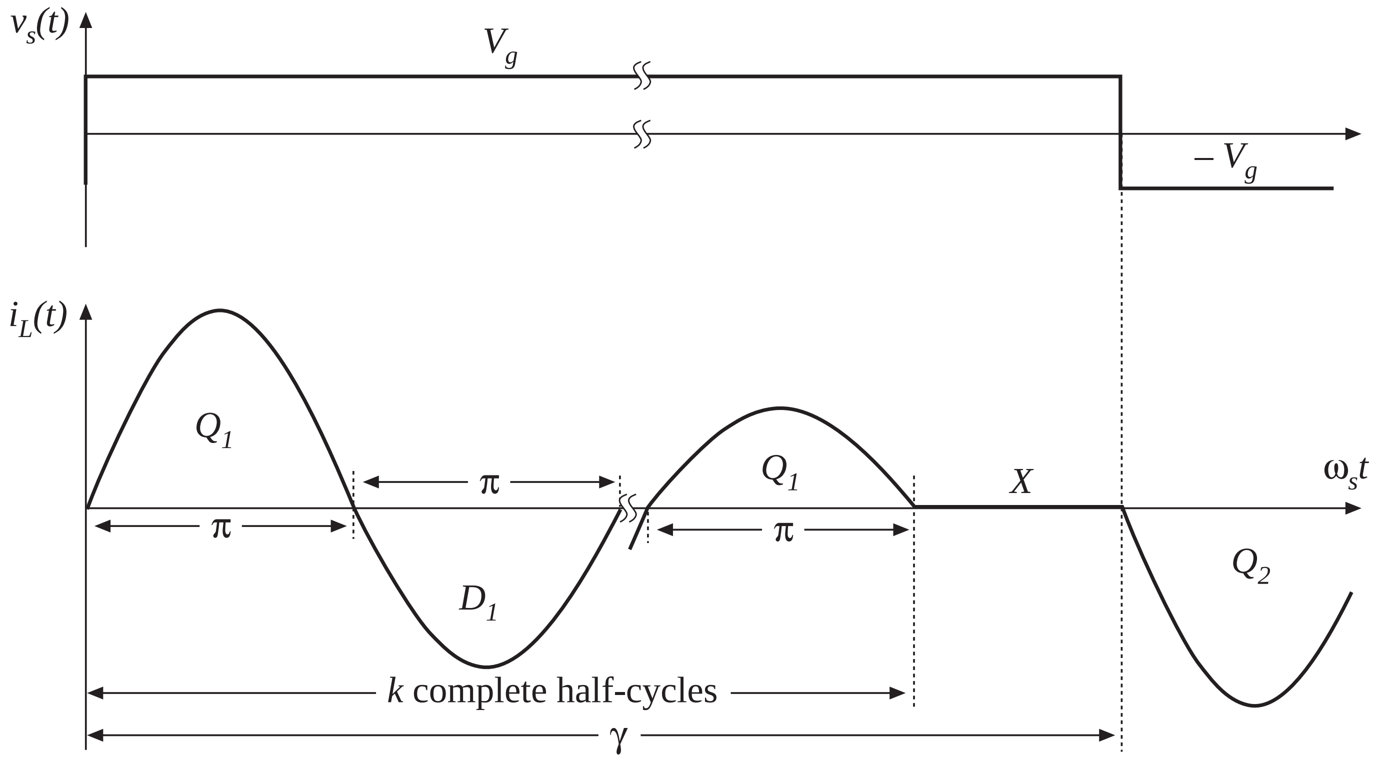
For a resistive load, eliminate J and solve for M vs. γ

$$M = \frac{\left(\frac{Q\gamma}{2}\right)}{\xi^4 \tan^2\left(\frac{\gamma}{2}\right) + \left(\frac{Q\gamma}{2}\right)^2} \left[(-1)^{k+1} + \sqrt{1 + \frac{\left[\xi^2 - \cos^2\left(\frac{\gamma}{2}\right)\right]\left[\xi^4 \tan^2\left(\frac{\gamma}{2}\right) + \left(\frac{Q\gamma}{2}\right)^2\right]}{\left(\frac{Q\gamma}{2}\right)^2 \cos^2\left(\frac{\gamma}{2}\right)}} \right]$$

Exact, closed-form, valid for any CCM

Discontinuous conduction mode

Type k DCM: during each half-switching-period, the tank rings for k complete half-cycles. The output diodes then become reverse-biased for the remainder of the half-switching-period.



Steady-state solution: type k DCM, odd k

$$M = \frac{1}{k}$$

Conditions for operation in type k DCM, odd k :

$$f_s < \frac{f_0}{k}$$

$$\frac{2(k+1)}{\gamma} > J > \frac{2(k-1)}{\gamma}$$

Steady-state solution: type k DCM, even k

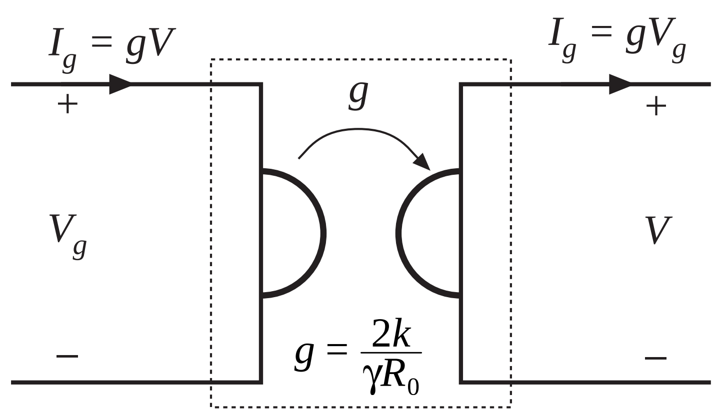
$$J = \frac{2k}{\gamma}$$

Conditions for operation in type k DCM, even k :

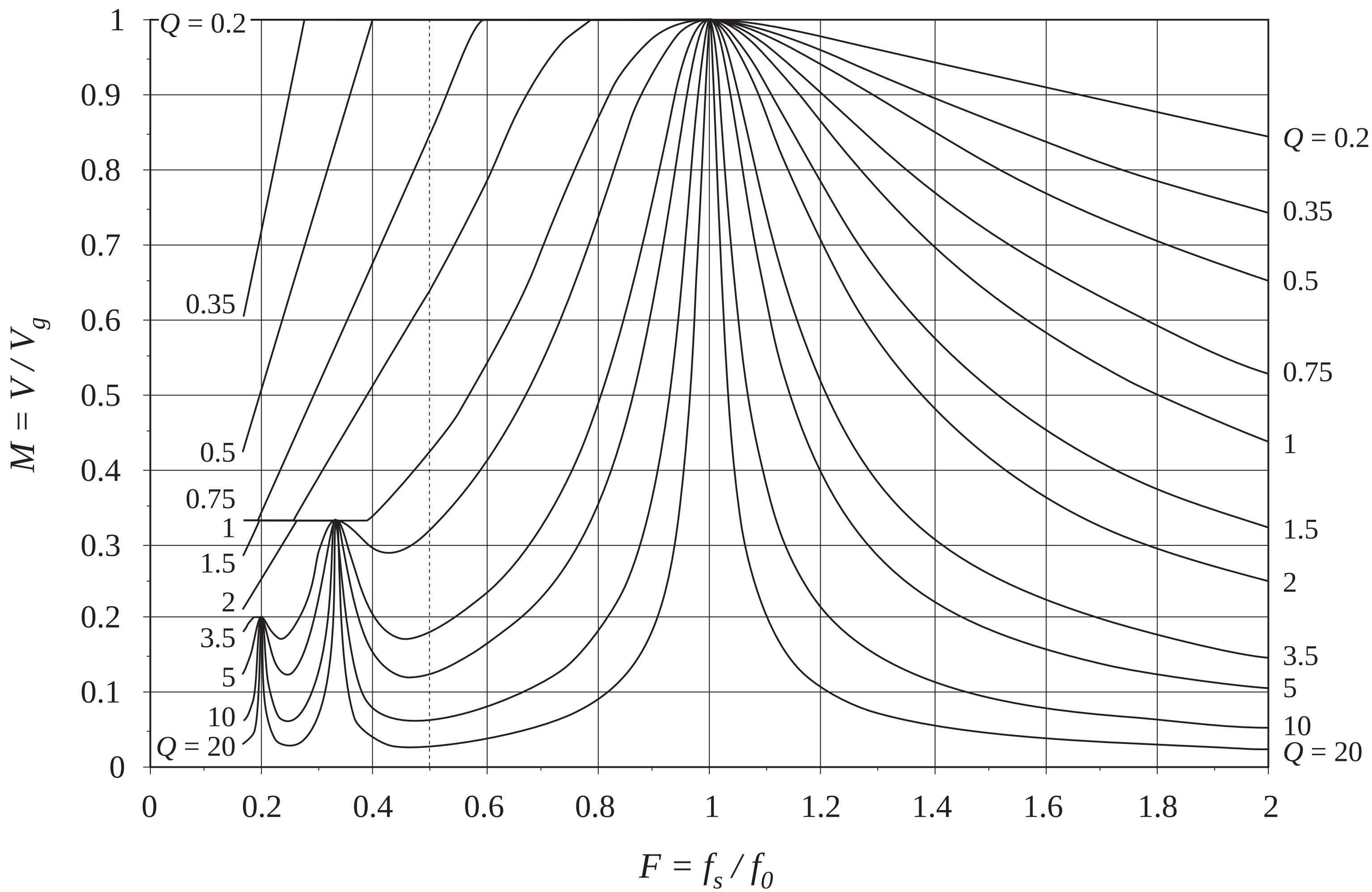
$$f_s < \frac{f_0}{k}$$

$$\frac{1}{k-1} > M > \frac{1}{k+1}$$

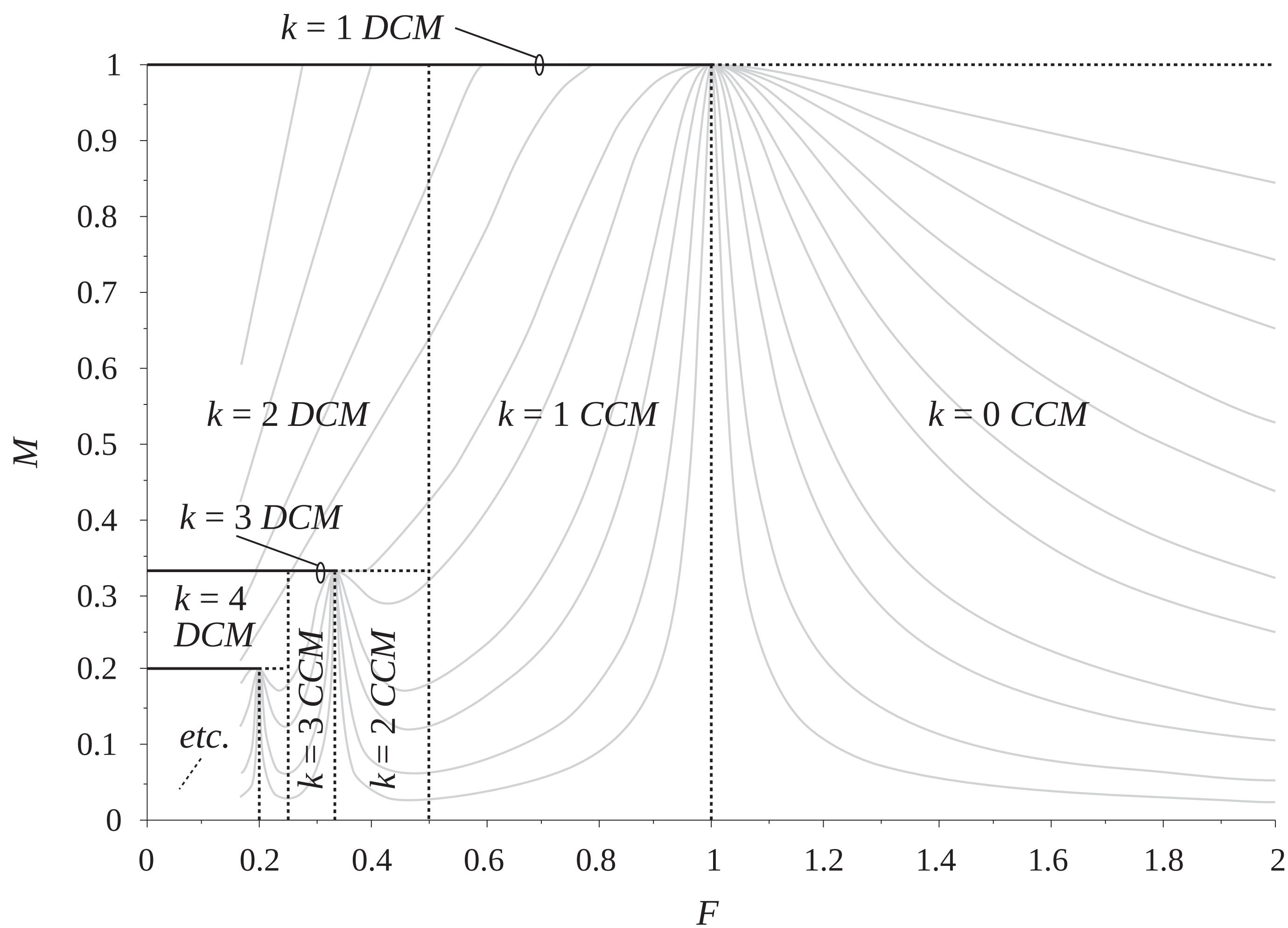
gyrator model, SRC operating in an even DCM:



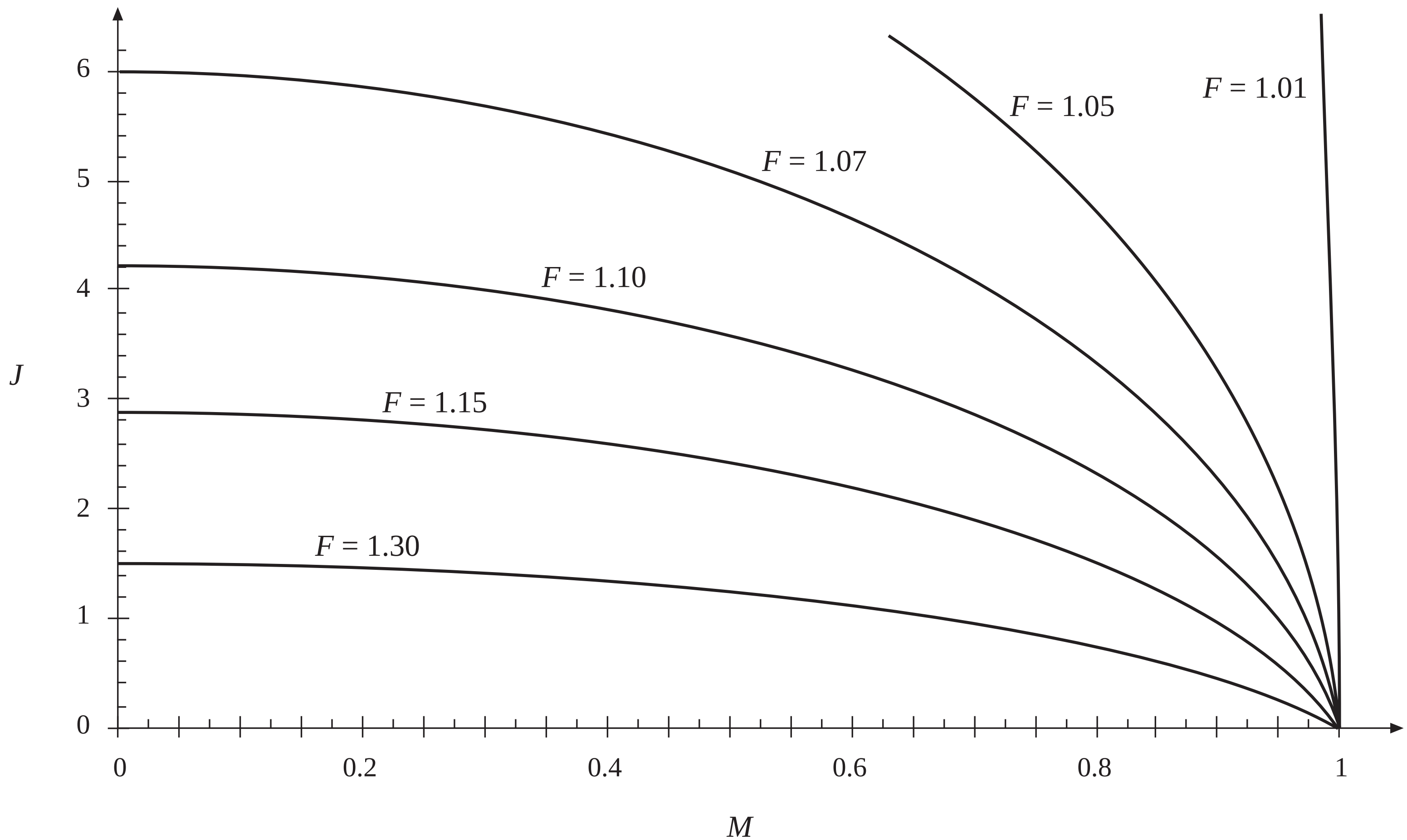
Control plane characteristics, SRC



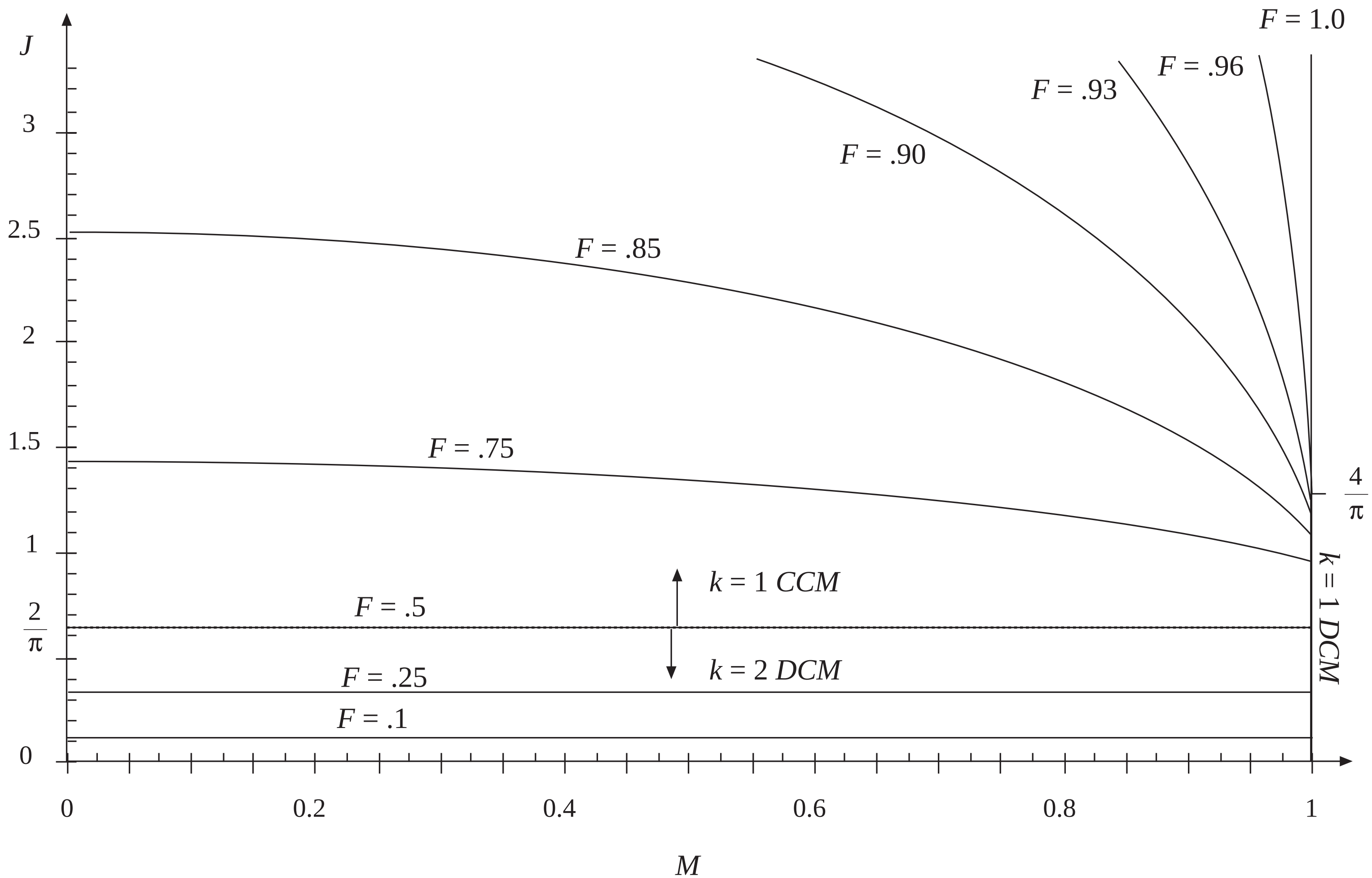
Mode boundaries, SRC



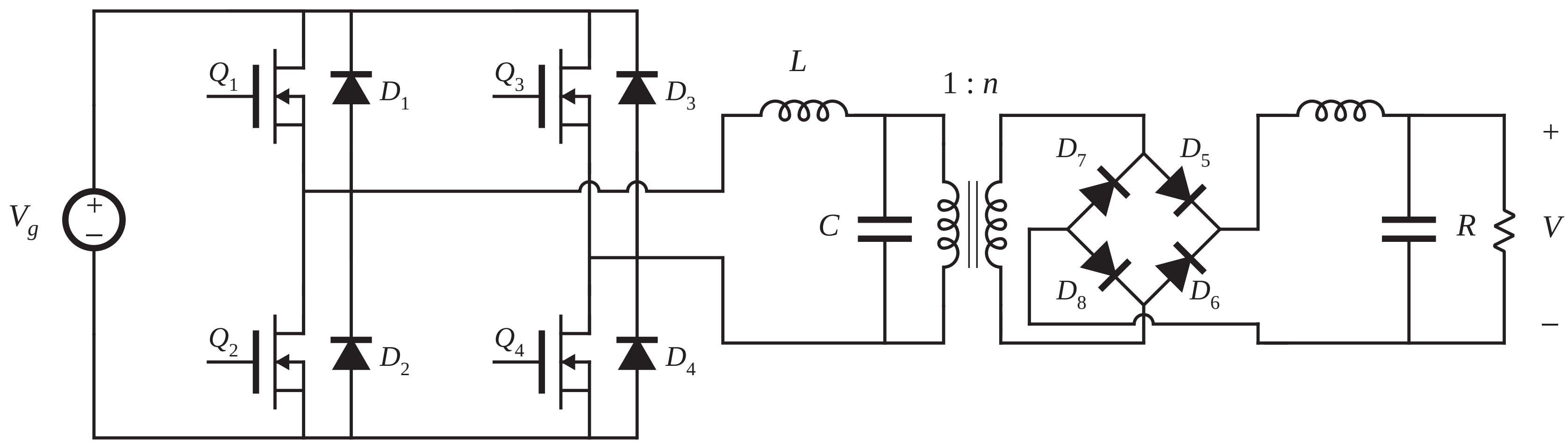
Output characteristics, SRC above resonance



Output characteristics, SRC below resonance



19.3.2 Exact characteristics of the parallel resonant converter



Normalized load voltage and current:

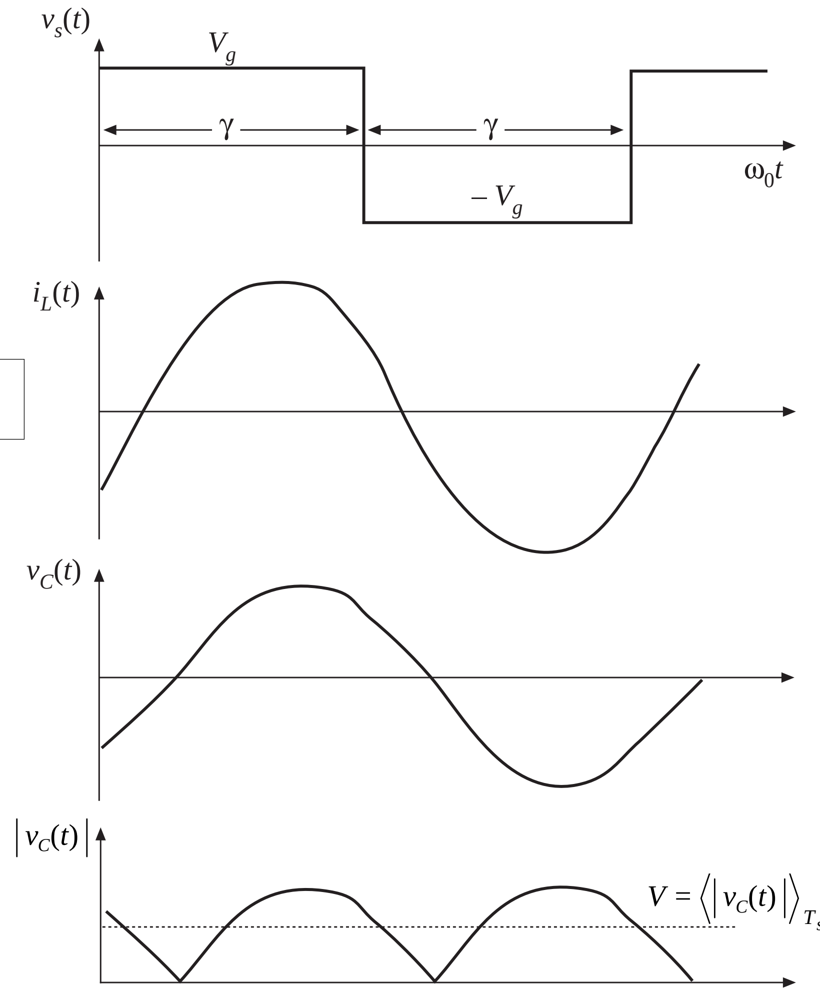
$$M = \frac{V}{nV_g} \qquad J = \frac{InR_0}{V_g}$$

Parallel resonant converter in CCM

CCM closed-form solution

$$M = \left(\frac{2}{\gamma}\right)\left(\varphi - \frac{\sin(\varphi)}{\cos\left(\frac{\gamma}{2}\right)}\right)$$

$$\varphi = \begin{cases} -\cos^{-1}\left(\cos\left(\frac{\gamma}{2}\right) + J\sin\left(\frac{\gamma}{2}\right)\right) & \text{for } 0 < \gamma < \pi \\ +\cos^{-1}\left(\cos\left(\frac{\gamma}{2}\right) + J\sin\left(\frac{\gamma}{2}\right)\right) & \text{for } \pi < \gamma < 2\pi \end{cases}$$



Parallel resonant converter in DCM

Mode boundary

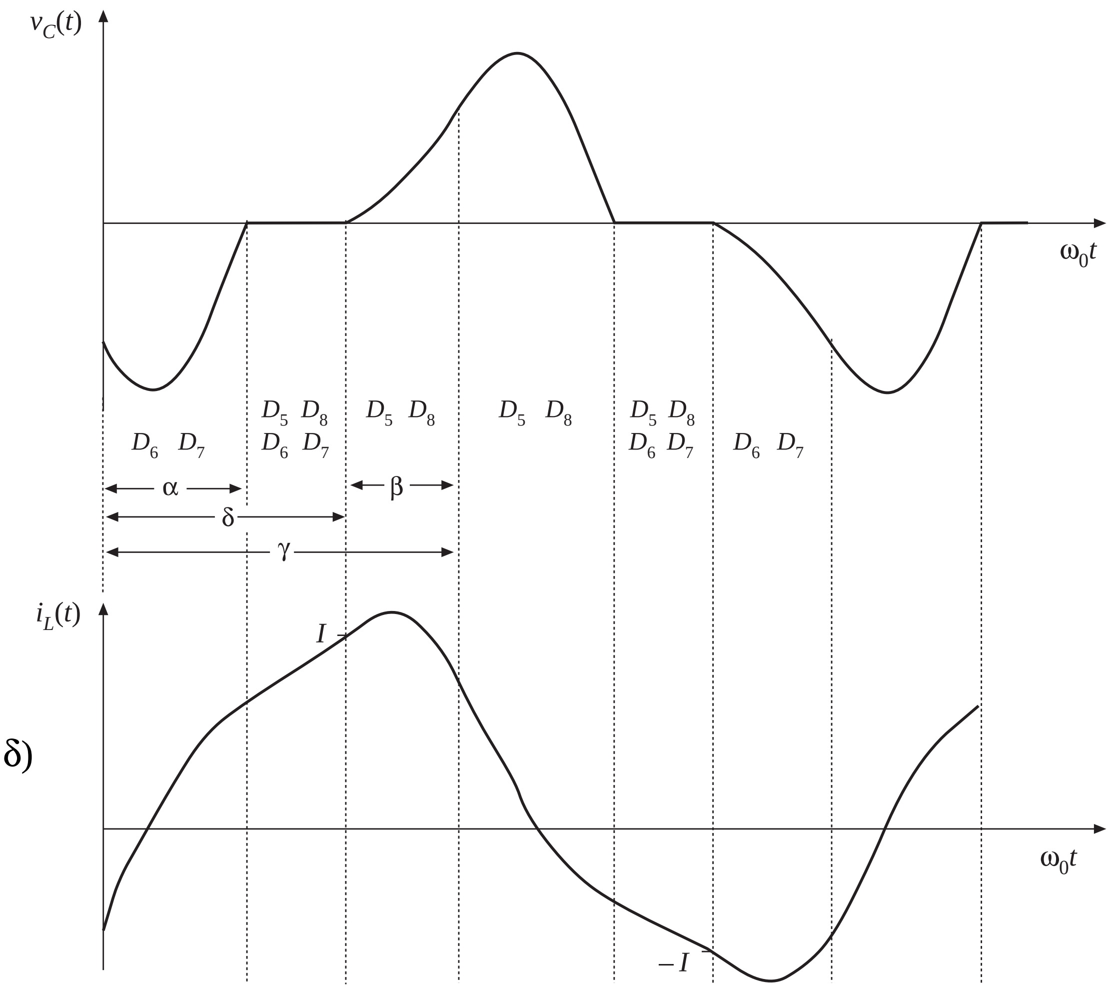
$$J > J_{crit}(\gamma) \qquad \text{for DCM}$$
$$J < J_{crit}(\gamma) \qquad \text{for CCM}$$

$$J_{crit}(\gamma) = -\frac{1}{2}\sin(\gamma) + \sqrt{\sin^2\left(\frac{\gamma}{2}\right) + \frac{1}{4}\sin^2(\gamma)}$$

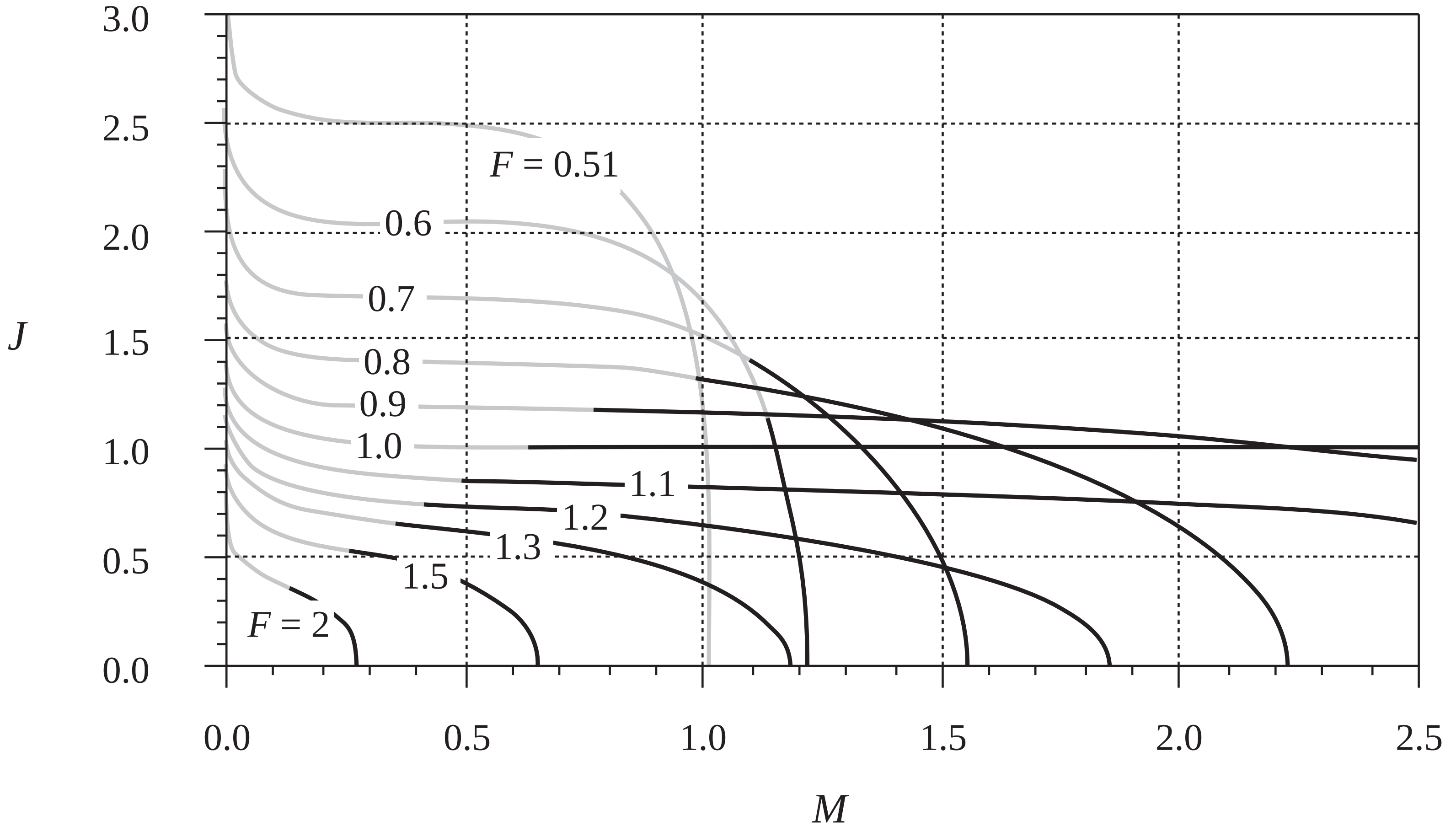
DCM equations

$$M_{C0} = 1 - \cos(\beta)$$
$$J_{L0} = J + \sin(\beta)$$
$$\cos(\alpha + \beta) - 2\cos(\alpha) = -1$$
$$-\sin(\alpha + \beta) + 2\sin(\alpha) + (\delta - \alpha) = 2J$$
$$\beta + \delta = \gamma$$
$$M = 1 + \left(\frac{2}{\gamma}\right)(J - \delta)$$

(require iteration)



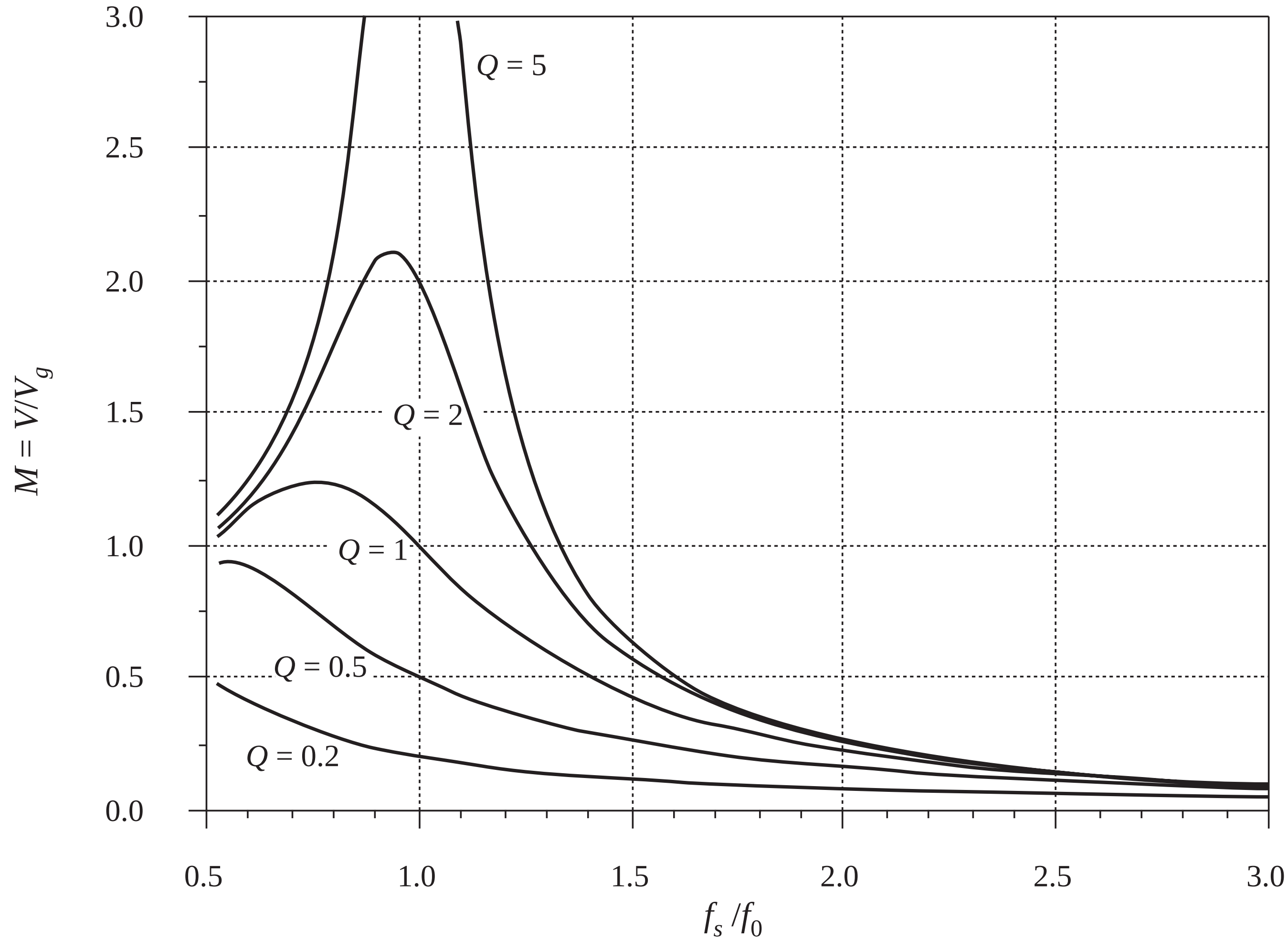
Output characteristics of the PRC



Solid curves: CCM   Shaded curves: DCM

Control characteristics of the PRC

with resistive load



19.4 Soft switching

Soft switching can mitigate some of the mechanisms of switching loss and possibly reduce the generation of EMI

Semiconductor devices are switched on or off at the zero crossing of their voltage or current waveforms:

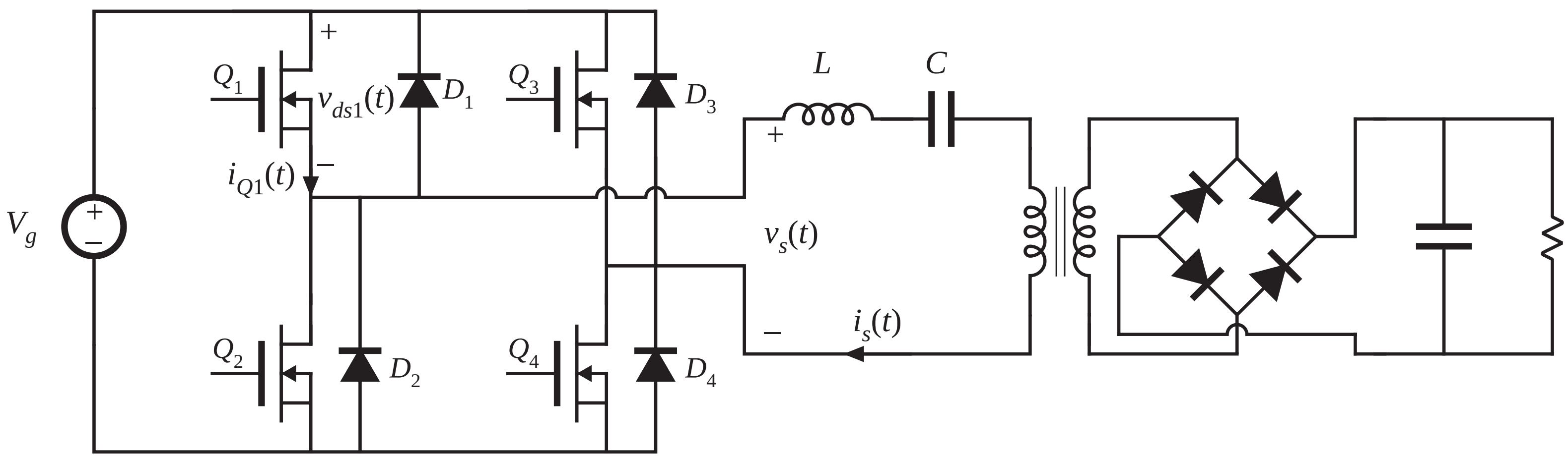
> *Zero-current switching*: transistor turn-off transition occurs at zero current. Zero-current switching eliminates the switching loss caused by IGBT current tailing and by stray inductances. It can also be used to commutate SCR's.
>
> *Zero-voltage switching*: transistor turn-on transition occurs at zero voltage. Diodes may also operate with zero-voltage switching. Zero-voltage switching eliminates the switching loss induced by diode stored charge and device output capacitances.

Zero-voltage switching is usually preferred in modern converters.

Zero-voltage transition converters are modified PWM converters, in which an inductor charges and discharges the device capacitances. Zero-voltage switching is then obtained.

19.4.1 Operation of the full bridge below resonance: Zero-current switching

Series resonant converter example



Operation below resonance: input tank current leads voltage

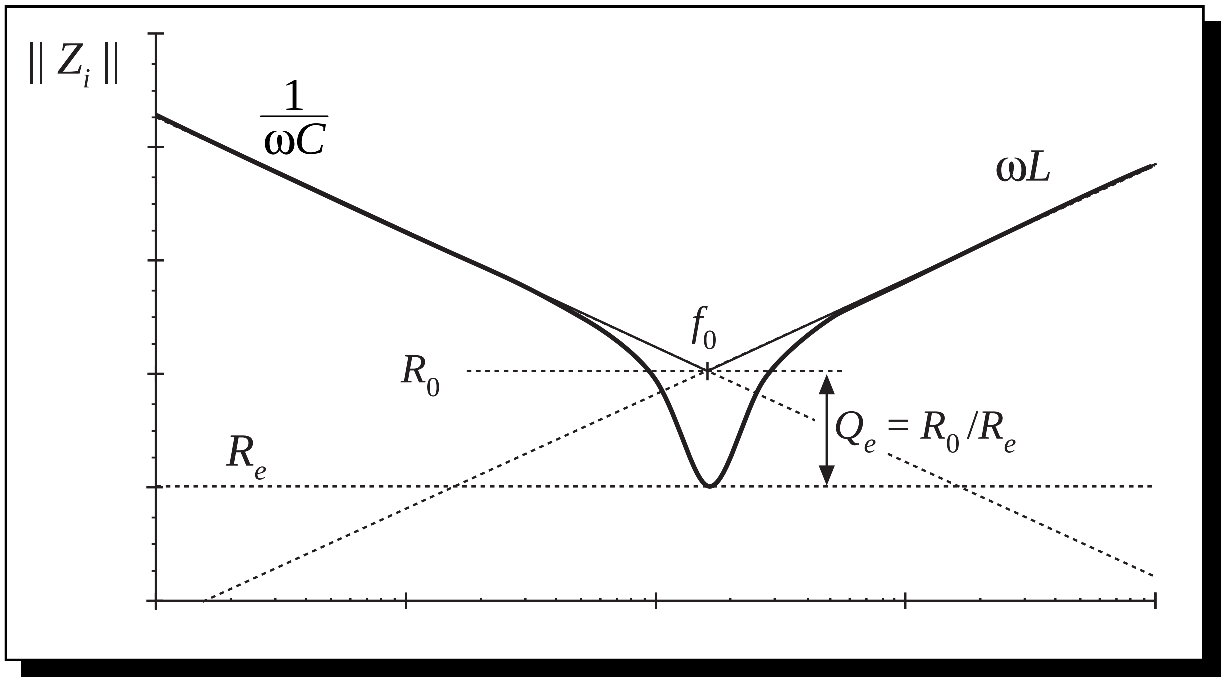
Zero-current switching (ZCS) occurs

Tank input impedance

Operation below resonance: tank input impedance Z_i is dominated by tank capacitor.

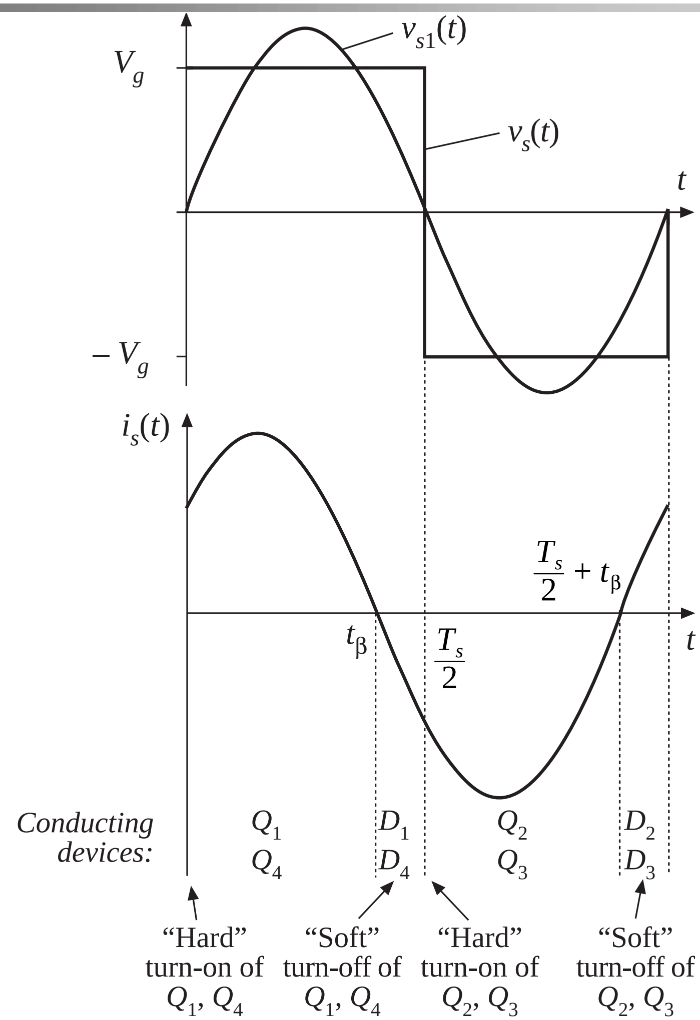
$\angle Z_i$ is positive, and tank input current leads tank input voltage.

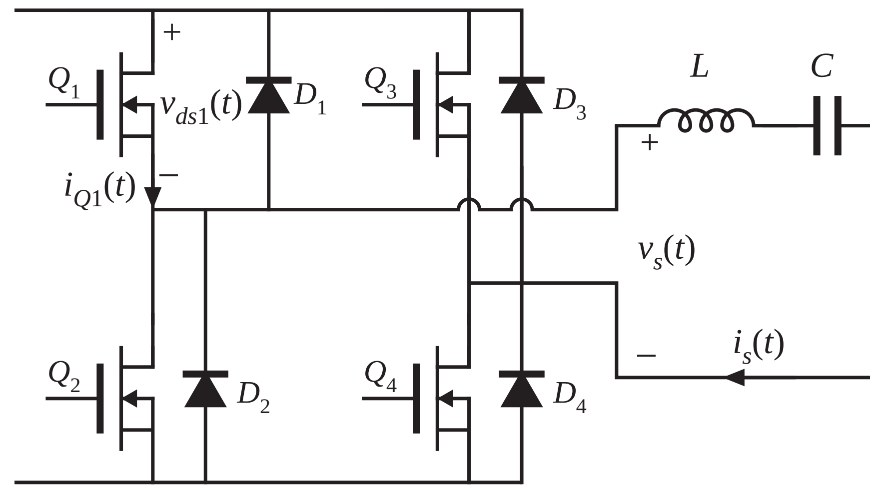
Zero crossing of the tank input current waveform $i_s(t)$ occurs before the zero crossing of the voltage $v_s(t)$.



Switch network waveforms, below resonance

Zero-current switching

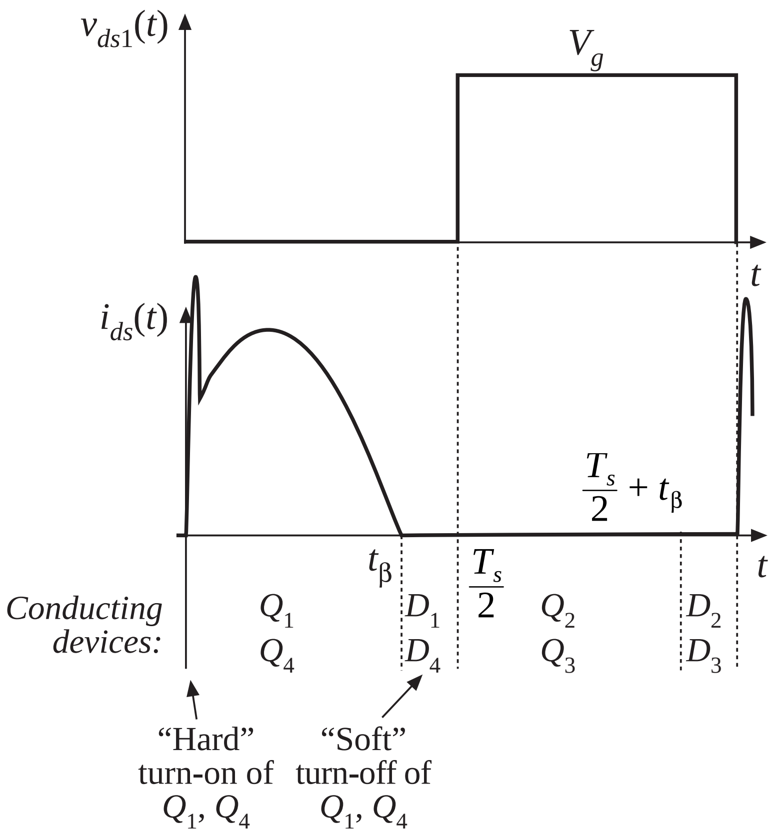


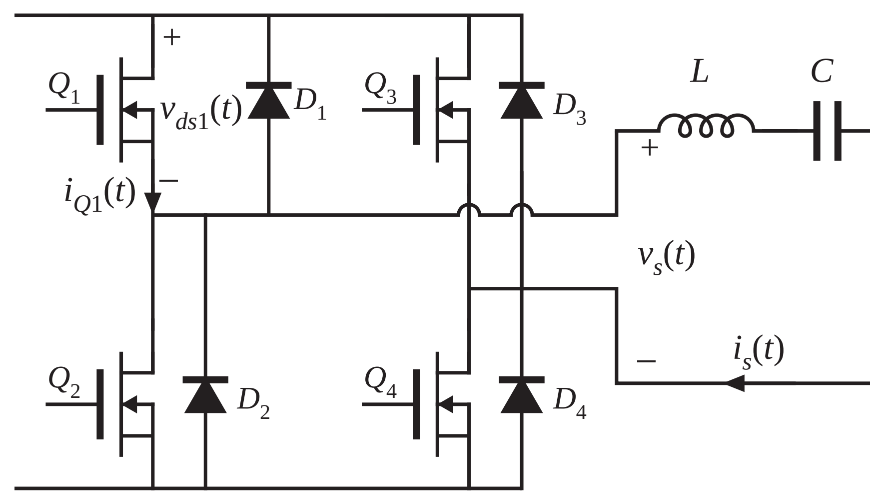


Conduction sequence: Q_1–D_1–Q_2–D_2

Q_1 is turned off during D_1 conduction interval, without loss

ZCS turn-on transition: hard switching

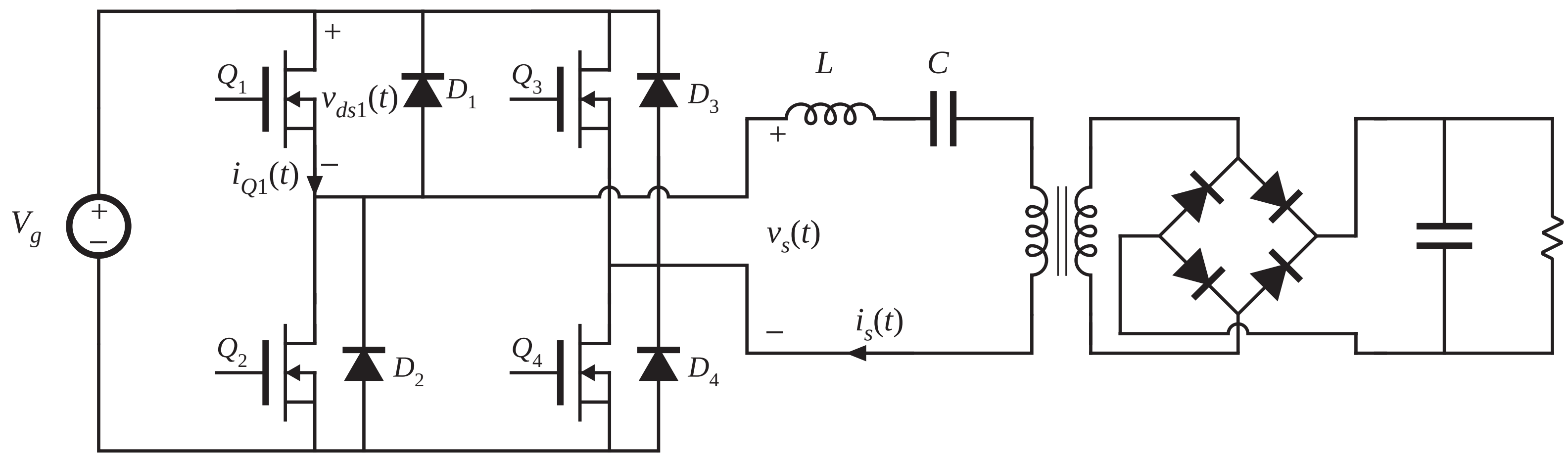




Q_1 turns on while D_2 is conducting. Stored charge of D_2 and of semiconductor output capacitances must be removed. Transistor turn-on transition is identical to hard-switched PWM, and switching loss occurs.

19.4.2 Operation of the full bridge below resonance: Zero-voltage switching

Series resonant converter example



Operation above resonance: input tank current lags voltage

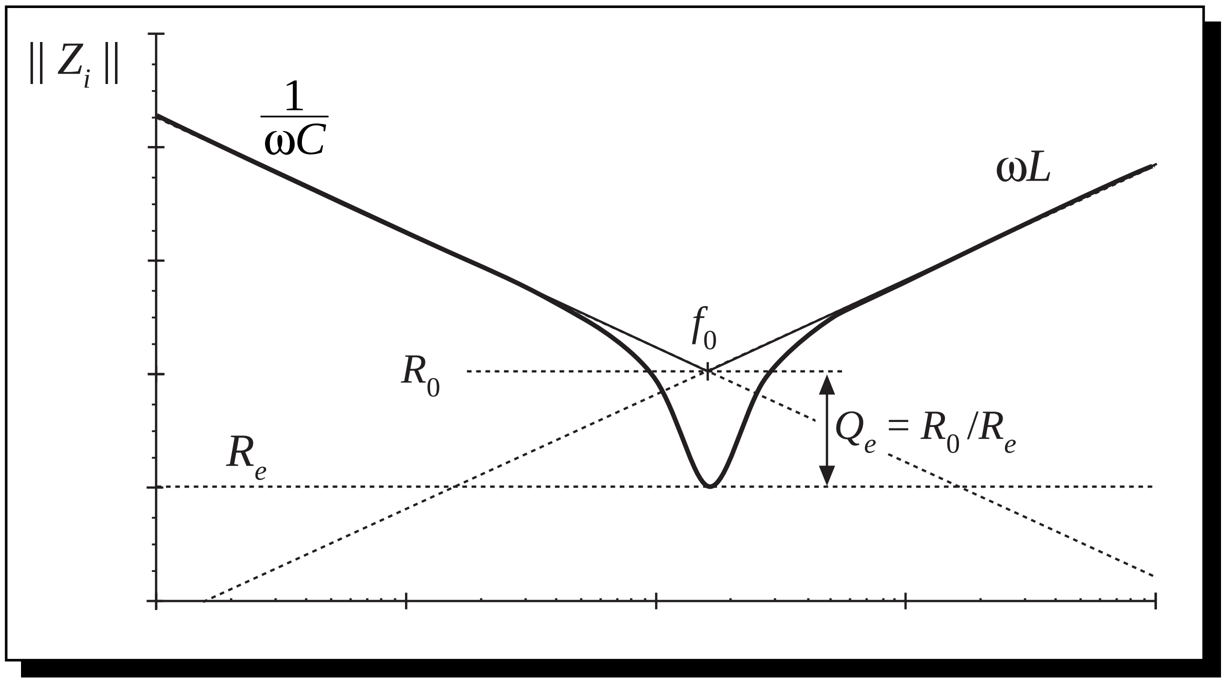
Zero-voltage switching (ZVS) occurs

Tank input impedance

Operation above resonance: tank input impedance Z_i is dominated by tank inductor.

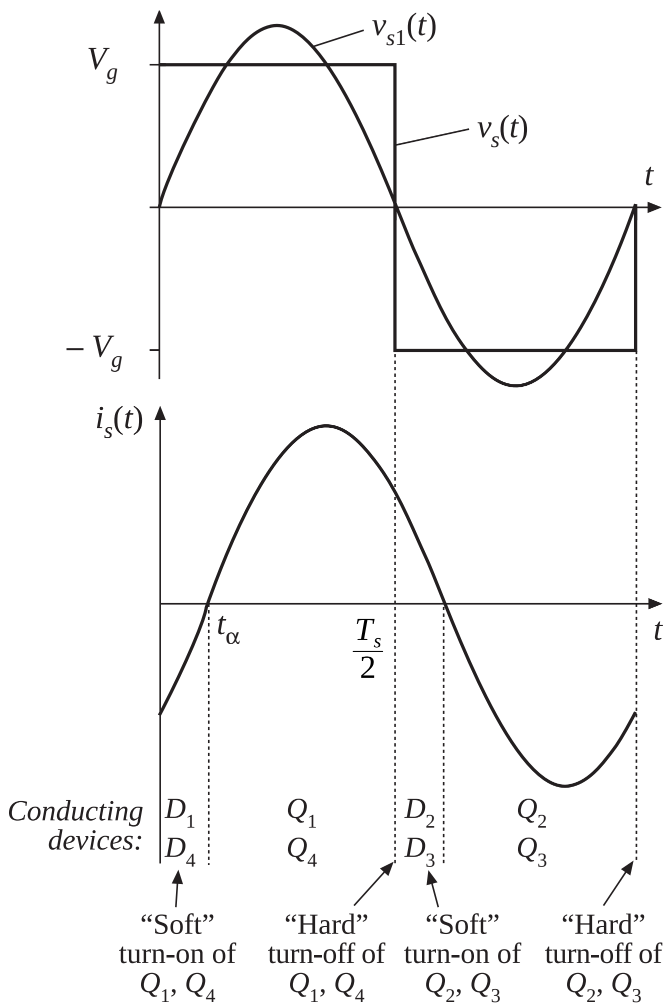
$\angle Z_i$ is negative, and tank input current lags tank input voltage.

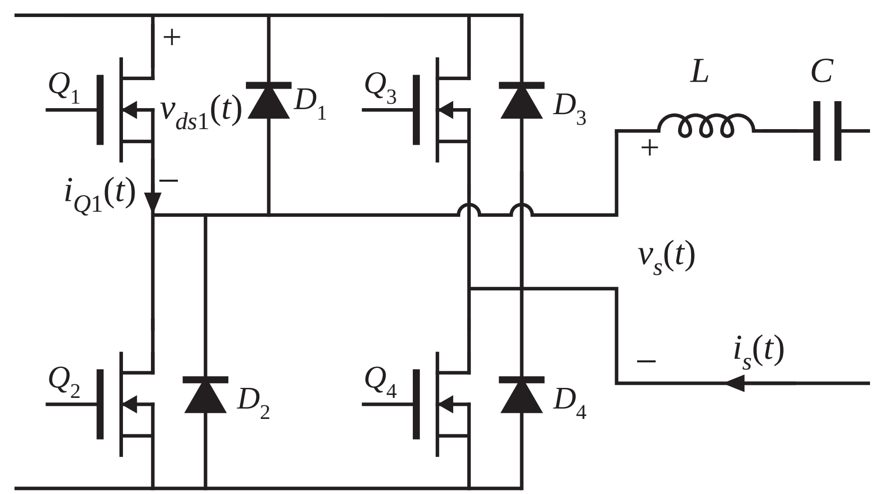
Zero crossing of the tank input current waveform $i_s(t)$ occurs after the zero crossing of the voltage $v_s(t)$.



Switch network waveforms, above resonance

Zero-voltage switching

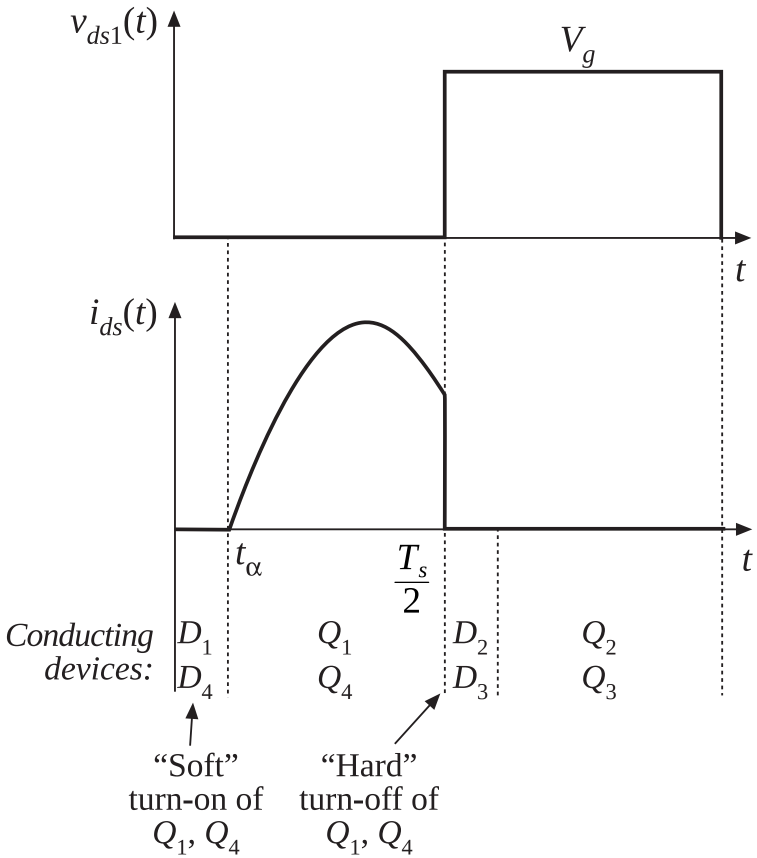


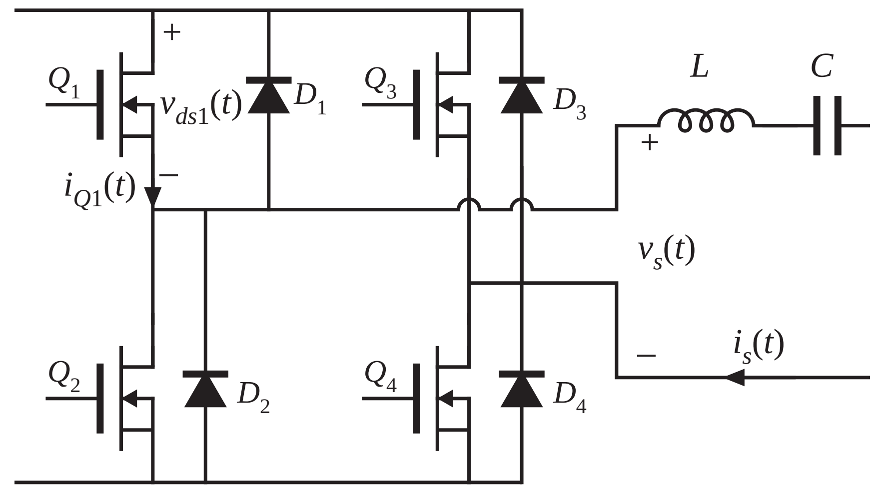


Conduction sequence: D_1–Q_1–D_2–Q_2

Q_1 is turned on during D_1 conduction interval, without loss

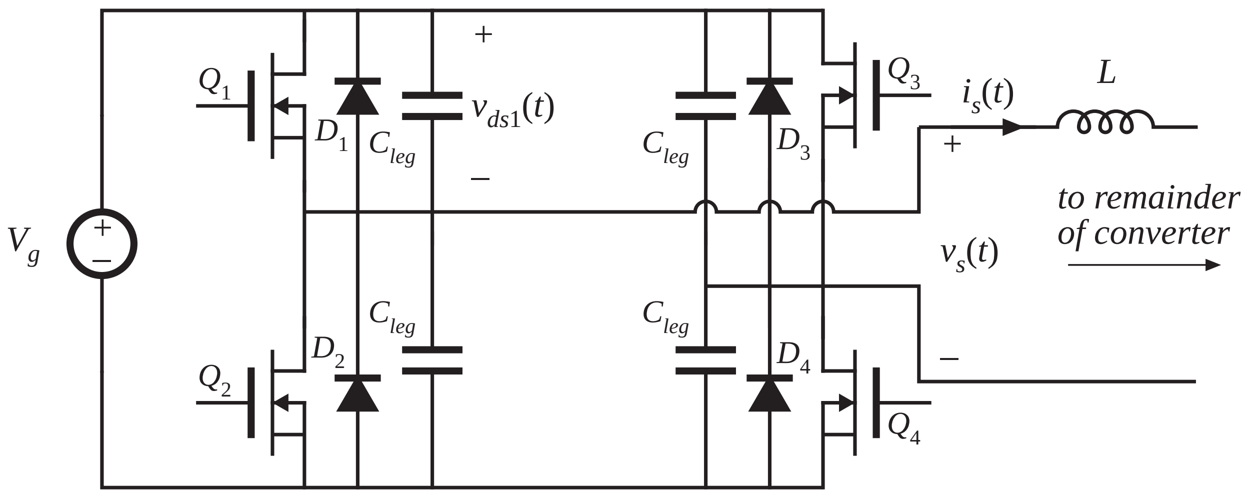
ZVS turn-off transition: hard switching?



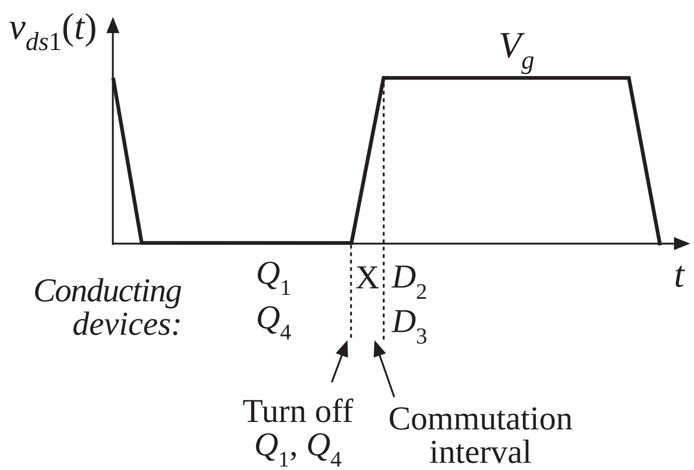


When Q_1 turns off, D_2 must begin conducting. Voltage across Q_1 must increase to V_g. Transistor turn-off transition is identical to hard-switched PWM. Switching loss may occur (but see next slide).

Soft switching at the ZVS turn-off transition



- Introduce small capacitors C_{leg} across each device (or use device output capacitances).
- Introduce delay between turn-off of Q_1 and turn-on of Q_2.

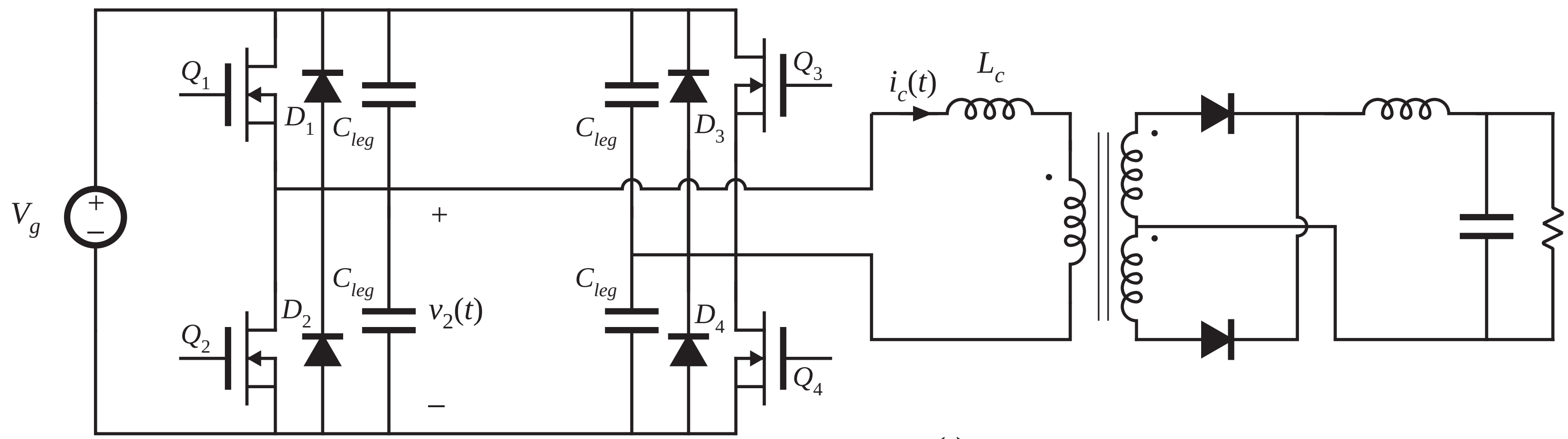


Tank current $i_s(t)$ charges and discharges C_{leg}. Turn-off transition becomes lossless. During commutation interval, no devices conduct.

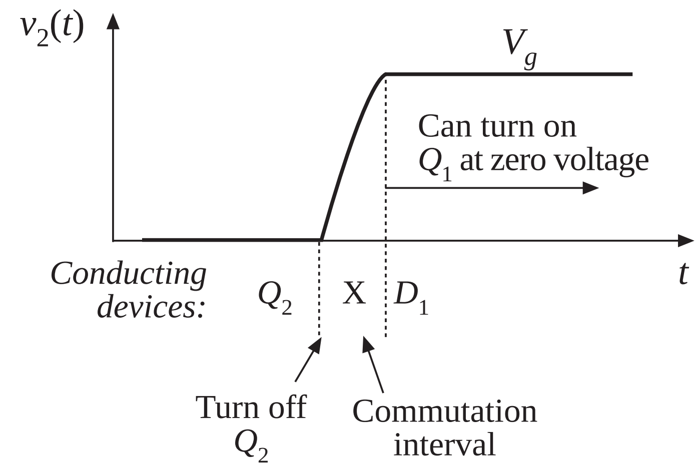
So zero-voltage switching exhibits low switching loss: losses due to diode stored charge and device output capacitances are eliminated.

19.4.3 The zero-voltage transition converter

Basic version based on full-bridge PWM buck converter



- Can obtain ZVS of all primary-side MOSFETs and diodes
- Secondary-side diodes switch at zero-current, with loss
- Phase-shift control



19.5 Load-dependent properties of resonant converters

Resonant inverter design objectives:

1. Operate with a specified load characteristic and range of operating points
   - With a nonlinear load, must properly match inverter output characteristic to load characteristic
2. Obtain zero-voltage switching or zero-current switching
   - Preferably, obtain these properties at all loads
   - Could allow ZVS property to be lost at light load, if necessary
3. Minimize transistor currents and conduction losses
   - To obtain good efficiency at light load, the transistor current should scale proportionally to load current (in resonant converters, it often doesn't!)

Topics of Discussion

Section 19.5

Inverter output *i-v* characteristics

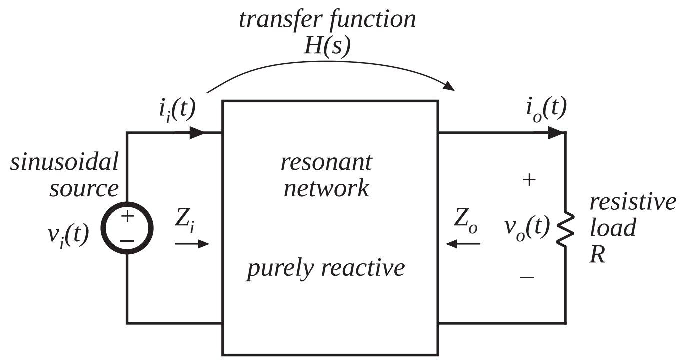
Two theorems

- Dependence of transistor current on load current
- Dependence of zero-voltage/zero-current switching on load resistance
- Simple, intuitive frequency-domain approach to design of resonant converter

Examples and interpretation

- Series
- Parallel
- LCC

Inverter output characteristics



Let H_∞ be the open-circuit ($R \rightarrow \infty$) transfer function:

$$\left.\frac{v_o(j\omega)}{v_i(j\omega)}\right|_{R \rightarrow \infty} = H_\infty(j\omega)$$

and let Z_{o0} be the output impedance (with $v_i \rightarrow$ short-circuit). Then,

$$v_o(j\omega) = H_\infty(j\omega)\, v_i(j\omega)\, \frac{R}{R + Z_{o0}(j\omega)}$$

The output voltage magnitude is:

$$\left\| v_o \right\|^2 = v_o v_o^* = \frac{\left\| H_\infty \right\|^2 \left\| v_i \right\|^2}{\left(1 + \left\| Z_{o0} \right\|^2 / R^2\right)}$$

with $\quad R = \left\| v_o \right\| / \left\| i_o \right\|$

This result can be rearranged to obtain

$$\left\| v_o \right\|^2 + \left\| i_o \right\|^2 \left\| Z_{o0} \right\|^2 = \left\| H_\infty \right\|^2 \left\| v_i \right\|^2$$

Hence, at a given frequency, the output characteristic (i.e., the relation between $\|v_o\|$ and $\|i_o\|$) of any resonant inverter of this class is elliptical.

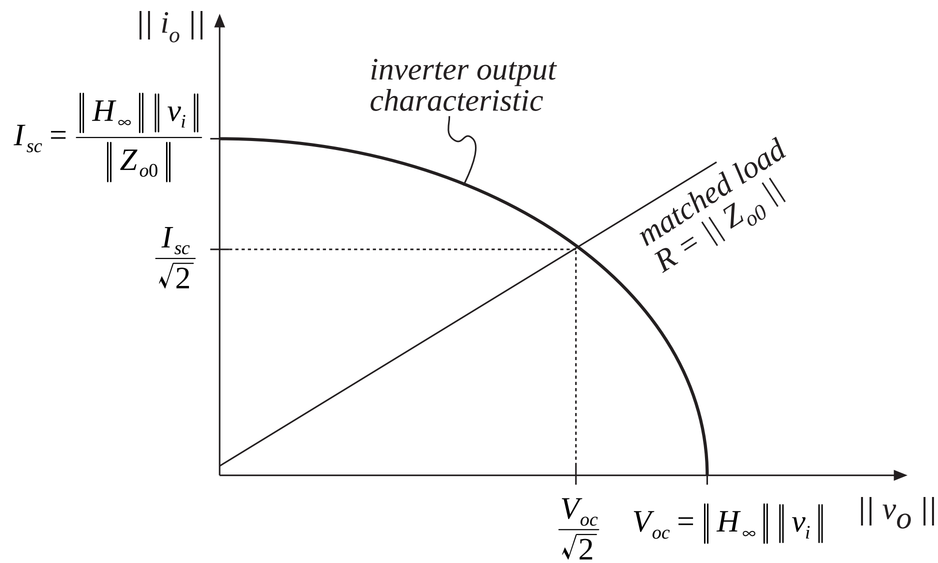
Inverter output characteristics

General resonant inverter output characteristics are elliptical, of the form

$$\frac{\left\| v_o \right\|^2}{V_{oc}^2} + \frac{\left\| i_o \right\|^2}{I_{sc}^2} = 1$$

with $$V_{oc} = \left\| H_\infty \right\| \left\| v_i \right\|$$

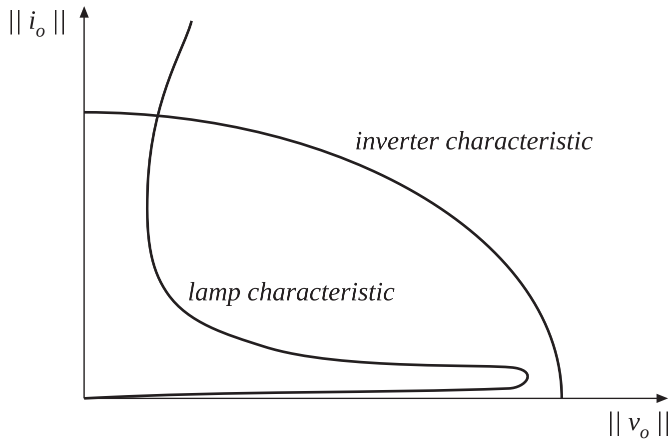
$$I_{sc} = \frac{\left\| H_\infty \right\| \left\| v_i \right\|}{\left\| Z_{o0} \right\|}$$



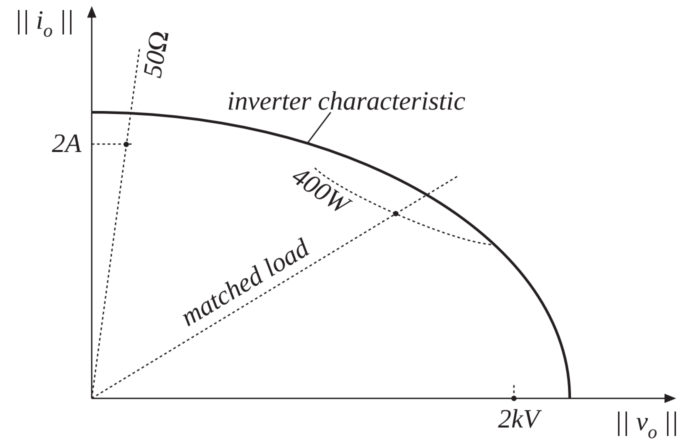
This result is valid provided that *(i)* the resonant network is purely reactive, and *(ii)* the load is purely resistive.

Matching ellipse
to application requirements

Electronic ballast

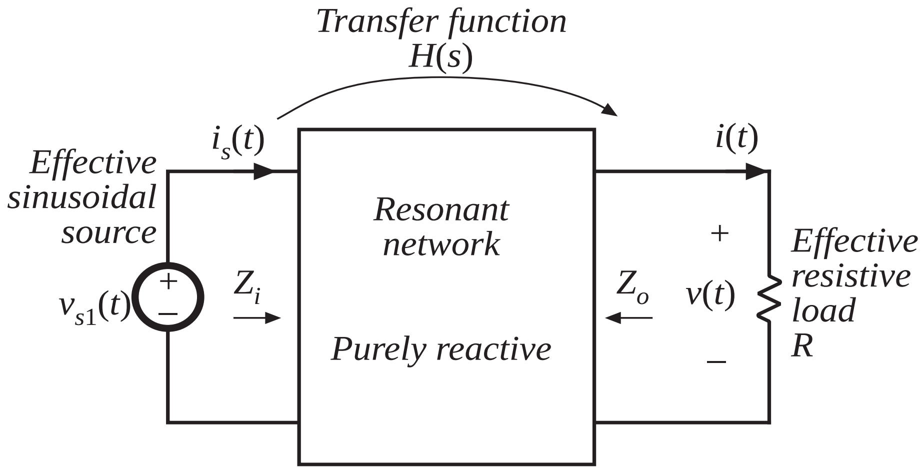


Electrosurgical generator



Input impedance of the resonant tank network

$$Z_i(s) = Z_{i0}(s)\,\frac{1 + \dfrac{R}{Z_{o0}(s)}}{1 + \dfrac{R}{Z_{o\infty}(s)}} = Z_{i\infty}(s)\,\frac{1 + \dfrac{Z_{o0}(s)}{R}}{1 + \dfrac{Z_{o\infty}(s)}{R}}$$



where

$$Z_{i0} = \left.\frac{v_i}{i_i}\right|_{R \to 0} \qquad Z_{i\infty} = \left.\frac{v_i}{i_i}\right|_{R \to \infty} \qquad Z_{o0} = \left.\frac{v_o}{-\,i_o}\right|_{v_i \to \text{short circuit}} \qquad Z_{o\infty} = \left.\frac{v_o}{-\,i_o}\right|_{v_i \to \text{open circuit}}$$

Other relations

Reciprocity

$$\frac{Z_{i0}}{Z_{i\infty}} = \frac{Z_{o0}}{Z_{o\infty}}$$

Tank transfer function

$$H(s) = \frac{H_\infty(s)}{1 + \frac{R}{Z_{o0}}}$$

where
$$H_\infty = \left.\frac{v_o(s)}{v_i(s)}\right|_{R \to \infty}$$

$$\| H_\infty \|^2 = Z_{o0} \left(\frac{1}{Z_{i0}} - \frac{1}{Z_{i\infty}} \right)$$

If the tank network is purely reactive, then each of its impedances and transfer functions have zero real parts:

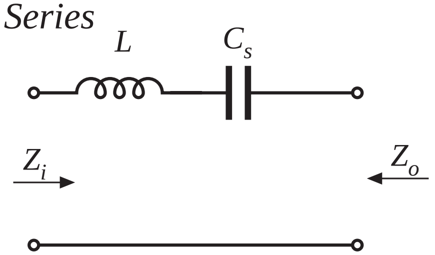
$$Z_{i0} = -Z_{i0}^*$$
$$Z_{i\infty} = -Z_{i\infty}^*$$
$$Z_{o0} = -Z_{o0}^*$$
$$Z_{o\infty} = -Z_{o\infty}^*$$
$$H_\infty = -H_\infty^*$$

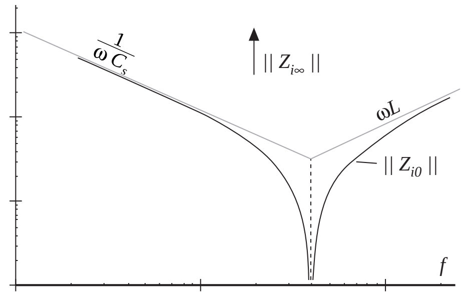
Hence, the input impedance magnitude is

$$\| Z_i \|^2 = Z_i Z_i^* = \| Z_{i0} \|^2 \frac{\left(1 + \frac{R^2}{\| Z_{o0} \|^2} \right)}{\left(1 + \frac{R^2}{\| Z_{o\infty} \|^2} \right)}$$

Z_{i0} *and* $Z_{i\infty}$ for 3 common inverters

Series

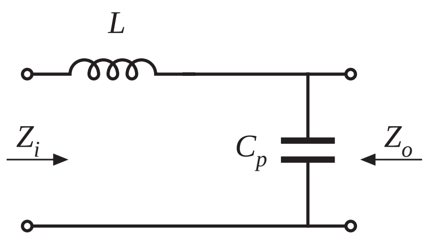


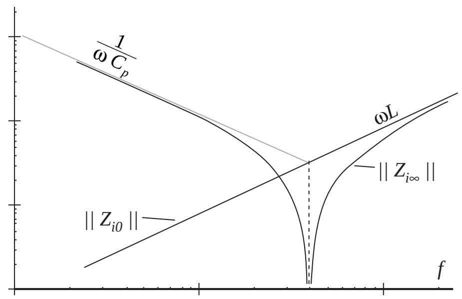


$$Z_{i0}(s) = sL + \frac{1}{sC_s}$$

$$Z_{i\infty}(s) = \infty$$

Parallel

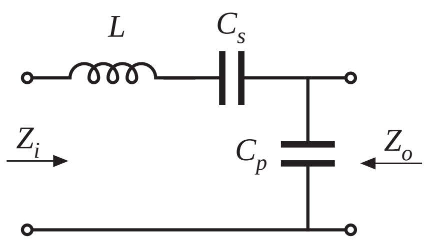


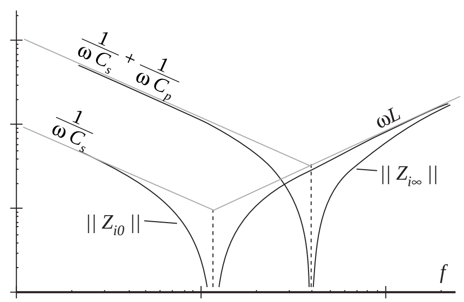


$$Z_{i0}(s) = sL$$

$$Z_{i\infty}(s) = sL + \frac{1}{sC_p}$$

LCC





$$Z_{i0}(s) = sL + \frac{1}{sC_s}$$

$$Z_{i\infty}(s) = sL + \frac{1}{sC_p} + \frac{1}{sC_s}$$

A Theorem relating transistor current variations to load resistance R

Theorem 1: If the tank network is purely reactive, then its input impedance $\| Z_i \|$ is a monotonic function of the load resistance R.

- So as the load resistance R varies from 0 to ∞, the resonant network input impedance $\| Z_i \|$ varies monotonically from the short-circuit value $\| Z_{i0} \|$ to the open-circuit value $\| Z_{i\infty} \|$.
- The impedances $\| Z_{i\infty} \|$ and $\| Z_{i0} \|$ are easy to construct.
- If you want to minimize the circulating tank currents at light load, maximize $\| Z_{i\infty} \|$.
- Note: for many inverters, $\| Z_{i\infty} \| < \| Z_{i0} \|$! The no-load transistor current is therefore greater than the short-circuit transistor current.

Proof of Theorem 1

Previously shown:

$$\| Z_i \|^2 = \| Z_{i0} \|^2 \; \frac{\left(1 + \dfrac{R}{\| Z_{o0} \|^2}\right)}{\left(1 + \dfrac{R}{\| Z_{o\infty} \|^2}\right)}$$

➡ Differentiate:

$$\frac{d \, \| Z_i \|^2}{d \, R} = 2 \, \| Z_{i0} \|^2 \; \frac{\left(\dfrac{1}{\| Z_{o0} \|^2} - \dfrac{1}{\| Z_{o\infty} \|^2}\right) R}{\left(1 + \dfrac{R^2}{\| Z_{o\infty} \|^2}\right)^2}$$

➡ Derivative has roots at:

$$(i) \quad R = 0$$

$$(ii) \quad R = \infty$$

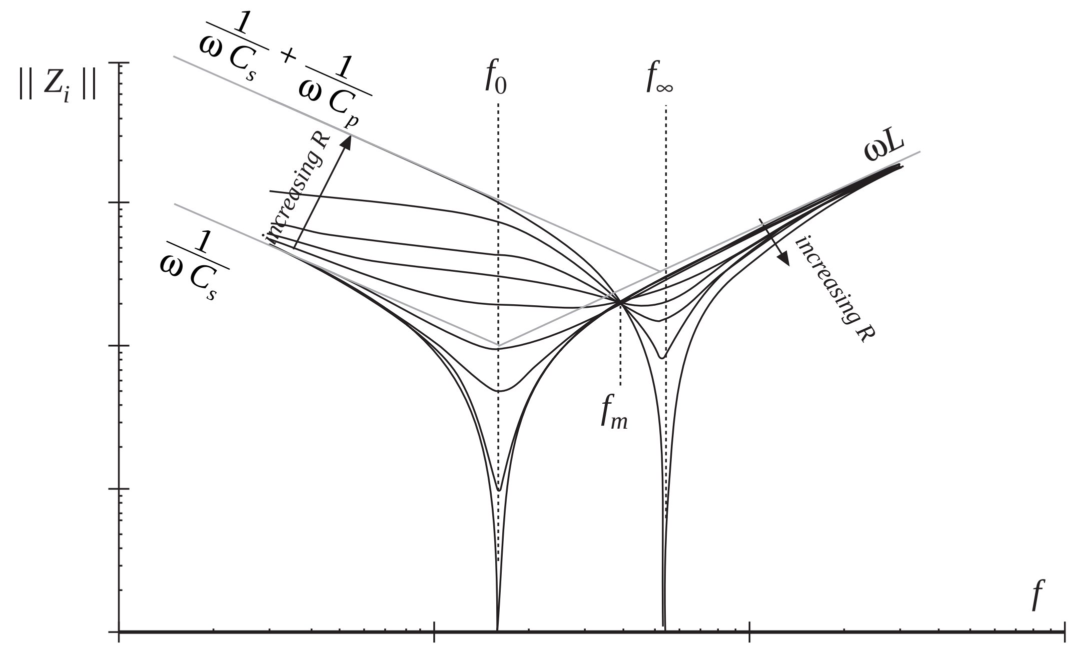
$$(iii) \quad \| Z_{o0} \| = \| Z_{o\infty} \|, \; or \; \| Z_{i0} \| = \| Z_{i\infty} \|$$

So the resonant network input impedance is a monotonic function of R, over the range $0 < R < \infty$.

In the special case $\| Z_{i0} \| = \| Z_{i\infty} \|$, $\| Z_i \|$ is independent of R.

Example: $\| Z_i \|$ of LCC

- for $f < f_m$, $\| Z_i \|$ increases with increasing R .
- for $f > f_m$, $\| Z_i \|$ decreases with increasing R .
- at a given frequency f, $\| Z_i \|$ is a monotonic function of R.
- It's not necessary to draw the entire plot: just construct $\| Z_{i0} \|$ and $\| Z_{i\infty} \|$.



Discussion: LCC

$\|\ Z_{i0}\ \|$ and $\|\ Z_{i\infty}\ \|$ both represent series resonant impedances, whose Bode diagrams are easily constructed.

$\|\ Z_{i0}\ \|$ and $\|\ Z_{i\infty}\ \|$ intersect at frequency f_m.

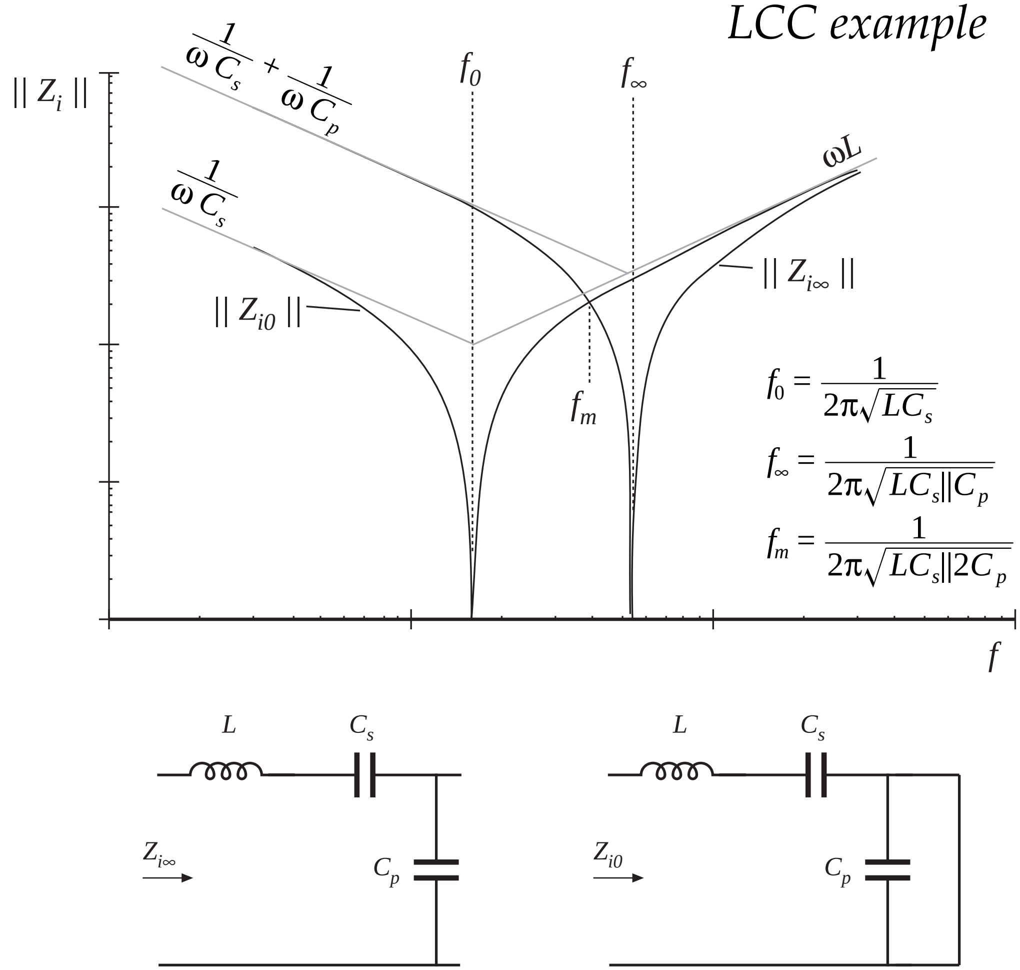
For f < f_m

then $\|\ Z_{i0}\ \| < \|\ Z_{i\infty}\ \|$; hence transistor current decreases as load current decreases

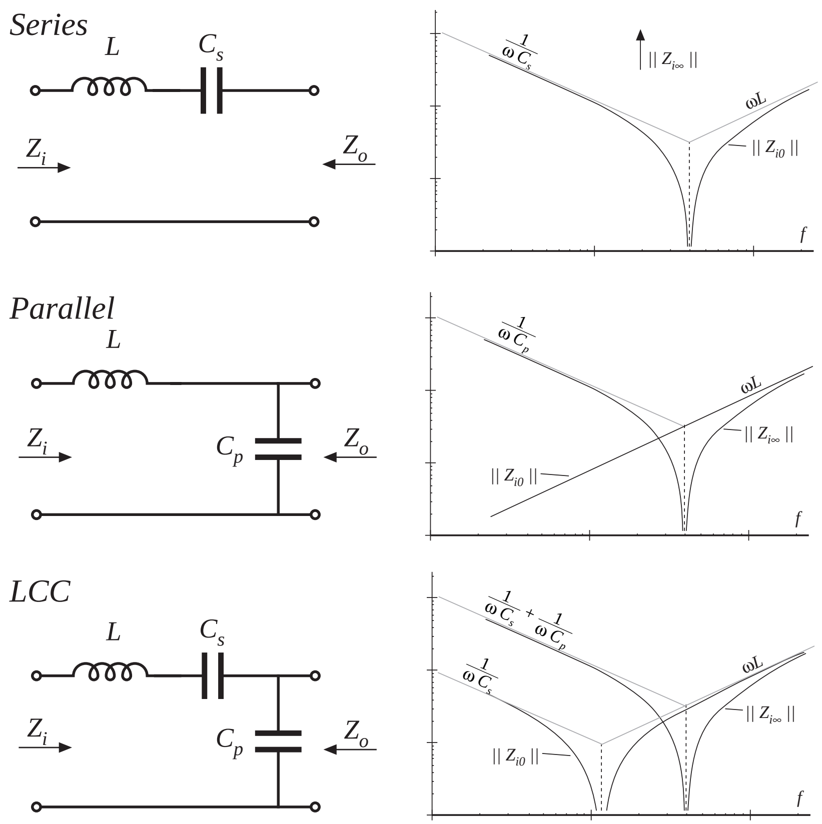
For f > f_m

then $\|\ Z_{i0}\ \| > \|\ Z_{i\infty}\ \|$; hence transistor current increases as load current decreases, and transistor current is greater than or equal to short-circuit current for all R

LCC example



Discussion -series and parallel



- No-load transistor current = 0, both above and below resonance.
- ZCS below resonance, ZVS above resonance

- Above resonance: no-load transistor current is *greater* than short-circuit transistor current. ZVS.
- Below resonance: no-load transistor current is less than short-circuit current (for $f < f_m$), but determined by $\| Z_{i\infty} \|$. ZCS.

A Theorem relating the ZVS/ZCS boundary to load resistance R

Theorem 2: If the tank network is purely reactive, then the boundary between zero-current switching and zero-voltage switching occurs when the load resistance *R* is equal to the critical value R_{crit}, given by

$$R_{crit} = \| Z_{o0} \| \sqrt{\frac{-Z_{i\infty}}{Z_{i0}}}$$

It is assumed that zero-current switching (ZCS) occurs when the tank input impedance is capacitive in nature, while zero-voltage switching (ZVS) occurs when the tank is inductive in nature. This assumption gives a necessary but not sufficient condition for ZVS when significant semiconductor output capacitance is present.

Proof of Theorem 2

Previously shown:

$$Z_i = Z_{i\infty} \frac{1 + \dfrac{Z_{o0}}{R}}{1 + \dfrac{Z_{o\infty}}{R}}$$

If ZCS occurs when Z_i is capacitive, while ZVS occurs when Z_i is inductive, then the boundary is determined by $\angle Z_i$ = 0. Hence, the critical load R_{crit} is the resistance which causes the imaginary part of Z_i to be zero:

$$\mathrm{Im}\left(Z_i(R_{crit})\right) = 0$$

Note that $Z_{i\infty}$, Z_{o0}, and $Z_{o\infty}$ have zero real parts. Hence,

$$\mathrm{Im}\left(Z_i(R_{crit})\right) = \mathrm{Im}\left(Z_{i\infty}\right) \mathrm{Re}\left(\frac{1 + \dfrac{Z_{o0}}{R_{crit}}}{1 + \dfrac{Z_{o\infty}}{R_{crit}}}\right)$$

$$= \mathrm{Im}\left(Z_{i\infty}\right) \mathrm{Re}\left(\frac{1 - \dfrac{Z_{o0}Z_{o\infty}}{R_{crit}^2}}{1 + \dfrac{\left\| Z_{o\infty} \right\|^2}{R_{crit}^2}}\right)$$

Solution for R_{crit} yields

$$R_{crit} = \left\| Z_{o0} \right\| \sqrt{\frac{-Z_{i\infty}}{Z_{i0}}}$$

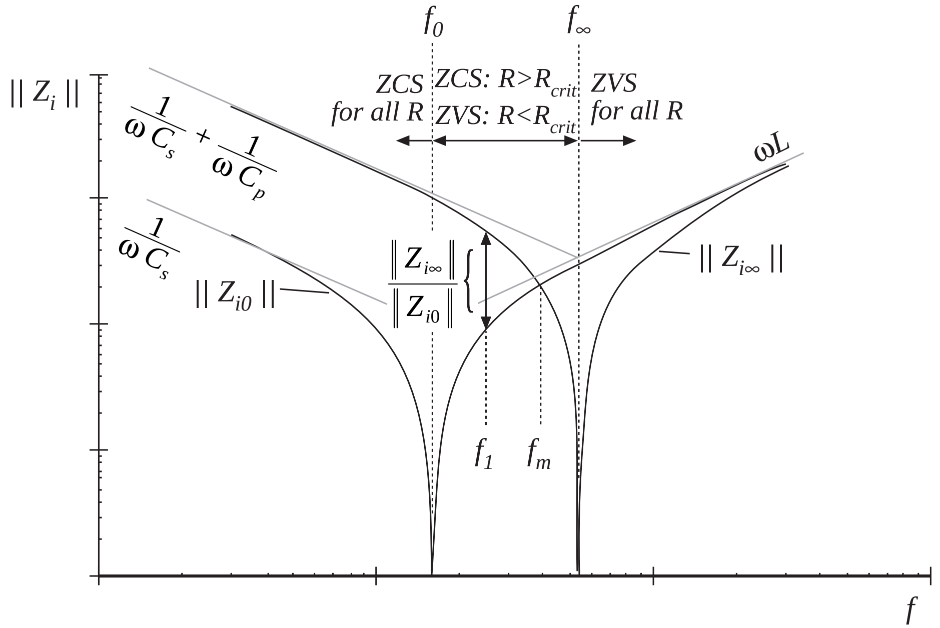
Discussion —Theorem 2

$$R_{crit} = \| Z_{o0} \| \sqrt{\frac{-Z_{i\infty}}{Z_{i0}}}$$

- Again, $Z_{i\infty}$, Z_{i0}, and Z_{o0} are pure imaginary quantities.
- If $Z_{i\infty}$ and Z_{i0} have the same phase (both inductive or both capacitive), then there is no real solution for R_{crit}.
- Hence, if at a given frequency $Z_{i\infty}$ and Z_{i0} are both capacitive, then ZCS occurs for all loads. If $Z_{i\infty}$ and Z_{i0} are both inductive, then ZVS occurs for all loads.
- If $Z_{i\infty}$ and Z_{i0} have opposite phase (one is capacitive and the other is inductive), then there is a real solution for R_{crit}. The boundary between ZVS and ZCS operation is then given by $R = R_{crit}$.
- Note that $R = \| Z_{o0} \|$ corresponds to operation at matched load with maximum output power. The boundary is expressed in terms of this matched load impedance, and the ratio $Z_{i\infty} / Z_{i0}$.

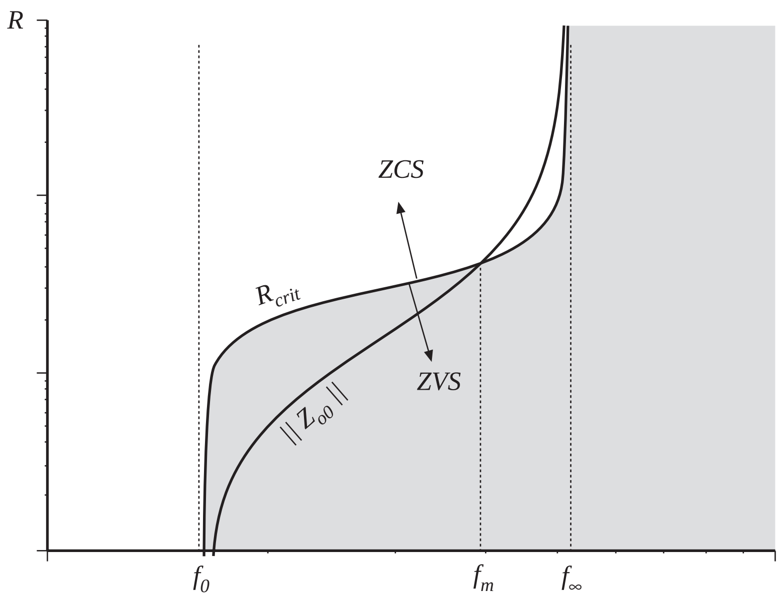
LCC example

- For $f > f_\infty$, ZVS occurs for all R.
- For $f < f_0$, ZCS occurs for all R.
- For $f_0 < f < f_\infty$, ZVS occurs for $R < R_{crit}$, and ZCS occurs for $R > R_{crit}$.
- Note that $R = \| Z_{o0} \|$ corresponds to operation at matched load with maximum output power. The boundary is expressed in terms of this matched load impedance, and the ratio $Z_{i\infty} / Z_{i0}$.

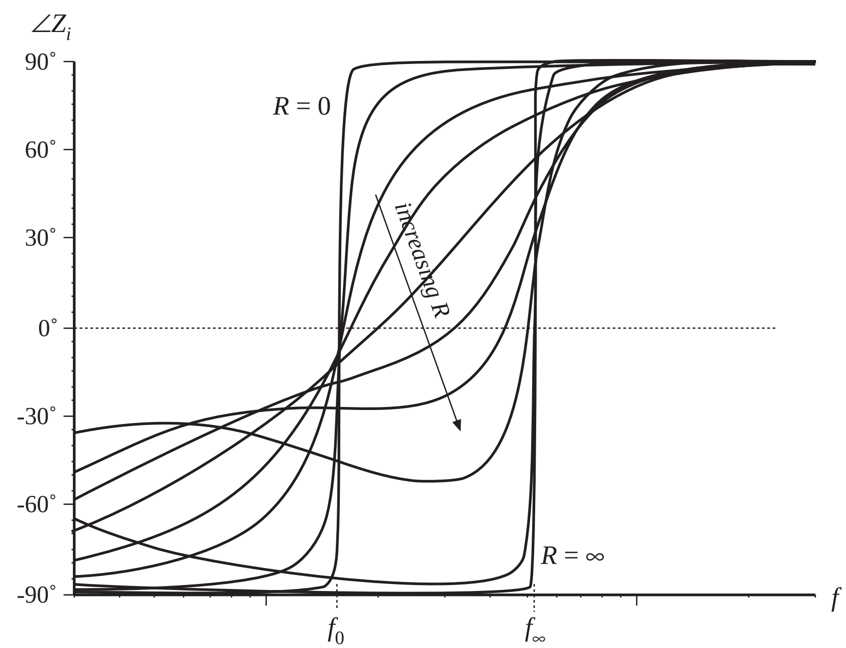


$$R_{crit} = \| Z_{o0} \| \sqrt{\frac{-Z_{i\infty}}{Z_{i0}}}$$

LCC example, continued



Typical dependence of R_{crit} and matched-load impedance $|| Z_{o0} ||$ on frequency f, LCC example.



Typical dependence of tank input impedance phase vs. load R and frequency, LCC example.

19.6 Summary of Key Points

1. The sinusoidal approximation allows a great deal of insight to be gained into the operation of resonant inverters and dc–dc converters. The voltage conversion ratio of dc–dc resonant converters can be directly related to the tank network transfer function. Other important converter properties, such as the output characteristics, dependence (or lack thereof) of transistor current on load current, and zero-voltage- and zero-current-switching transitions, can also be understood using this approximation. The approximation is accurate provided that the effective Q–factor is sufficiently large, and provided that the switching frequency is sufficiently close to resonance.
2. Simple equivalent circuits are derived, which represent the fundamental components of the tank network waveforms, and the dc components of the dc terminal waveforms.

Summary of key points

3. Exact solutions of the ideal dc–dc series and parallel resonant converters are listed here as well. These solutions correctly predict the conversion ratios, for operation not only in the fundamental continuous conduction mode, but in discontinuous and subharmonic modes as well.
4. Zero-voltage switching mitigates the switching loss caused by diode recovered charge and semiconductor device output capacitances. When the objective is to minimize switching loss and EMI, it is preferable to operate each MOSFET and diode with zero-voltage switching.
5. Zero-current switching leads to natural commutation of SCRs, and can also mitigate the switching loss due to current tailing in IGBTs.

Summary of key points

6. The input impedance magnitude $\| Z_i \|$, and hence also the transistor current magnitude, are monotonic functions of the load resistance R. The dependence of the transistor conduction loss on the load current can be easily understood by simply plotting $\| Z_i \|$ in the limiting cases as $R \rightarrow \infty$ and as $R \rightarrow 0$, or $\| Z_{i\infty} \|$ and $\| Z_{i0} \|$.
7. The ZVS/ZCS boundary is also a simple function of $Z_{i\infty}$ and Z_{i0}. If ZVS occurs at open-circuit and at short-circuit, then ZVS occurs for all loads. If ZVS occurs at short-circuit, and ZCS occurs at open-circuit, then ZVS is obtained at matched load provided that $\| Z_{i\infty} \| > \| Z_{i0} \|$.
8. The output characteristics of all resonant inverters considered here are elliptical, and are described completely by the open-circuit transfer function magnitude $\| H_\infty \|$, and the output impedance $\| Z_{o0} \|$. These quantities can be chosen to match the output characteristics to the application requirements.