Chapter 17

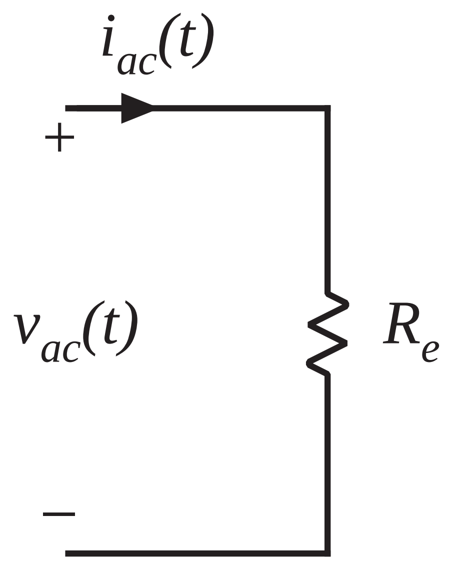
The Ideal Rectifier

17.1 Properties of the ideal rectifier

It is desired that the rectifier present a resistive load to the ac power system. This leads to

- unity power factor
- ac line current has same waveshape as voltage

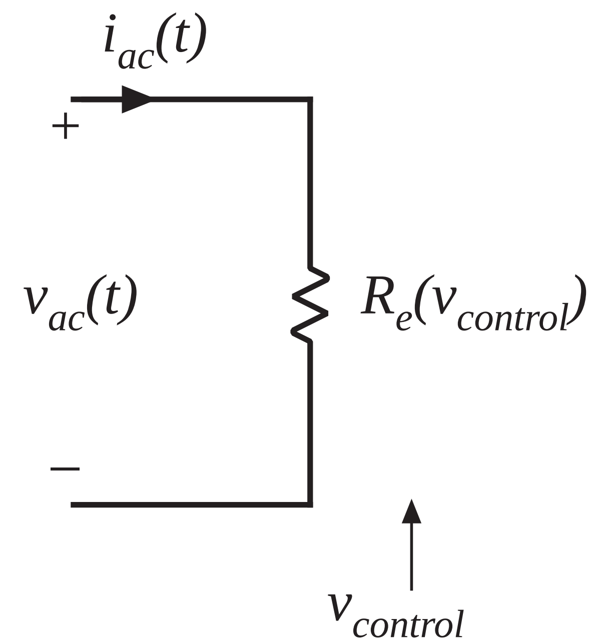
$$i_{ac}(t) = \frac{v_{ac}(t)}{R_e}$$



R_e is called the *emulated resistance*

Control of power throughput

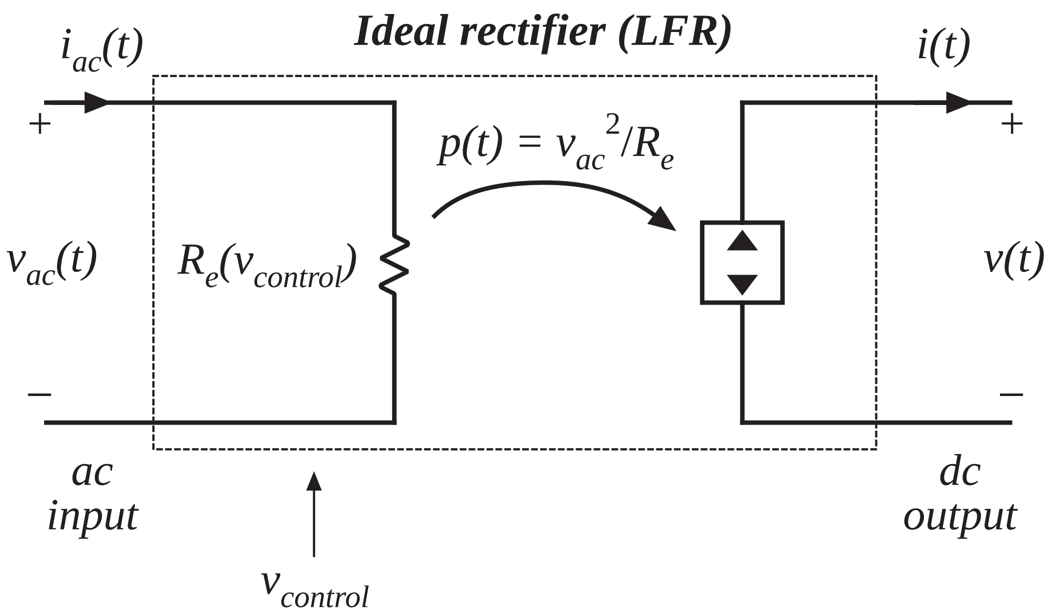
$$P_{av} = \frac{V_{ac,rms}^2}{R_e(v_{control})}$$



Power apparently "consumed" by R_e is actually transferred to rectifier dc output port. To control the amount of output power, it must be possible to adjust the value of R_e.

Output port model

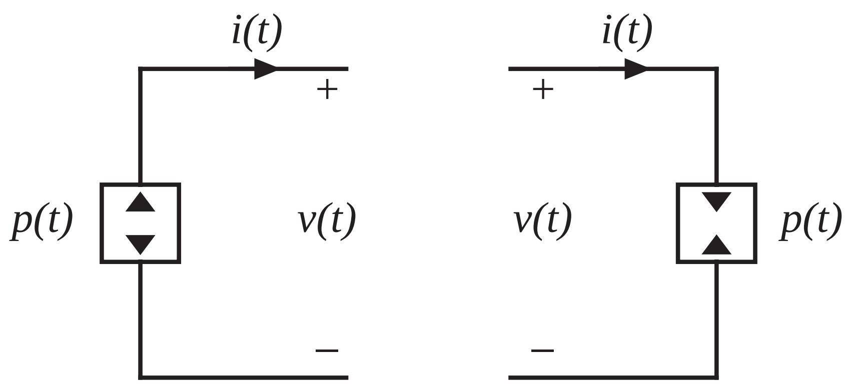
The ideal rectifier is lossless and contains no internal energy storage. Hence, the instantaneous input power equals the instantaneous output power. Since the instantaneous power is independent of the dc load characteristics, the output port obeys a power source characteristic.



$$p(t) = \frac{v_{ac}^2(t)}{R_e(v_{control}(t))}$$

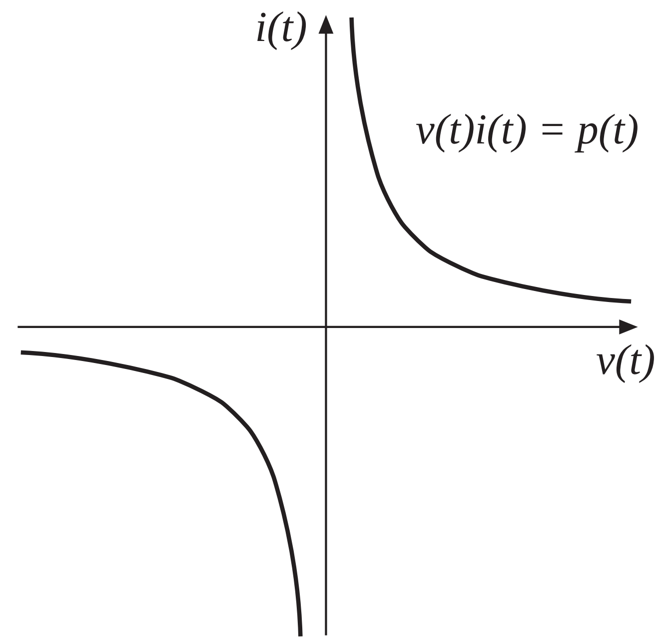
$$v(t)i(t) = p(t) = \frac{v_{ac}^2(t)}{R_e}$$

The dependent power source



power source

power sink



i-*v* characteristic

Equations of the ideal rectifier / LFR

Defining equations of the ideal rectifier:

$$i_{ac}(t) = \frac{v_{ac}(t)}{R_e(v_{control})}$$

$$v(t)i(t) = p(t)$$

$$p(t) = \frac{v_{ac}^2(t)}{R_e(v_{control}(t))}$$

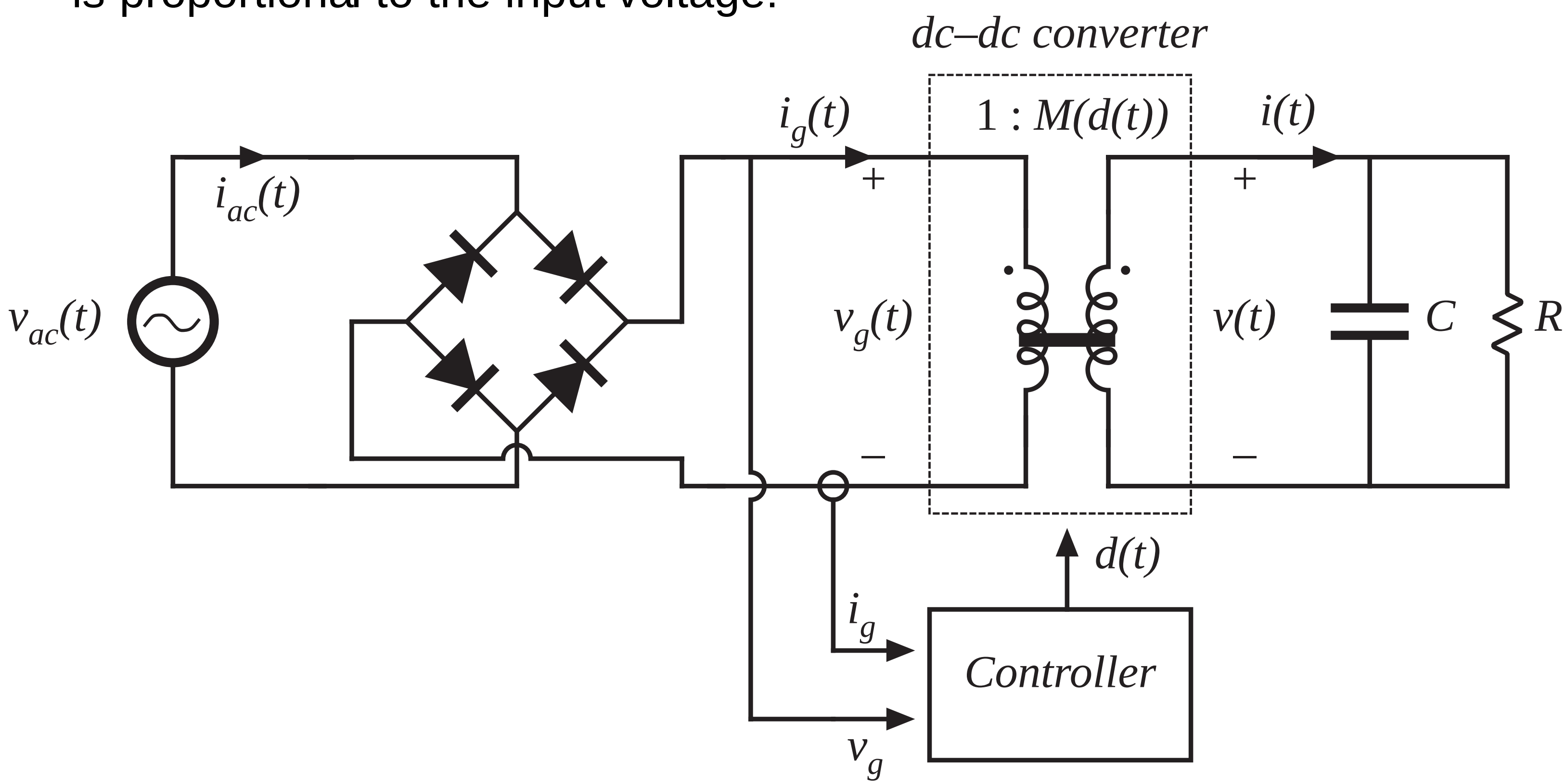
When connected to a resistive load of value R, the input and output rms voltages and currents are related as follows:

$$\frac{V_{rms}}{V_{ac,rms}} = \sqrt{\frac{R}{R_e}}$$

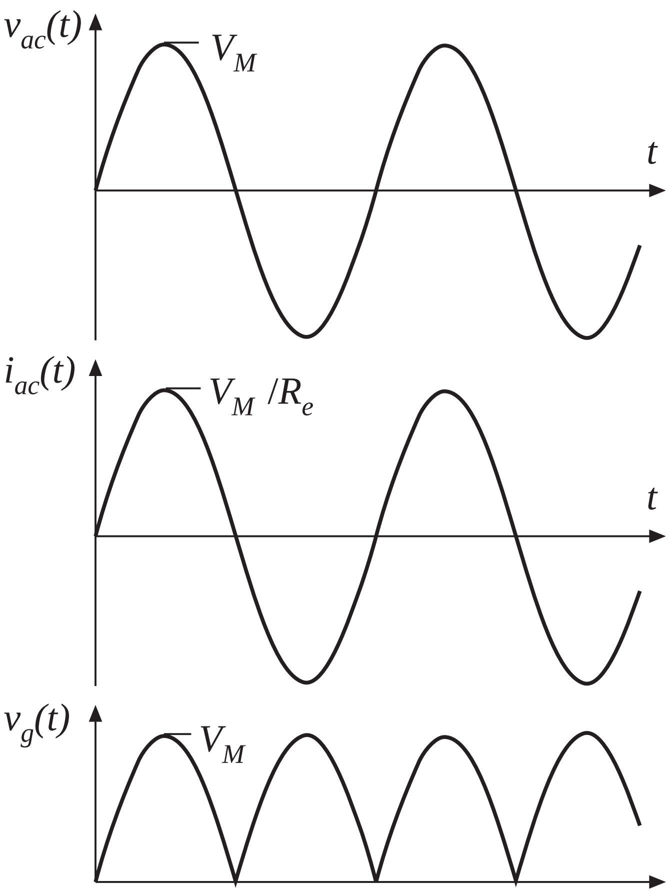
$$\frac{I_{ac,rms}}{I_{rms}} = \sqrt{\frac{R}{R_e}}$$

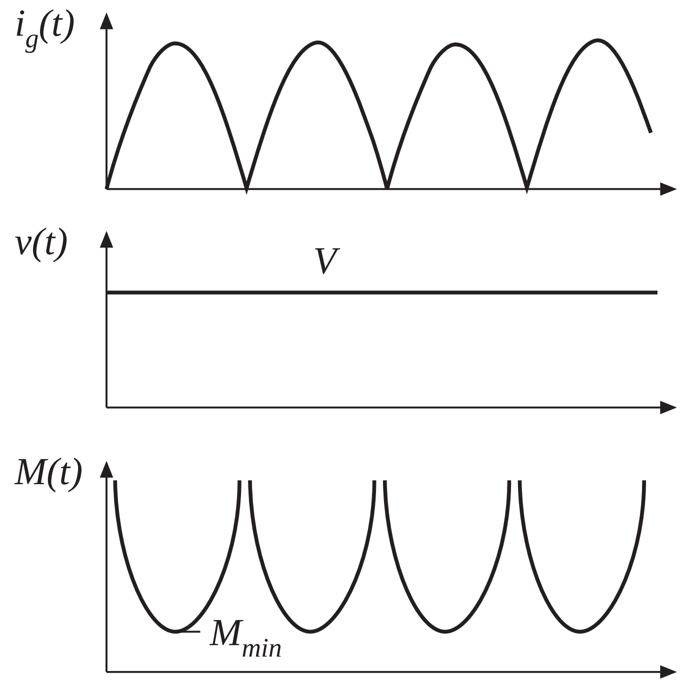
17.2 Realization of a near-ideal rectifier

Control the duty cycle of a dc-dc converter, such that the input current is proportional to the input voltage:



Waveforms





$$v_{ac}(t) = V_M \sin(\omega t)$$

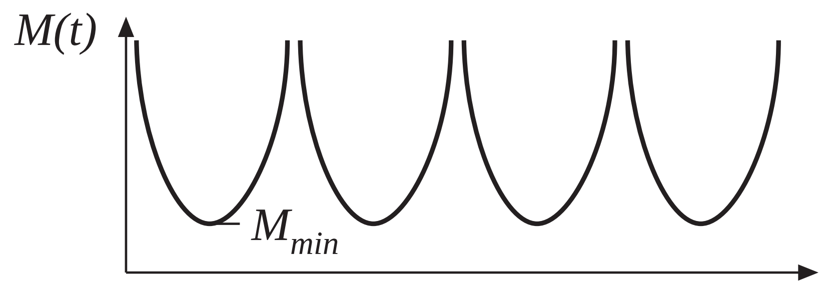
$$v_g(t) = V_M \left| \sin(\omega t) \right|$$

$$M(d(t)) = \frac{v(t)}{v_g(t)} = \frac{V}{V_M \left| \sin(\omega t) \right|}$$

$$M_{min} = \frac{V}{V_M}$$

Choice of converter

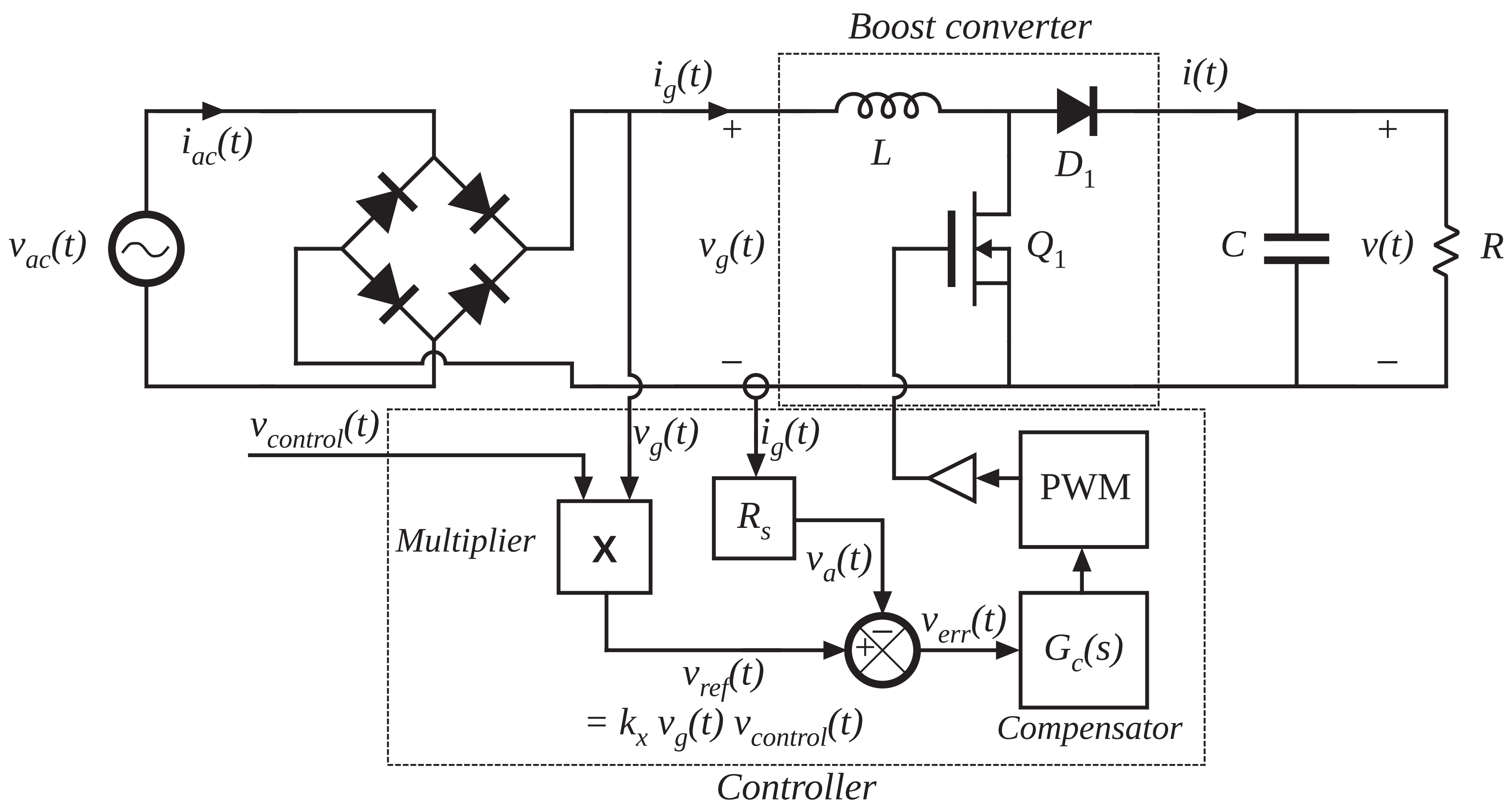
$$M(d(t)) = \frac{v(t)}{v_g(t)} = \frac{V}{V_M \left| \sin(\omega t) \right|}$$



- To avoid distortion near line voltage zero crossings, converter should be capable of producing $M(d(t))$ approaching infinity
- Above expression neglects converter dynamics
- Boost, buck-boost, Cuk, SEPIC, and other converters with similar conversion ratios are suitable
- We will see that the boost converter exhibits lowest transistor stresses. For this reason, it is most often chosen

Boost converter

with controller to cause input current to follow input voltage



Variation of duty cycle in boost rectifier

$$M(d(t)) = \frac{v(t)}{v_g(t)} = \frac{V}{V_M \left| \sin(\omega t) \right|}$$

Since $M \geq 1$ in the boost converter, it is required that $V \geq V_M$

If the converter operates in CCM, then

$$M(d(t)) = \frac{1}{1 - d(t)}$$

The duty ratio should therefore follow

$$d(t) = 1 - \frac{v_g(t)}{V} \qquad \text{in CCM}$$

CCM/DCM boundary, boost rectifier

Inductor current ripple is

$$\Delta i_g(t) = \frac{v_g(t) d(t) T_s}{2L}$$

Low-frequency (average) component of inductor current waveform is

$$\left\langle i_g(t) \right\rangle_{T_s} = \frac{v_g(t)}{R_e}$$

The converter operates in CCM when

$$\left\langle i_g(t) \right\rangle_{T_s} > \Delta i_g(t) \quad \Rightarrow \quad d(t) < \frac{2L}{R_e T_s}$$

Substitute CCM expression for $d(t)$:

$$R_e < \frac{2L}{T_s \left(1 - \frac{v_g(t)}{V} \right)} \quad \text{for CCM}$$

CCM/DCM boundary

$$R_e < \frac{2L}{T_s\left(1 - \frac{v_g(t)}{V}\right)} \quad \text{for CCM}$$

Note that $v_g(t)$ varies with time, between 0 and V_M. Hence, this equation may be satisfied at some points on the ac line cycle, and not at others. The converter always operates in CCM provided that

$$R_e < \frac{2L}{T_s}$$

The converter always operates in DCM provided that

$$R_e > \frac{2L}{T_s\left(1 - \frac{V_M}{V}\right)}$$

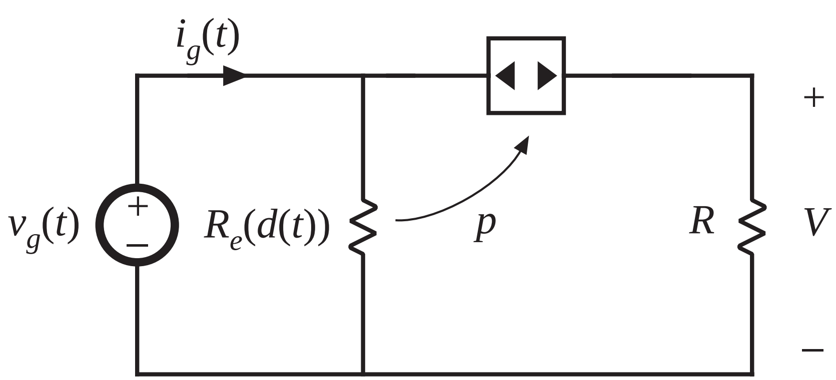
For R_e between these limits, the converter operates in DCM when $v_g(t)$ is near zero, and in CCM when $v_g(t)$ approaches V_M.

Static input characteristics of the boost converter

A plot of input current $i_g(t)$ vs input voltage $v_g(t)$, for various duty cycles $d(t)$. In CCM, the boost converter equilibrium equation is

$$\frac{v_g(t)}{V} = 1 - d(t)$$

The input characteristic in DCM is found by solution of the averaged DCM model (Fig. 10.12(b)):



Beware! This DCM $R_e(d)$ from Chapter 10 is not the same as the rectifier emulated resistance $R_e = v_g/i_g$

Solve for input current:

$$i_g(t) = \frac{v_g(t)}{R_e(d(t))} + \frac{p(t)}{V - v_g(t)}$$

$$\text{with} \quad p(t) = \frac{v_g^2(t)}{R_e(d(t))}$$

$$R_e(d(t)) = \frac{2L}{d^2(t)T_s}$$

Static input characteristics of the boost converter

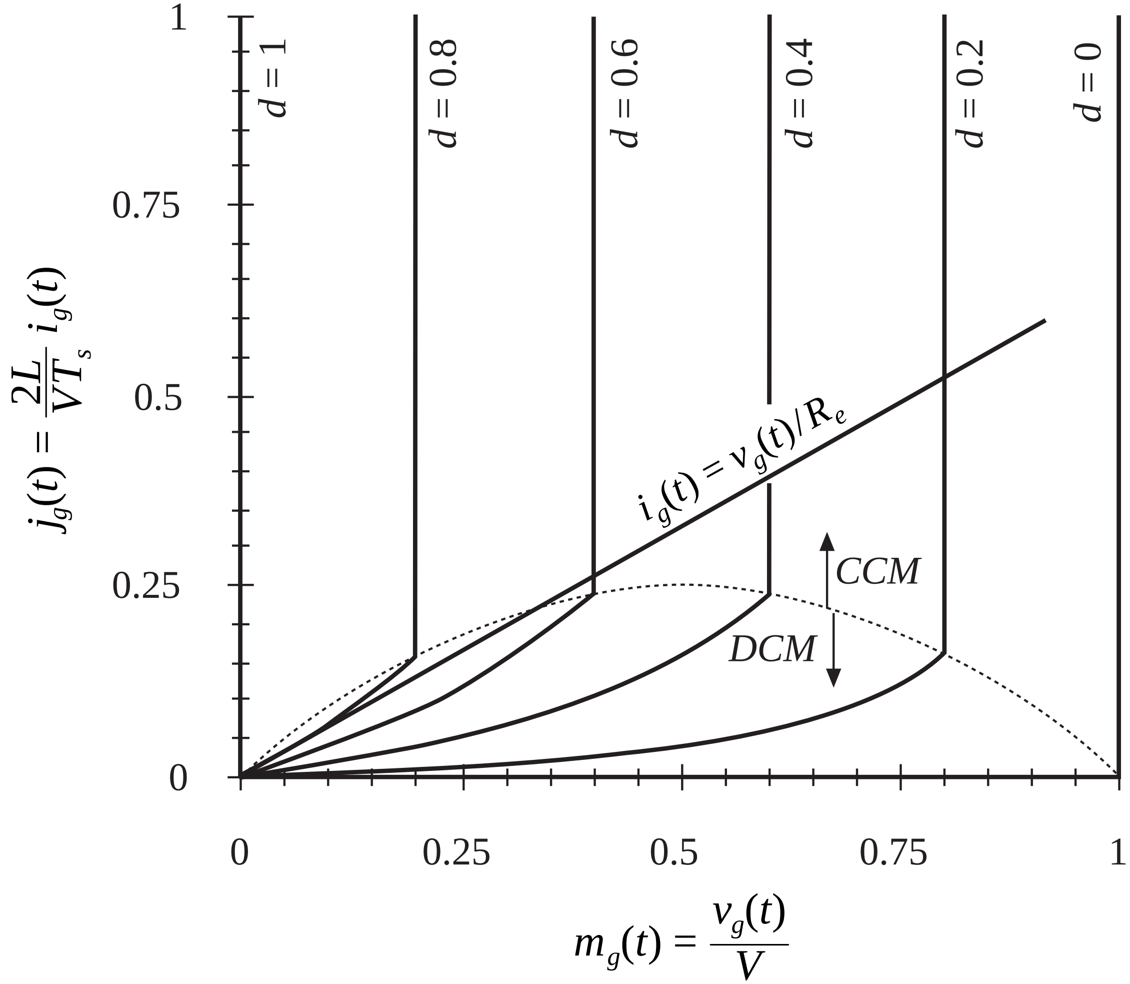
Now simplify DCM current expression, to obtain

$$\frac{2L}{VT_s}\, i_g(t) \left(1 - \frac{v_g(t)}{V}\right) = d^2(t)\, \frac{v_g(t)}{V}$$

CCM/DCM mode boundary, in terms of $v_g(t)$ and $i_g(t)$:

$$\frac{2L}{VT_s}\, i_g(t) > \left(\frac{v_g(t)}{V}\right)\left(1 - \frac{v_g(t)}{V}\right)$$

Boost input characteristics
with superimposed resistive characteristic



CCM:

$$\frac{v_g(t)}{V} = 1 - d(t)$$

DCM:

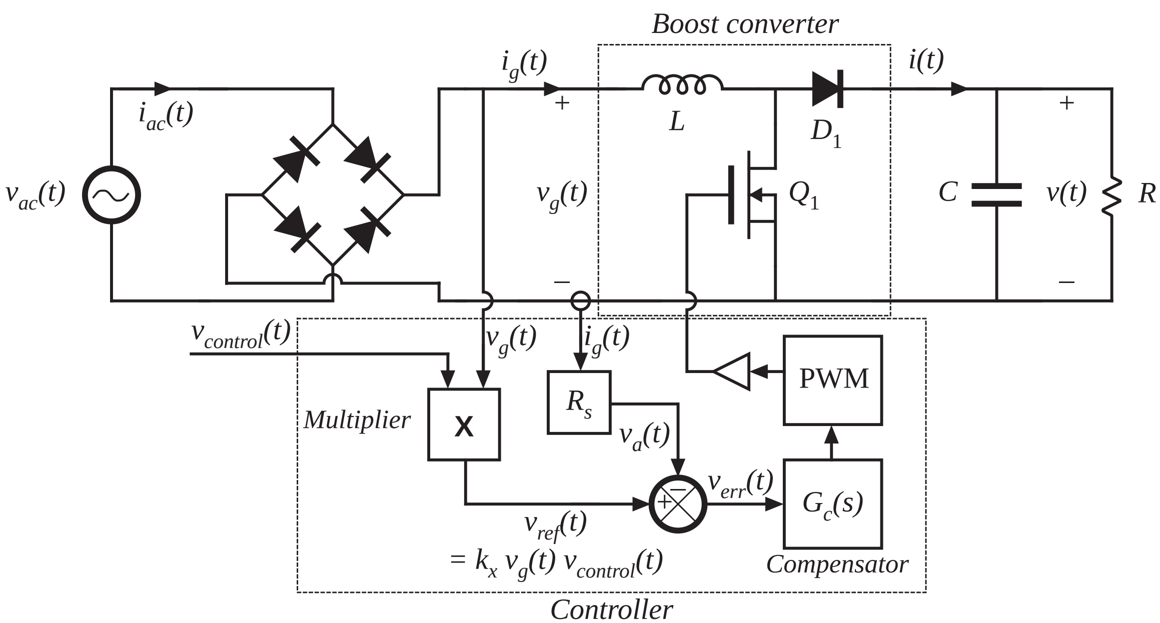
$$\frac{2L}{VT_s}\, i_g(t) \left(1 - \frac{v_g(t)}{V}\right) = d^2(t)\, \frac{v_g(t)}{V}$$

CCM when

$$\frac{2L}{VT_s}\, i_g(t) > \left(\frac{v_g(t)}{V}\right)\left(1 - \frac{v_g(t)}{V}\right)$$

R_e of the multiplying (average current) controller

Solve circuit to find R_e:



Current sensor gain

$$v_a(t) = i_g(t) R_s$$

when the error signal is small,

$$v_a(t) \approx v_{ref}(t)$$

multiplier equation

$$v_{ref}(t) = k_x v_g(t) v_{control}(t)$$

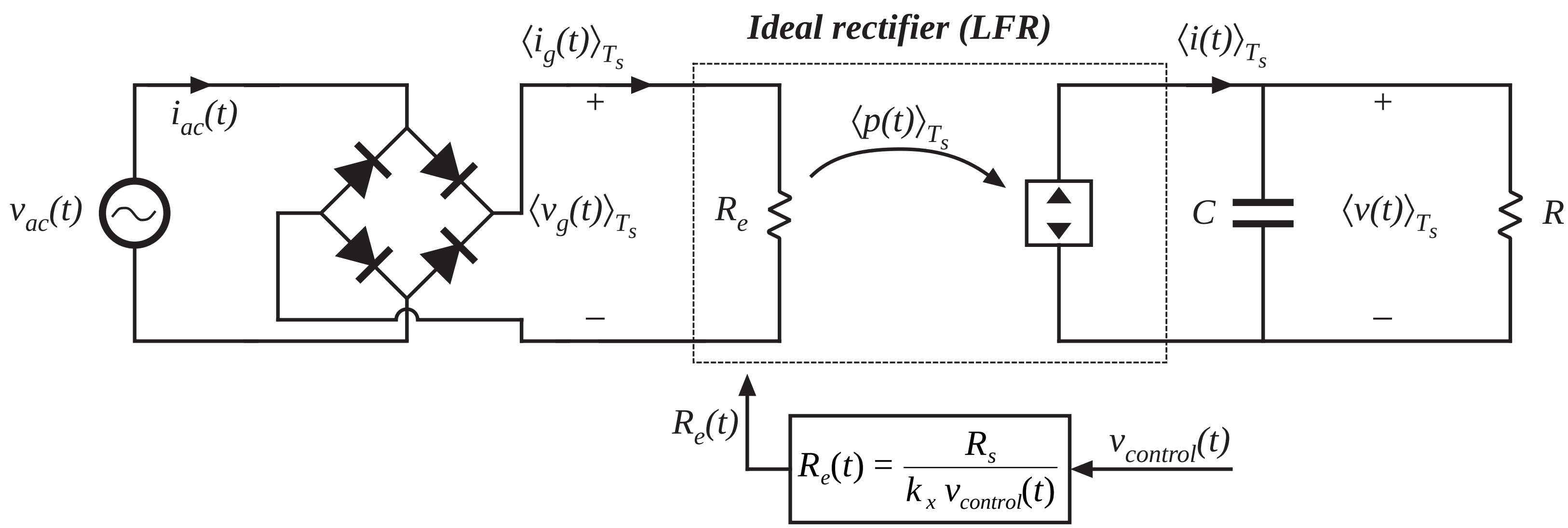
then R_e is

$$R_e = \frac{v_g(t)}{i_g(t)} = \frac{\left(\dfrac{v_{ref}(t)}{k_x v_{control}(t)}\right)}{\left(\dfrac{v_a(t)}{R_s}\right)}$$

simplify:

$$R_e(v_{control}(t)) = \frac{R_s}{k_x v_{control}(t)}$$

Low frequency system model



$$R_e(v_{control}(t)) = \frac{R_s}{k_x v_{control}(t)}$$

This model also applies to other converters that are controlled in the same manner, including buck-boost, Cuk, and SEPIC.

Open-loop DCM approach

We found in Chapter 10 that the buck-boost, SEPIC, and Cuk converters, when operated open-loop in DCM, inherently behave as loss-free resistors. This suggests that they could also be used as near-ideal rectifiers, without need for a multiplying controller.

Advantage: simple control

Disadvantages: higher peak currents, larger input current EMI

Like other DCM applications, this approach is usually restricted to low power (< 200W).

The boost converter can also be operated in DCM as a low harmonic rectifier. Input characteristic is

$$\left\langle i_g(t) \right\rangle_{T_s} = \frac{v_g(t)}{R_e} + \frac{v_g^2(t)}{R_e \left(v(t) - v_g(t) \right)}$$

Input current contains harmonics. If v is sufficiently greater than v_g, then harmonics are small.

17.3 Single-phase converter systems containing ideal rectifiers

- It is usually desired that the output voltage $v(t)$ be regulated with high accuracy, using a wide-bandwidth feedback loop
- For a given constant load characteristic, the instantaneous load current and power are then also constant:

$$p_{load}(t) = v(t)i(t) = VI$$

- The instantaneous input power of a single-phase ideal rectifier is not constant:

$$p_{ac}(t) = v_g(t)i_g(t)$$

with

$$v_g(t) = V_M \left| \sin(\omega t) \right| \qquad i_g(t) = \frac{v_g(t)}{R_e}$$

so

$$p_{ac}(t) = \frac{V_M^2}{R_e} \sin^2\left(\omega t\right) = \frac{V_M^2}{2R_e}\left(1 - \cos\left(2\omega t\right)\right)$$

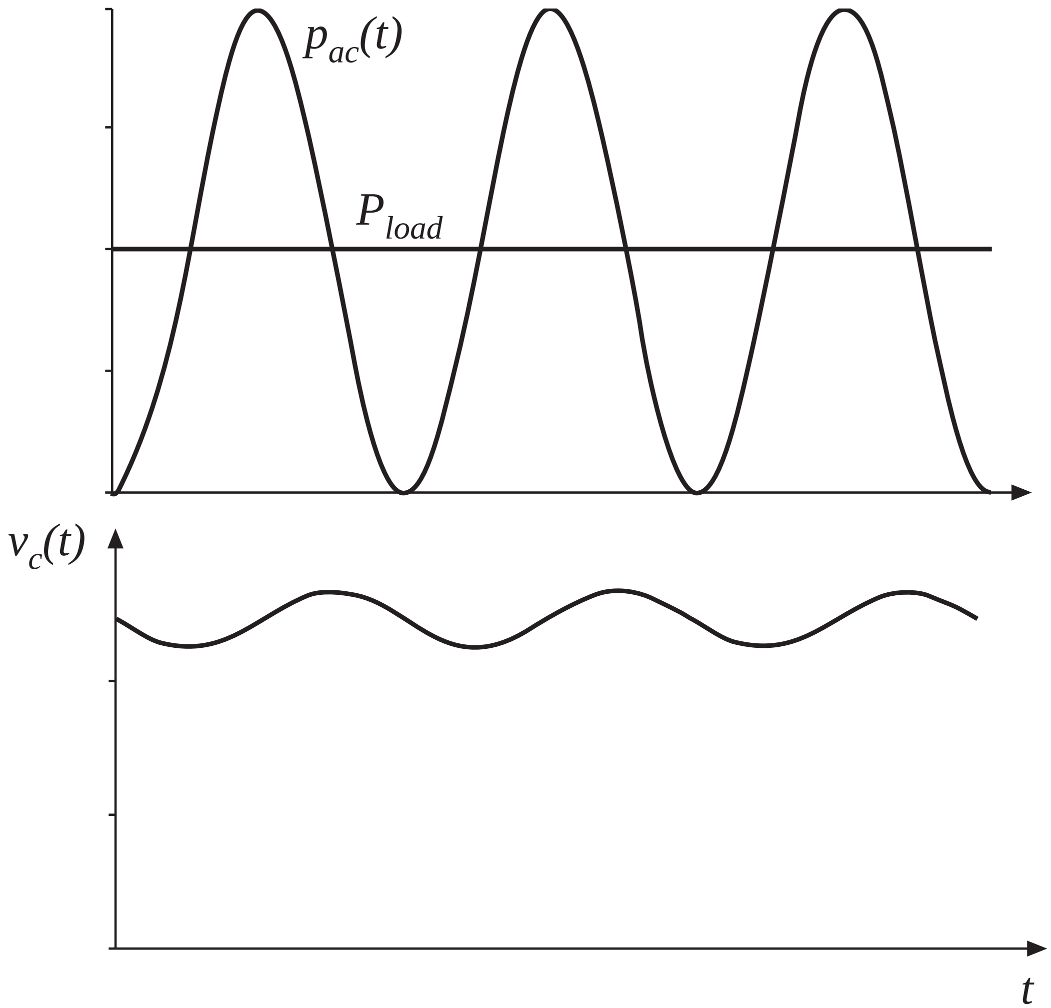
Power flow in single-phase ideal rectifier system

- Ideal rectifier is lossless, and contains no internal energy storage.
- Hence instantaneous input and output powers must be equal
- An energy storage element must be added
- Capacitor energy storage: instantaneous power flowing into capacitor is equal to difference between input and output powers:

$$p_C(t) = \frac{dE_C(t)}{dt} = \frac{d\left(\frac{1}{2}Cv_C^2(t)\right)}{dt} = p_{ac}(t) - p_{load}(t)$$

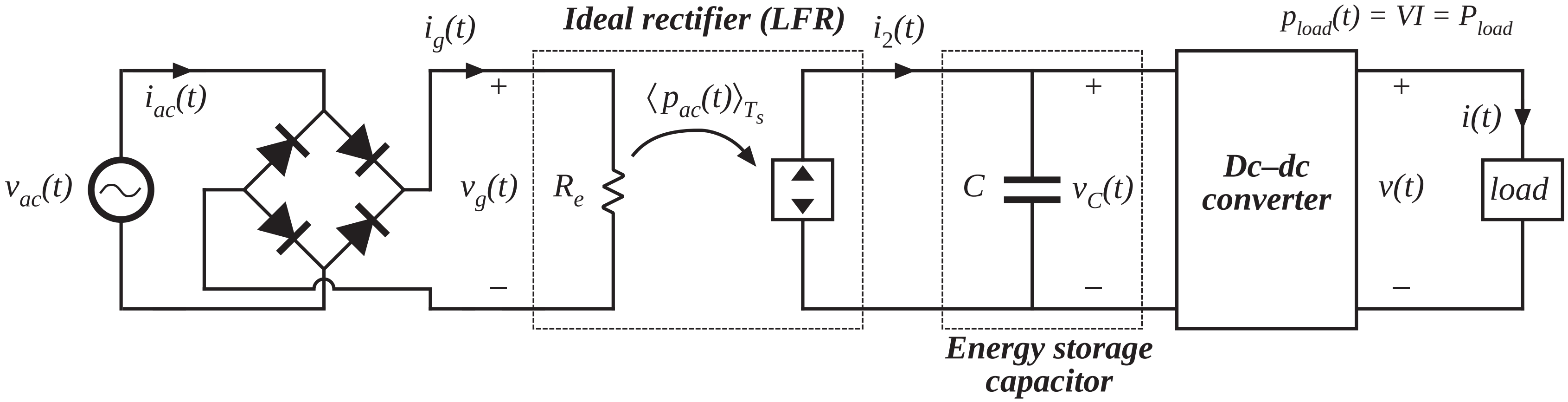
Energy storage capacitor voltage must be allowed to vary, in accordance with this equation

Capacitor energy storage in 1ø system



$$\frac{d\left(\frac{1}{2}\,C v_C^2(t)\right)}{dt} = p_{ac}(t) - p_{load}(t)$$

Single-phase system with internal energy storage



Energy storage capacitor voltage $v_C(t)$ must be independent of input and output voltage waveforms, so that it can vary according to

$$\frac{d\left(\frac{1}{2}\,Cv_C^2(t)\right)}{dt} = p_{ac}(t) - p_{load}(t)$$

This system is capable of

- Wide-bandwidth control of output voltage
- Wide-bandwidth control of input current waveform
- Internal independent energy storage

Hold up time

Internal energy storage allows the system to function in other situations where the instantaneous input and output powers differ.

A common example: continue to supply load power in spite of failure of ac line for short periods of time.

Hold up time: the duration which the dc output voltage $v(t)$ remains regulated after $v_{ac}(t)$ has become zero

A typical hold-up time requirement: supply load for one complete missing ac line cycle, or 20msec in a 50Hz system

During the hold-up time, the load power is supplied entirely by the energy storage capacitor

Energy storage element

Instead of a capacitor, and inductor or higher-order LC network could store the necessary energy.

But, inductors are not good energy-storage elements

Example

100V 100μF capacitor

100A 100μH inductor

each store 1 Joule of energy

But the capacitor is considerably smaller, lighter, and less expensive

So a single big capacitor is the best solution

Inrush current

A problem caused by the large energy storage capacitor: the large inrush current observed during system startup, necessary to charge the capacitor to its equilibrium value.

Boost converter is not capable of controlling this inrush current.

- Even with $d = 0$, a large current flows through the boost converter diode to the capacitor, as long as $v(t) < v_g(t)$.
- Additional circuitry is needed to limit the magnitude of this inrush current.

Converters having buck-boost characteristics are capable of controlling the inrush current. Unfortunately, these converters exhibit higher transistor stresses.

Universal input

The capability to operate from the ac line voltages and frequencies found everywhere in the world:

50Hz and 60Hz

Nominal rms line voltages of 100V to 260V:

100V, 110V, 115V, 120V, 132V, 200V, 220V, 230V, 240V, 260V

Regardless of the input voltage and frequency, the near-ideal rectifier produces a constant nominal dc output voltage. With a boost converter, this voltage is 380 or 400V.

Low-frequency model of dc-dc converter

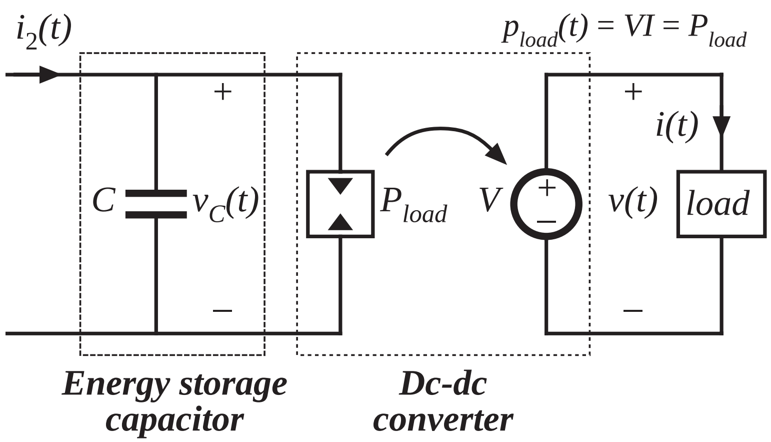
Dc-dc converter produces well-regulated dc load voltage V.

Load therefore draws constant current I.

Load power is therefore the constant value $P_{load} = VI$.

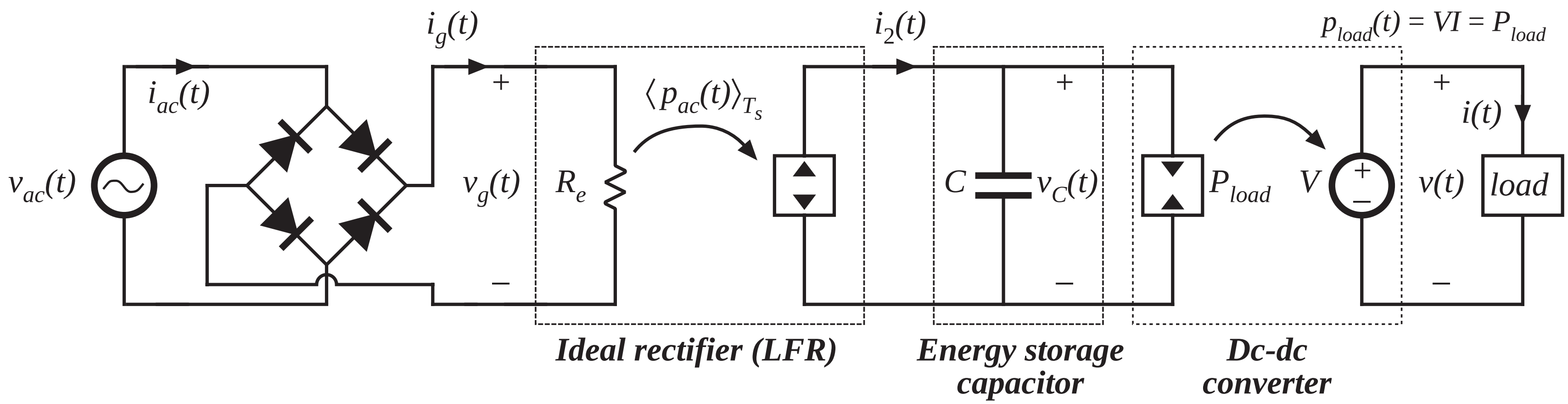
To the extent that dc-dc converter losses can be neglected, then dc-dc converter input power is P_{load} , regardless of capacitor voltage $v_c(t)$.

Dc-dc converter input port behaves as a power sink. A low frequency converter model is



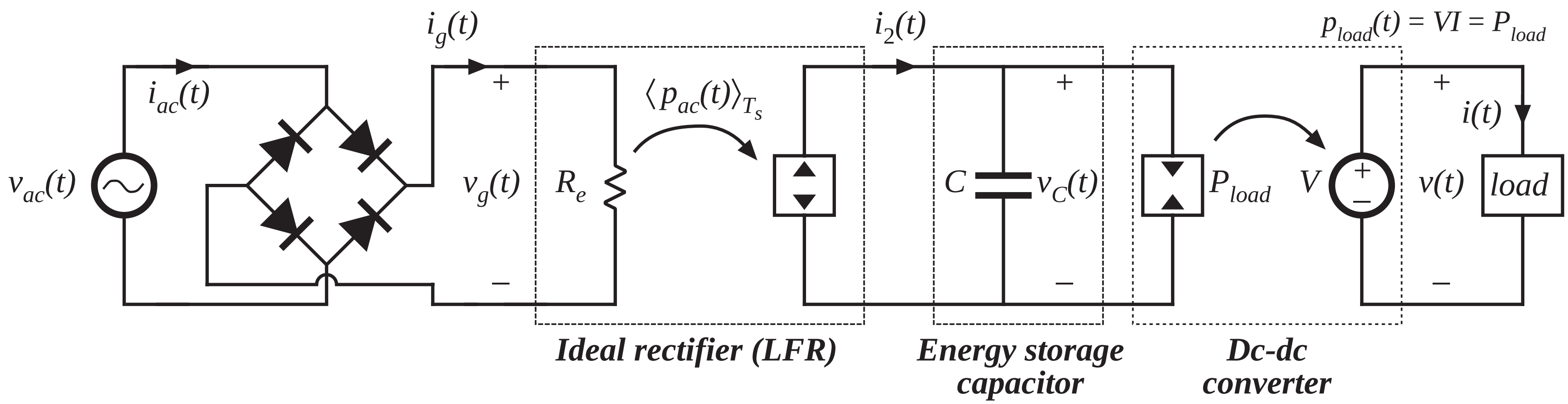
Low-frequency energy storage process, 1ø system

A complete low-frequency system model:



- Difference between rectifier output power and dc-dc converter input power flows into capacitor
- In equilibrium, average rectifier and load powers must be equal
- But the system contains no mechanism to accomplish this
- An additional feeback loop is necessary, to adjust R_e such that the rectifier average power is equal to the load power

Obtaining average power balance

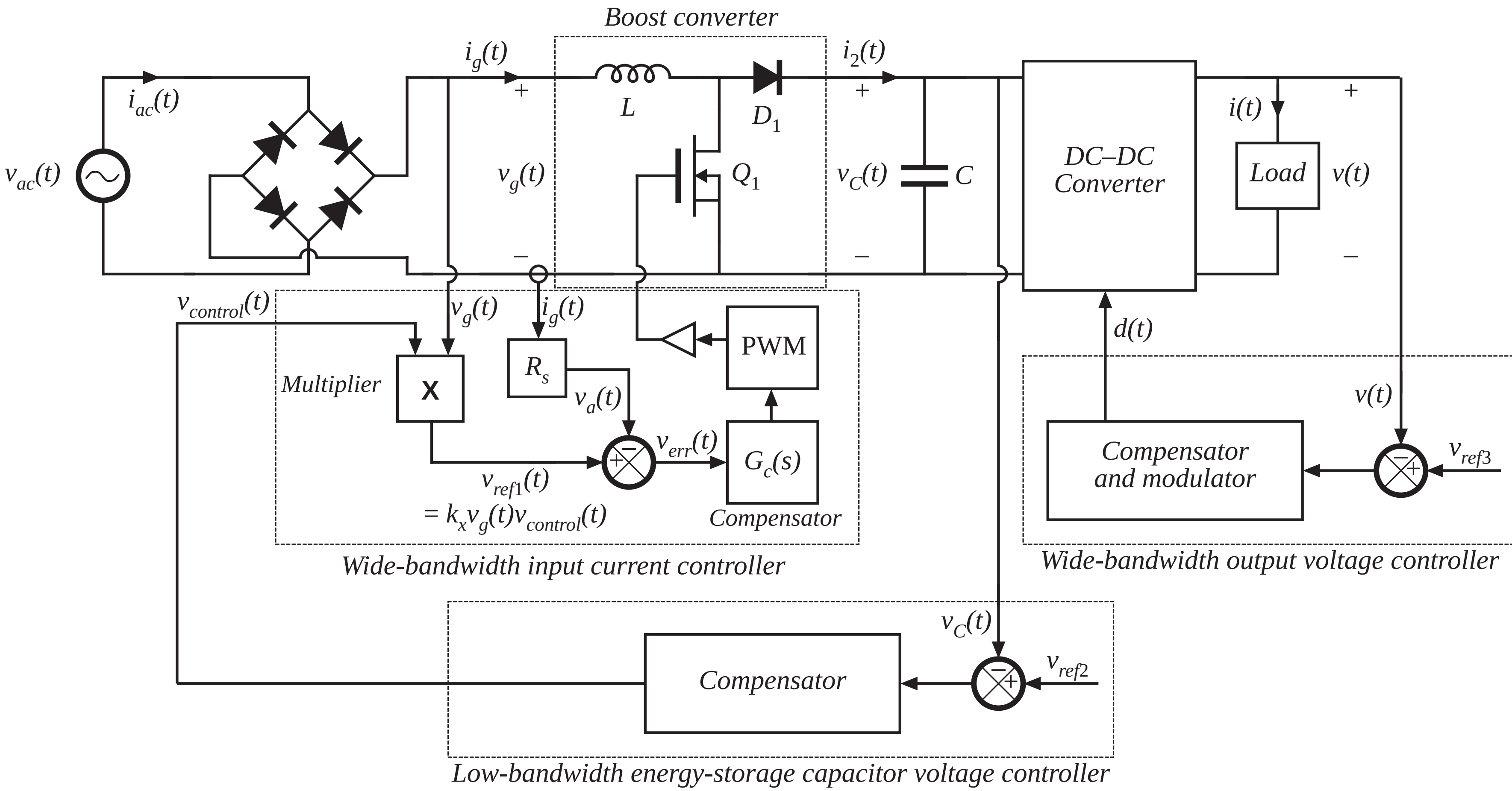


If the load power exceeds the average rectifier power, then there is a net discharge in capacitor energy and voltage over one ac line cycle.

There is a net increase in capacitor charge when the reverse is true.

This suggests that rectifier and load powers can be balanced by regulating the energy storage capacitor voltage.

A complete 1ø system
containing three feedback loops



Bandwidth of capacitor voltage loop

- The energy-storage-capacitor voltage feedback loop causes the dc component of $v_c(t)$ to be equal to some reference value
- Average rectifier power is controlled by variation of R_e.
- R_e must not vary too quickly; otherwise, ac line current harmonics are generated
- Extreme limit: loop has infinite bandwidth, and $v_c(t)$ is perfectly regulated to be equal to a constant reference value
  - Energy storage capacitor voltage then does not change, and this capacitor does not store or release energy
  - Instantaneous load and ac line powers are then equal
  - Input current becomes

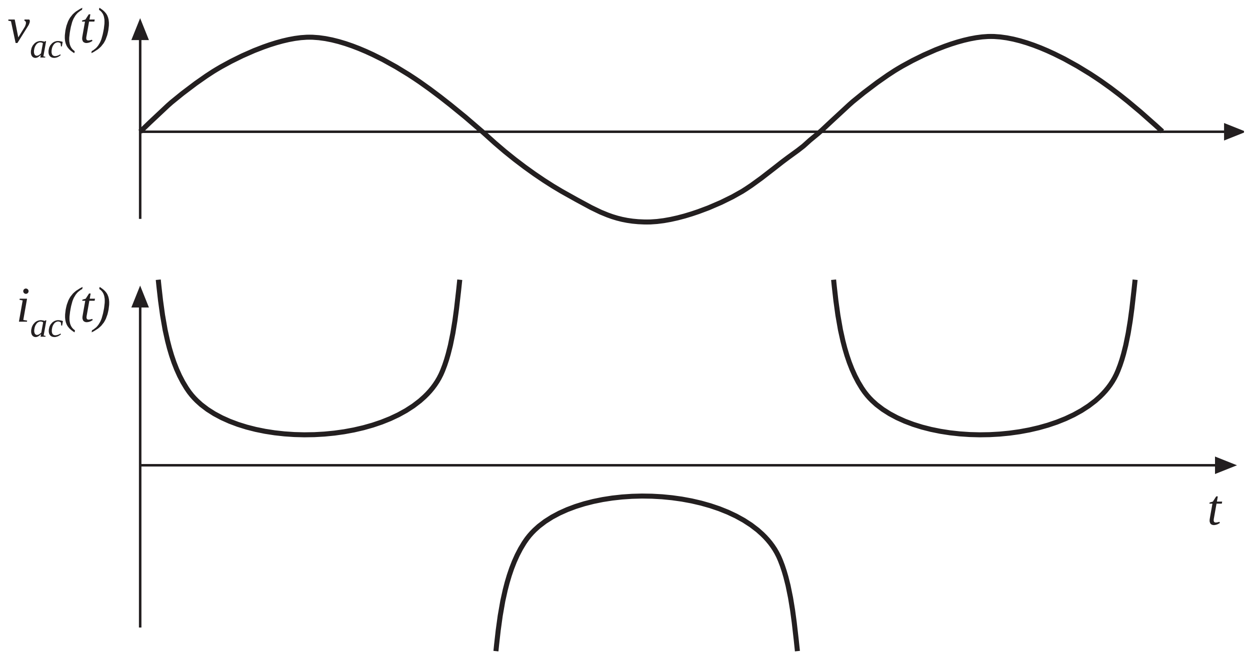
$$i_{ac}(t) = \frac{p_{ac}(t)}{v_{ac}(t)} = \frac{p_{load}(t)}{v_{ac}(t)} = \frac{P_{load}}{V_M \sin\left(\omega t\right)}$$

Input current waveform, extreme limit

$$i_{ac}(t) = \frac{p_{ac}(t)}{v_{ac}(t)} = \frac{p_{load}(t)}{v_{ac}(t)} = \frac{P_{load}}{V_M \sin\left(\omega t\right)}$$

THD → ∞

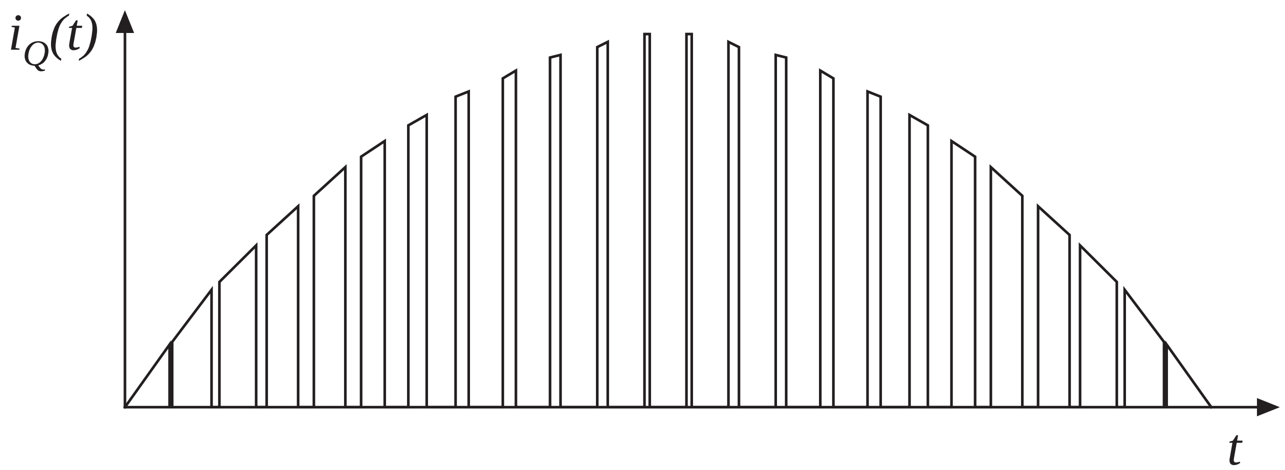
Power factor → 0



So bandwidth of capacitor voltage loop must be limited, and THD increases rapidly with increasing bandwidth

17.4 RMS values of rectifier waveforms

Doubly-modulated transistor current waveform, boost rectifier:



Computation of rms value of this waveform is complex and tedious

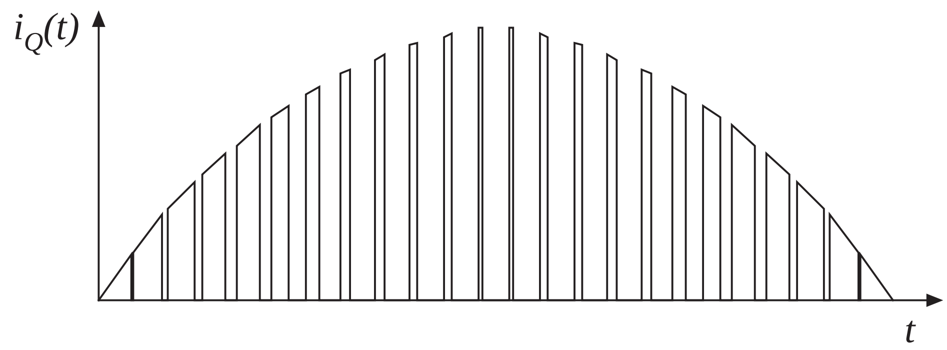
Approximate here using double integral

Generate tables of component rms and average currents for various rectifier converter topologies, and compare

RMS transistor current

RMS transistor current is

$$I_{Qrms} = \sqrt{\frac{1}{T_{ac}} \int_0^{T_{ac}} i_Q^2(t)\, dt}$$



Express as sum of integrals over all switching periods contained in one ac line period:

$$I_{Qrms} = \sqrt{\frac{1}{T_{ac}}\, T_s \sum_{n=1}^{T_{ac}/T_s} \left(\frac{1}{T_s} \int_{(n-1)T_s}^{nT_s} i_Q^2(t)\, dt \right)}$$

Quantity in parentheses is the value of i_Q^2, averaged over the n^{th} switching period.

Approximation of RMS expression

$$I_{Qrms} = \sqrt{\frac{1}{T_{ac}} T_s \sum_{n=1}^{T_{ac}/T_s} \left(\frac{1}{T_s} \int_{(n-1)T_s}^{nT_s} i_Q^2(t)\,dt \right)}$$

When $T_s << T_{ac}$, then the summation can be approximated by an integral, which leads to the double-average:

$$I_{Qrms} \approx \sqrt{\frac{1}{T_{ac}} \lim_{T_s \to 0} \left[T_s \sum_{n=1}^{T_{ac}/T_s} \left(\frac{1}{T_s} \int_{(n-1)T_s}^{nT_s} i_Q^2(\tau)\,d\tau \right) \right]}$$

$$= \sqrt{\frac{1}{T_{ac}} \int_0^{T_{ac}} \frac{1}{T_s} \int_t^{t+T_s} i_Q^2(\tau)\,d\tau\,dt}$$

$$= \sqrt{\left\langle \left\langle i_Q^2(t) \right\rangle_{T_s} \right\rangle_{T_{ac}}}$$

17.4.1 Boost rectifier example

For the boost converter, the transistor current $i_Q(t)$ is equal to the input current when the transistor conducts, and is zero when the transistor is off. The average over one switching period of $i_Q^2(t)$ is therefore

$$\left\langle i_Q^2 \right\rangle_{T_s} = \frac{1}{T_s}\int_t^{t+T_s} i_Q^2(t)\,dt$$

$$= d(t)\, i_{ac}^2(t)$$

If the input voltage is

$$v_{ac}(t) = V_M \left| \sin \omega t \right|$$

then the input current will be given by

$$i_{ac}(t) = \frac{V_M}{R_e} \left| \sin \omega t \right|$$

and the duty cycle will ideally be

$$\frac{V}{v_{ac}(t)} = \frac{1}{1 - d(t)}$$

(this neglects converter dynamics)

Boost rectifier example

Duty cycle is therefore

$$d(t) = 1 - \frac{V_M}{V} \left| \sin \omega t \right|$$

Evaluate the first integral:

$$\left\langle i_Q^2 \right\rangle_{T_s} = \frac{V_M^2}{R_e^2} \left(1 - \frac{V_M}{V} \left| \sin \omega t \right| \right) \sin^2\left(\omega t\right)$$

Now plug this into the RMS formula:

$$I_{Qrms} = \sqrt{\frac{1}{T_{ac}} \int_0^{T_{ac}} \left\langle i_Q^2 \right\rangle_{T_s} dt}$$

$$= \sqrt{\frac{1}{T_{ac}} \int_0^{T_{ac}} \frac{V_M^2}{R_e^2} \left(1 - \frac{V_M}{V} \left| \sin \omega t \right| \right) \sin^2\left(\omega t\right) dt}$$

$$I_{Qrms} = \sqrt{\frac{2}{T_{ac}} \frac{V_M^2}{R_e^2} \int_0^{T_{ac}/2} \left(\sin^2\left(\omega t\right) - \frac{V_M}{V} \sin^3\left(\omega t\right) \right) dt}$$

Integration of powers of sin θ over complete half-cycle

$$\frac{1}{\pi}\int_0^{\pi} sin^n(\theta)d\theta = \begin{cases} \dfrac{2}{\pi}\dfrac{2\cdot4\cdot6\cdots(n-1)}{1\cdot3\cdot5\cdots n} & \text{if } n \text{ is odd} \\ \dfrac{1\cdot3\cdot5\cdots(n-1)}{2\cdot4\cdot6\cdots n} & \text{if } n \text{ is even} \end{cases}$$

n	$\frac{1}{\pi}\int_0^{\pi} sin^n(\theta)d\theta$
1	$\frac{2}{\pi}$
2	$\frac{1}{2}$
3	$\frac{4}{3\pi}$
4	$\frac{3}{8}$
5	$\frac{16}{15\pi}$
6	$\frac{15}{48}$

Boost example: Transistor RMS current

$$I_{Qrms} = \frac{V_M}{\sqrt{2}R_e}\sqrt{1 - \frac{8}{3\pi}\frac{V_M}{V}} = I_{ac\,rms}\sqrt{1 - \frac{8}{3\pi}\frac{V_M}{V}}$$

Transistor RMS current is minimized by choosing V as small as possible: $V = V_M$. This leads to

$$I_{Qrms} = 0.39 I_{ac\,rms}$$

When the dc output voltage is not too much greater than the peak ac input voltage, the boost rectifier exhibits very low transistor current. Efficiency of the boost rectifier is then quite high, and 95% is typical in a 1kW application.

Table of rectifier current stresses for various topologies

Table 17.2 Summary of rectifier current stresses for several converter topologies

	rms	*Average*	*Peak*
CCM boost			
Transistor	$I_{ac\,rms}\sqrt{1-\frac{8}{3\pi}\frac{V_M}{V}}$	$I_{ac\,rms}\frac{2\sqrt{2}}{\pi}\left(1-\frac{\pi}{8}\frac{V_M}{V}\right)$	$I_{ac\,rms}\sqrt{2}$
Diode	$I_{dc}\sqrt{\frac{16}{3\pi}\frac{V}{V_M}}$	I_{dc}	$2\,I_{dc}\frac{V}{V_M}$
Inductor	$I_{ac\,rms}$	$I_{ac\,rms}\frac{2\sqrt{2}}{\pi}$	$I_{ac\,rms}\sqrt{2}$
CCM flyback, with n:1 isolation transformer and input filter			
Transistor, xfmr primary	$I_{ac\,rms}\sqrt{1+\frac{8}{3\pi}\frac{V_M}{nV}}$	$I_{ac\,rms}\frac{2\sqrt{2}}{\pi}$	$I_{ac\,rms}\sqrt{2}\left(1+\frac{V}{n}\right.$
L_1	$I_{ac\,rms}$	$I_{ac\,rms}\frac{2\sqrt{2}}{\pi}$	$I_{ac\,rms}\sqrt{2}$
C_1	$I_{ac\,rms}\sqrt{\frac{8}{3\pi}\frac{V_M}{nV}}$	0	$I_{ac\,rms}\sqrt{2}\max\left(1,\frac{V_M}{nV}\right.$
Diode, xfmr secondary	$I_{dc}\sqrt{\frac{3}{2}+\frac{16}{3\pi}\frac{nV}{V_M}}$	I_{dc}	$2I_{dc}\left(1+\frac{nV}{V_M}\right)$

Table of rectifier current stresses
continued

CCM SEPIC, nonisolated			
Transistor	$I_{ac\,rms}\sqrt{1+\frac{8}{3\pi}\frac{V_M}{V}}$	$I_{ac\,rms}\frac{2\sqrt{2}}{\pi}$	$I_{ac\,rms}\sqrt{2}\left(1+\frac{V_{l}}{V}\right.$
L_1	$I_{ac\,rms}$	$I_{ac\,rms}\frac{2\sqrt{2}}{\pi}$	$I_{ac\,rms}\sqrt{2}$
C_1	$I_{ac\,rms}\sqrt{\frac{8}{3\pi}\frac{V_M}{V}}$	0	$I_{ac\,rms}\max\left(1,\frac{V_{l}}{V}\right.$
L_2	$I_{ac\,rms}\frac{V_M}{V}\frac{\sqrt{3}}{2}$	$\frac{I_{ac\,rms}}{\sqrt{2}}\frac{V_M}{V}$	$I_{ac\,rms}\frac{V_M}{V}\sqrt{2}$
Diode	$I_{dc}\sqrt{\frac{3}{2}+\frac{16}{3\pi}\frac{V}{V_M}}$	I_{dc}	$2I_{dc}\left(1+\frac{V}{V_M}\right)$
CCM SEPIC, with n:1 isolation transformer			
transistor	$I_{ac\,rms}\sqrt{1+\frac{8}{3\pi}\frac{V_M}{nV}}$	$I_{ac\,rms}\frac{2\sqrt{2}}{\pi}$	$I_{ac\,rms}\sqrt{2}\left(1+\frac{V_{l}}{nV}\right.$
L_1	$I_{ac\,rms}$	$I_{ac\,rms}\frac{2\sqrt{2}}{\pi}$	$I_{ac\,rms}\sqrt{2}$
C_1, xfmr primary	$I_{ac\,rms}\sqrt{\frac{8}{3\pi}\frac{V_M}{nV}}$	0	$I_{ac\,rms}\sqrt{2}\max\left(1,\right.$
Diode, xfmr secondary	$I_{dc}\sqrt{\frac{3}{2}+\frac{16}{3\pi}\frac{nV}{V_M}}$	I_{dc}	$2I_{dc}\left(1+\frac{nV}{V_M}\right)$

with, in all cases, $\frac{I_{ac\,rms}}{I_{dc}}=\sqrt{2}\frac{V}{V_M}$, ac input voltage = $V_M\sin(\omega t)$

dc output voltage = V

Comparison of rectifier topologies

Boost converter

- Lowest transistor rms current, highest efficiency
- Isolated topologies are possible, with higher transistor stress
- No limiting of inrush current
- Output voltage must be greater than peak input voltage

Buck-boost, SEPIC, and Cuk converters

- Higher transistor rms current, lower efficiency
- Isolated topologies are possible, without increased transistor stress
- Inrush current limiting is possible
- Output voltage can be greater than or less than peak input voltage

Comparison of rectifier topologies

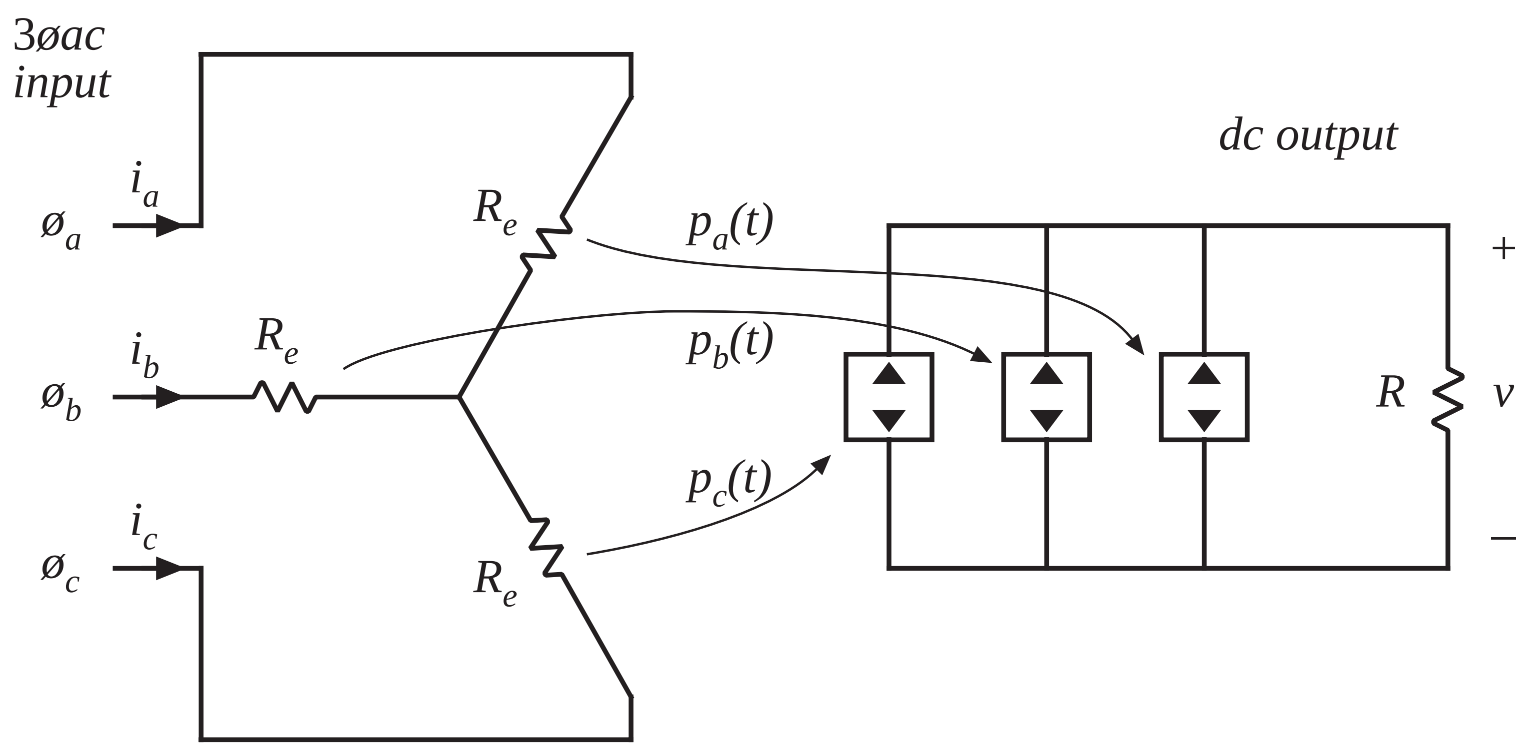
1kW, 240Vrms example. Output voltage: 380Vdc. Input current: 4.2Arms

Converter	Transistor rms current	Transistor voltage	Diode rms current	Transistor rms current, 120V	Diode rms current, 120V
Boost	2 A	380 V	3.6 A	6.6 A	5.1 A
Nonisolated SEPIC	5.5 A	719 V	4.85 A	9.8 A	6.1 A
Isolated SEPIC	5.5 A	719 V	36.4 A	11.4 A	42.5 A

Isolated SEPIC example has 4:1 turns ratio, with 42V 23.8A dc load

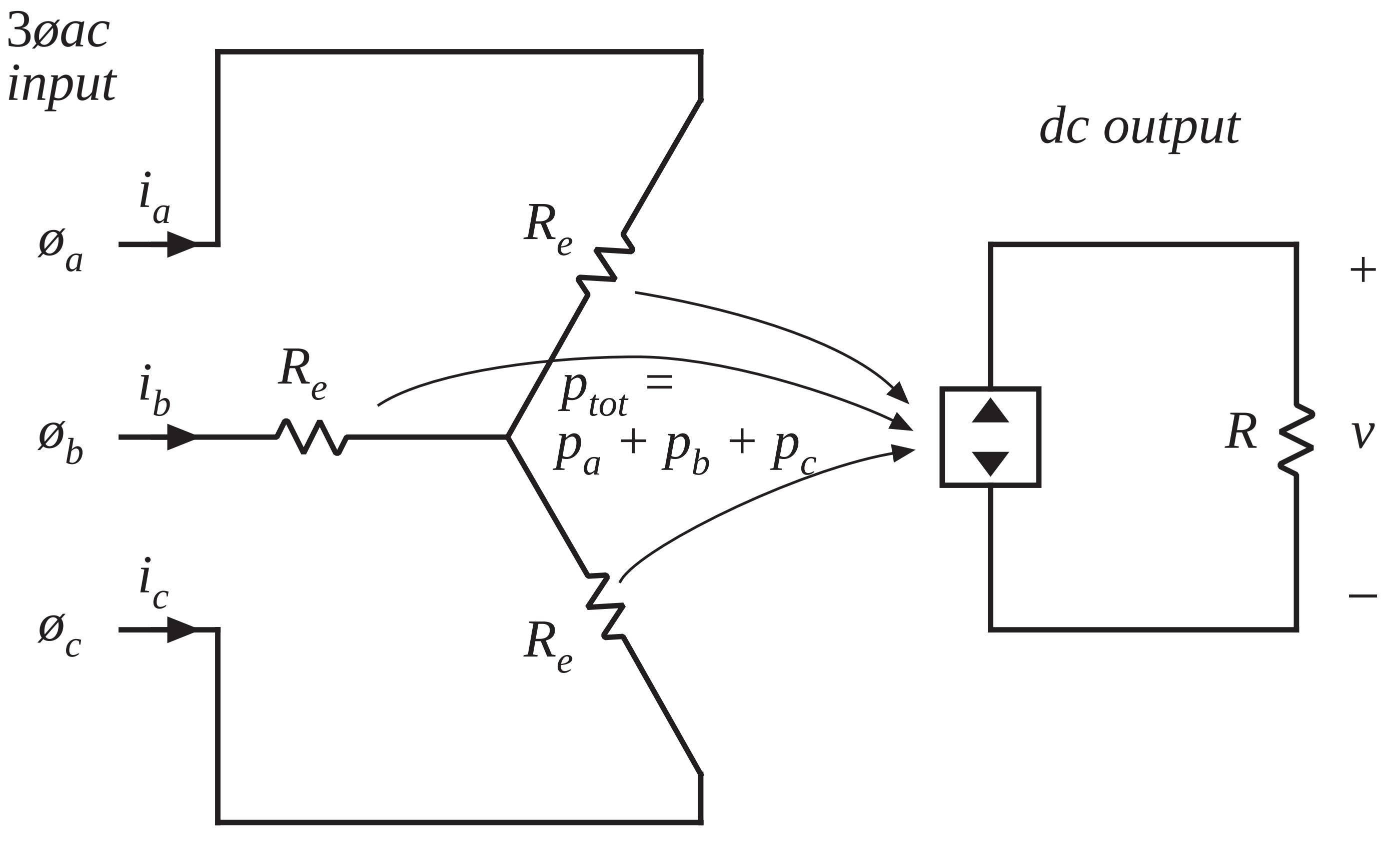
17.5 Ideal three-phase rectifiers

Ideal 3ø rectifier, modeled as three 1ø ideal rectifiers:



Ideal 3ø rectifier model

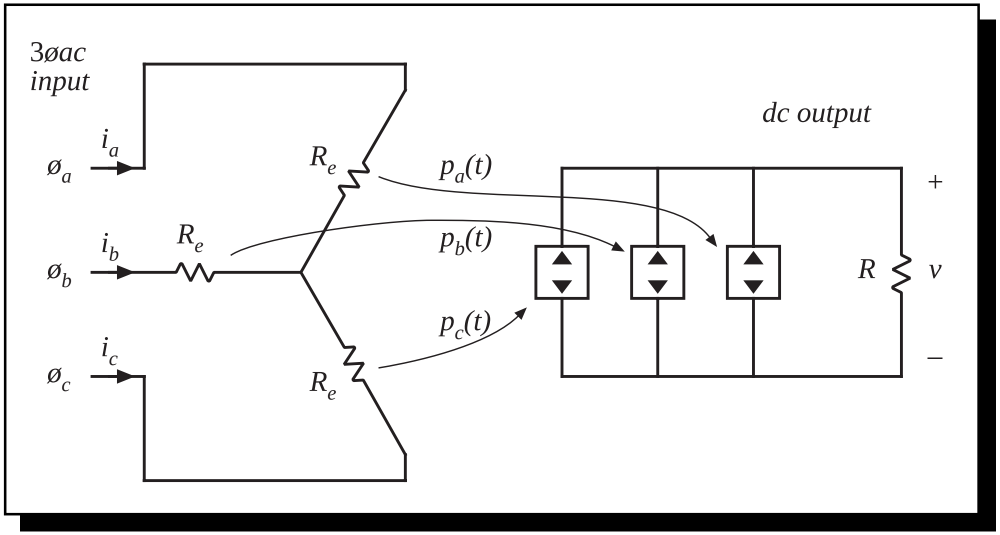
Combine parallel-connected power sources into a single source $p_{tot}(t)$:



Value of $p_{tot}(t)$

Ac input voltages:

$$v_{an}(t) = V_M \sin\left(\omega t\right)$$
$$v_{bn}(t) = V_M \sin\left(\omega t - 120°\right)$$
$$v_{cn}(t) = V_M \sin\left(\omega t - 240°\right)$$



Instantaneous phase powers:

$$p_a(t) = \frac{v_{an}^2(t)}{R_e} = \frac{V_M^2}{2R_e}\left(1 - \cos\left(2\omega t\right)\right)$$
$$p_b(t) = \frac{v_{bn}^2(t)}{R_e} = \frac{V_M^2}{2R_e}\left(1 - \cos\left(2\omega t - 240°\right)\right)$$
$$p_c(t) = \frac{v_{cn}^2(t)}{R_e} = \frac{V_M^2}{2R_e}\left(1 - \cos\left(2\omega t - 120°\right)\right)$$

Total 3ø instantaneous power:

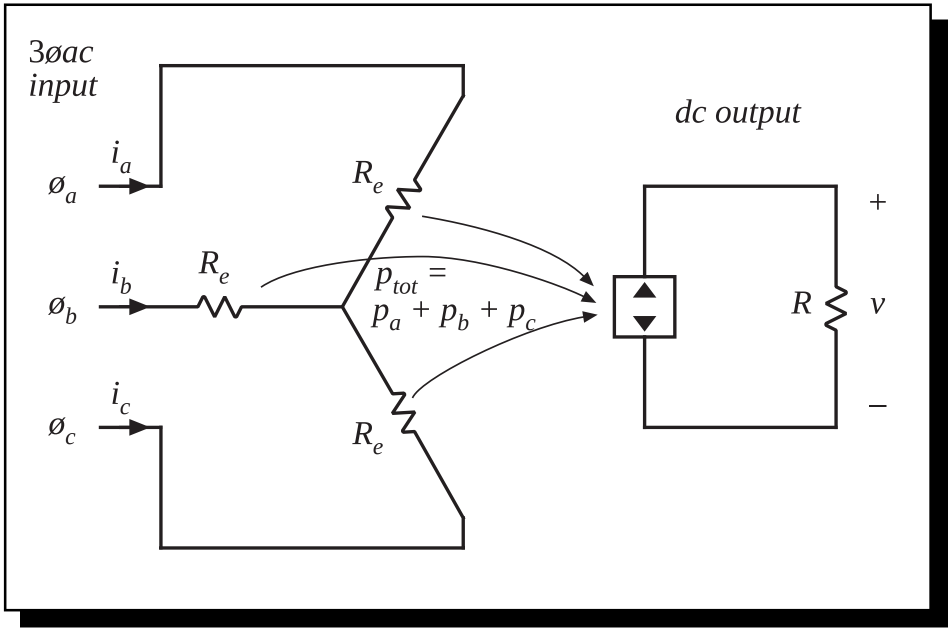
$$p_{tot}(t) = p_a(t) + p_b(t) + p_c(t) = \frac{3}{2}\,\frac{V_M^2}{R_e}$$

- 2nd harmonic terms add to zero
- total 3ø power $p_{tot}(t)$ is constant

Instantaneous power in ideal 3ø rectifier

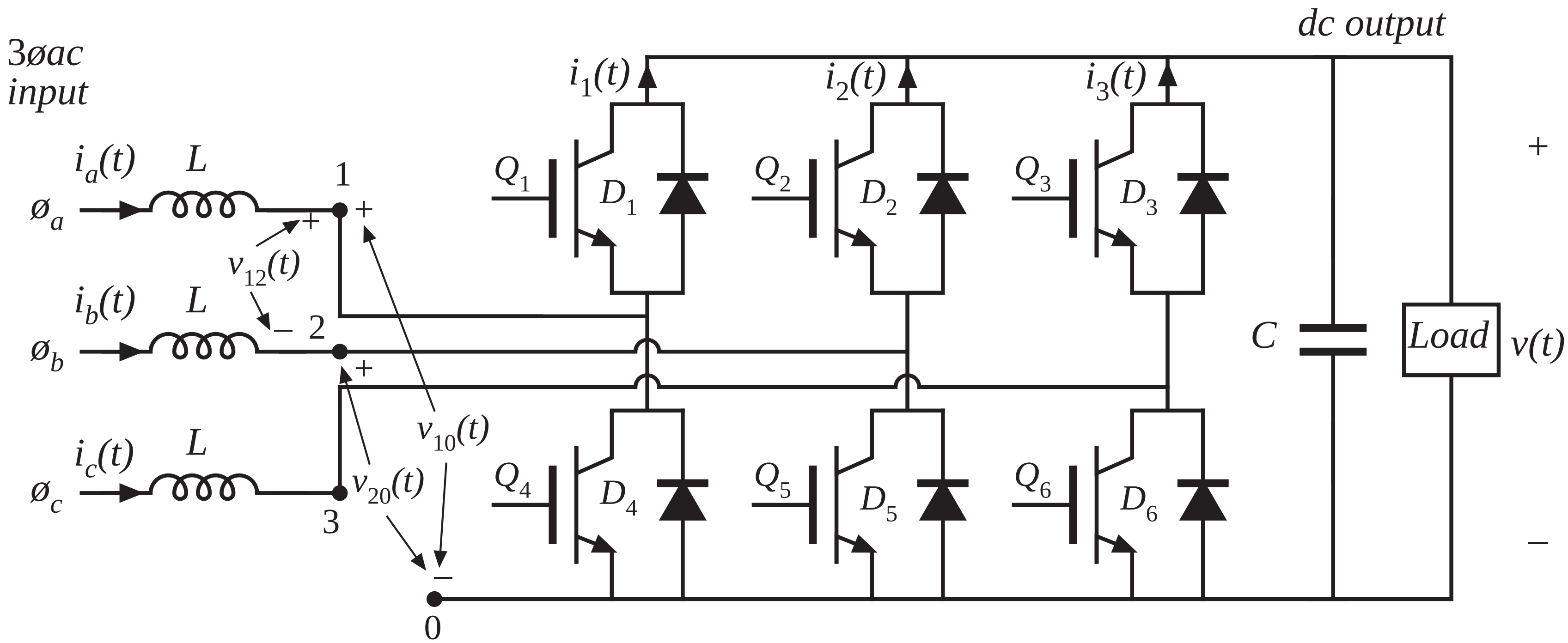
$$p_{tot}(t) = p_a(t) + p_b(t) + p_c(t) = \frac{3}{2}\,\frac{V_M^2}{R_e}$$

- In a balanced system, the ideal 3ø rectifier supplies constant power to its dc output
- a constant power load can be supplied, without need for low-frequency internal energy storage



17.5.1 Three-phase rectifiers operating in CCM

3øac–dc boost rectifier



- Uses six current-bidirectional switches
- Operation of each individual phase is similar to the 1ø boost rectifier

The 3øac–dc boost rectifier

- Voltage-source inverter, operated backwards as a rectifier
- Converter is capable of bidirectional power flow
- Dc output voltage V must be greater than peak ac line-line voltage $V_{L,pk}$.
- Ac input currents are nonpulsating. In CCM, input EMI filtering is relatively easy
- Very low RMS transistor currents and conduction loss
- The leading candidate to replace uncontrolled 3ø rectifiers
- Requires six active devices
- Cannot regulate output voltage down to zero:

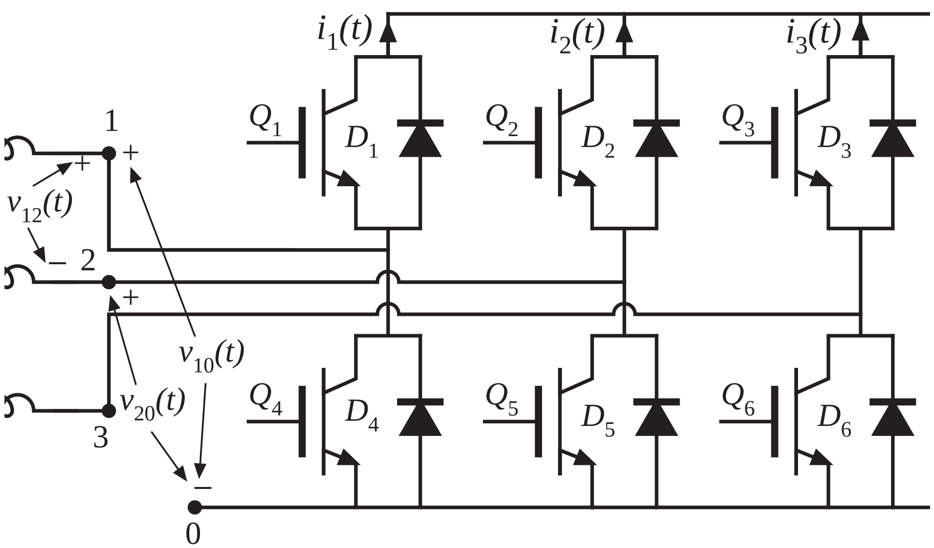
    no current limiting

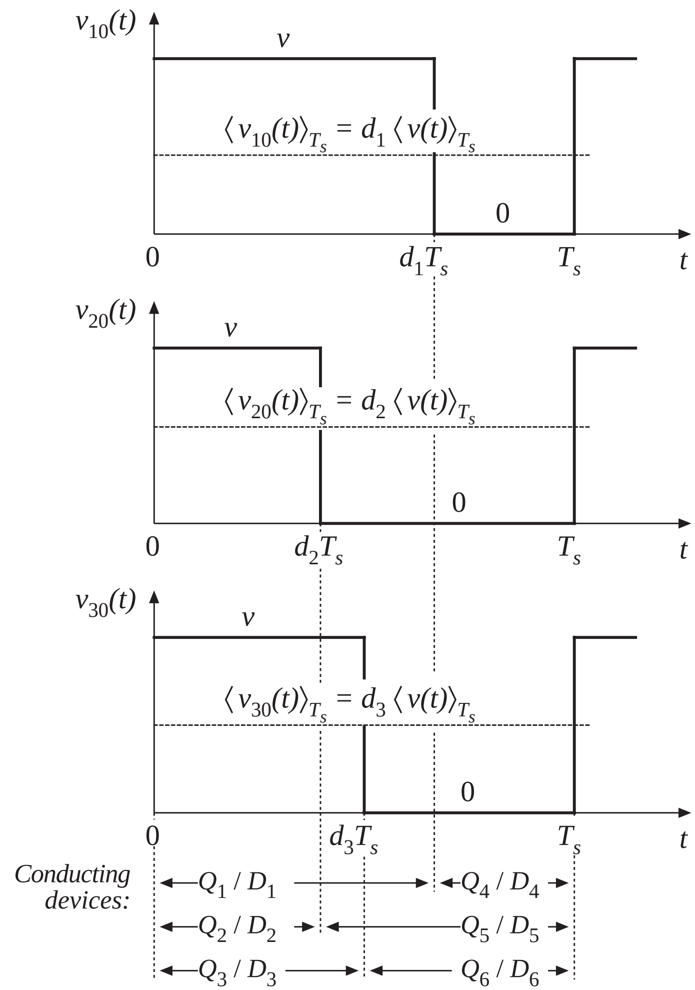
    cannot replace traditional buck-type controlled rectifiers

Control of switches in CCM 3øac-dc boost rectifier

Pulse-width modulation:

Drive lower transistors ($Q_4 - Q_6$) with complements of duty cycles of respective upper transistors ($Q_1 - Q_3$). Each phase operates independently, with its own duty cycle.





Average switch waveforms

Average the switch voltages:

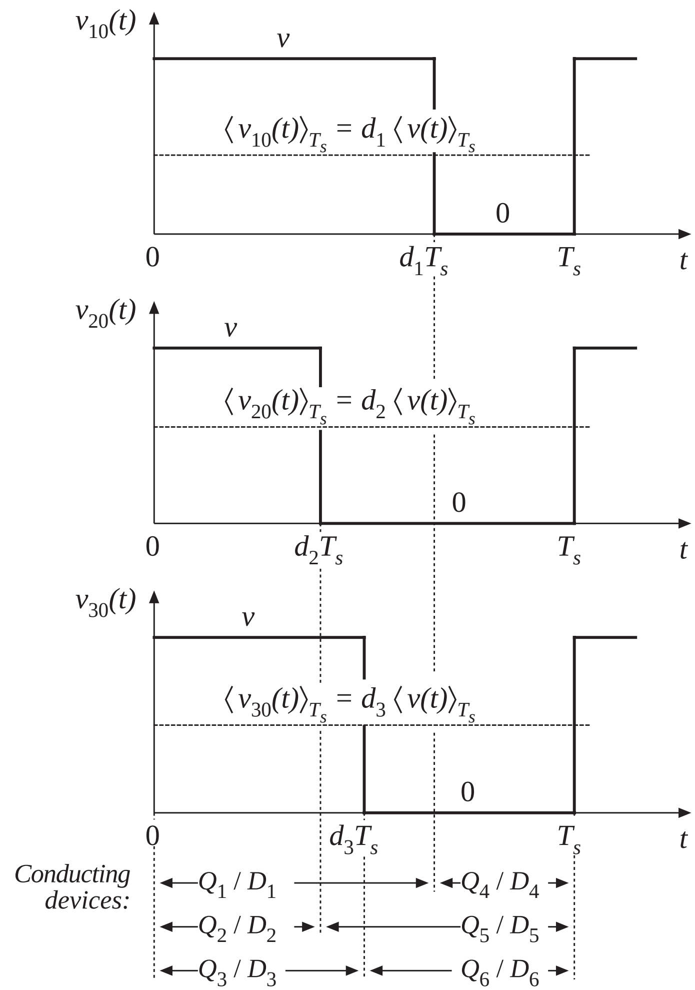
$$\left\langle v_{10}(t) \right\rangle_{T_s} = d_1(t) \left\langle v(t) \right\rangle_{T_s}$$
$$\left\langle v_{20}(t) \right\rangle_{T_s} = d_2(t) \left\langle v(t) \right\rangle_{T_s}$$
$$\left\langle v_{30}(t) \right\rangle_{T_s} = d_3(t) \left\langle v(t) \right\rangle_{T_s}$$

Average line-line voltages:

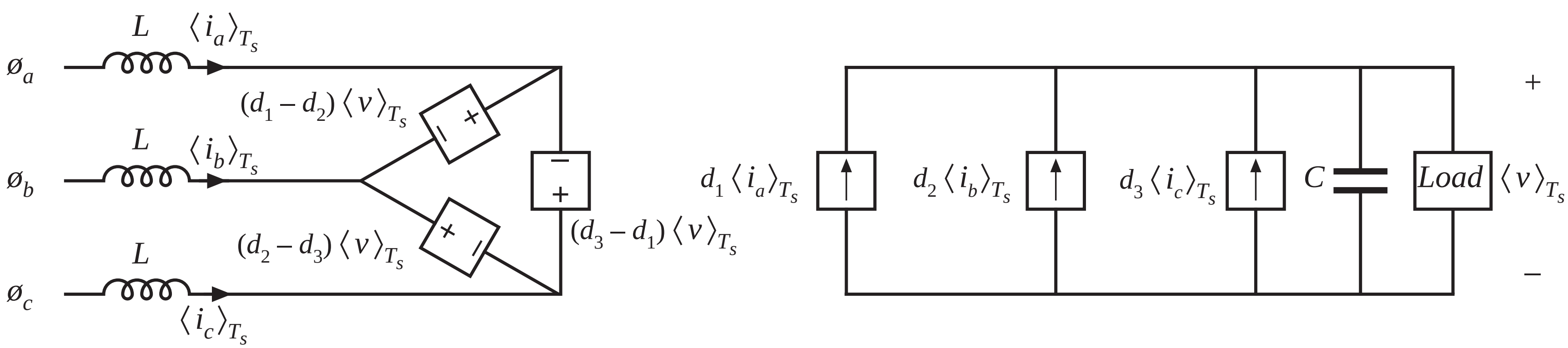
$$\left\langle v_{12}(t) \right\rangle_{T_s} = \left\langle v_{10}(t) \right\rangle_{T_s} - \left\langle v_{20}(t) \right\rangle_{T_s} = \left(d_1(t) - d_2(t) \right) \left\langle v(t) \right\rangle_{T_s}$$
$$\left\langle v_{23}(t) \right\rangle_{T_s} = \left\langle v_{20}(t) \right\rangle_{T_s} - \left\langle v_{30}(t) \right\rangle_{T_s} = \left(d_2(t) - d_3(t) \right) \left\langle v(t) \right\rangle_{T_s}$$
$$\left\langle v_{31}(t) \right\rangle_{T_s} = \left\langle v_{30}(t) \right\rangle_{T_s} - \left\langle v_{10}(t) \right\rangle_{T_s} = \left(d_3(t) - d_1(t) \right) \left\langle v(t) \right\rangle_{T_s}$$

Average switch output-side currents:

$$\left\langle i_1(t) \right\rangle_{T_s} = d_1(t) \left\langle i_a(t) \right\rangle_{T_s}$$
$$\left\langle i_2(t) \right\rangle_{T_s} = d_2(t) \left\langle i_b(t) \right\rangle_{T_s}$$
$$\left\langle i_3(t) \right\rangle_{T_s} = d_3(t) \left\langle i_c(t) \right\rangle_{T_s}$$



Averaged circuit model



$$\left\langle v_{12}(t)\right\rangle_{T_s} = \left\langle v_{10}(t)\right\rangle_{T_s} - \left\langle v_{20}(t)\right\rangle_{T_s} = \left(d_1(t) - d_2(t)\right)\left\langle v(t)\right\rangle_{T_s}$$
$$\left\langle v_{23}(t)\right\rangle_{T_s} = \left\langle v_{20}(t)\right\rangle_{T_s} - \left\langle v_{30}(t)\right\rangle_{T_s} = \left(d_2(t) - d_3(t)\right)\left\langle v(t)\right\rangle_{T_s}$$
$$\left\langle v_{31}(t)\right\rangle_{T_s} = \left\langle v_{30}(t)\right\rangle_{T_s} - \left\langle v_{10}(t)\right\rangle_{T_s} = \left(d_3(t) - d_1(t)\right)\left\langle v(t)\right\rangle_{T_s}$$

$$\left\langle i_1(t)\right\rangle_{T_s} = d_1(t)\left\langle i_a(t)\right\rangle_{T_s}$$
$$\left\langle i_2(t)\right\rangle_{T_s} = d_2(t)\left\langle i_b(t)\right\rangle_{T_s}$$
$$\left\langle i_3(t)\right\rangle_{T_s} = d_3(t)\left\langle i_c(t)\right\rangle_{T_s}$$

Q: How to vary $d(t)$ such that the desired ac and dc waveforms are obtained?

Solution is not unique.

Sinusoidal PWM

A simple modulation scheme: Sinusoidal PWM

Vary duty cycles sinusoidally, in synchronism with ac line

$$d_1(t) = D_0 + \tfrac{1}{2} D_m \sin\left(\omega t - \varphi\right)$$

$$d_2(t) = D_0 + \tfrac{1}{2} D_m \sin\left(\omega t - \varphi - 120°\right)$$

$$d_3(t) = D_0 + \tfrac{1}{2} D_m \sin\left(\omega t - \varphi - 240°\right)$$

where

ω is the ac line frequency

D_0 is a dc bias

D_m is the *modulation index*

For $D_0 = 0.5$, D_m in the above equations must be less than 1.

The modulation index is defined as one-half of the peak amplitude of the fundamental component of the duty cycle modulation. In some other modulation schemes, it is possible that $D_m > 1$.

Solution, linear sinusoidal PWM

If the switching frequency is high, then the inductors can be small and have negligible effect at the ac line frequency. The averaged switch voltage and ac line voltage are then equal:

$$\left\langle v_{12}(t) \right\rangle_{T_s} = \left(d_1(t) - d_2(t) \right) \left\langle v(t) \right\rangle_{T_s} \approx v_{ab}(t)$$

Substitute expressions for duty cycle and ac line voltage variations:

$$\tfrac{1}{2} D_m \left[\sin\left(\omega t - \varphi\right) - \sin\left(\omega t - \varphi - 120°\right) \right] \left\langle v(t) \right\rangle_{T_s} = V_M \left[\sin\left(\omega t\right) - \sin\left(\omega t - 120°\right) \right]$$

For small L, φ tends to zero. The expression then becomes

$$\tfrac{1}{2} D_m V = V_M$$

Solve for the output voltage:

$$V = \frac{2V_M}{D_m} \qquad\qquad V = \frac{2}{\sqrt{3}} \frac{V_{L,pk}}{D_m} = 1.15 \frac{V_{L,pk}}{D_m}$$

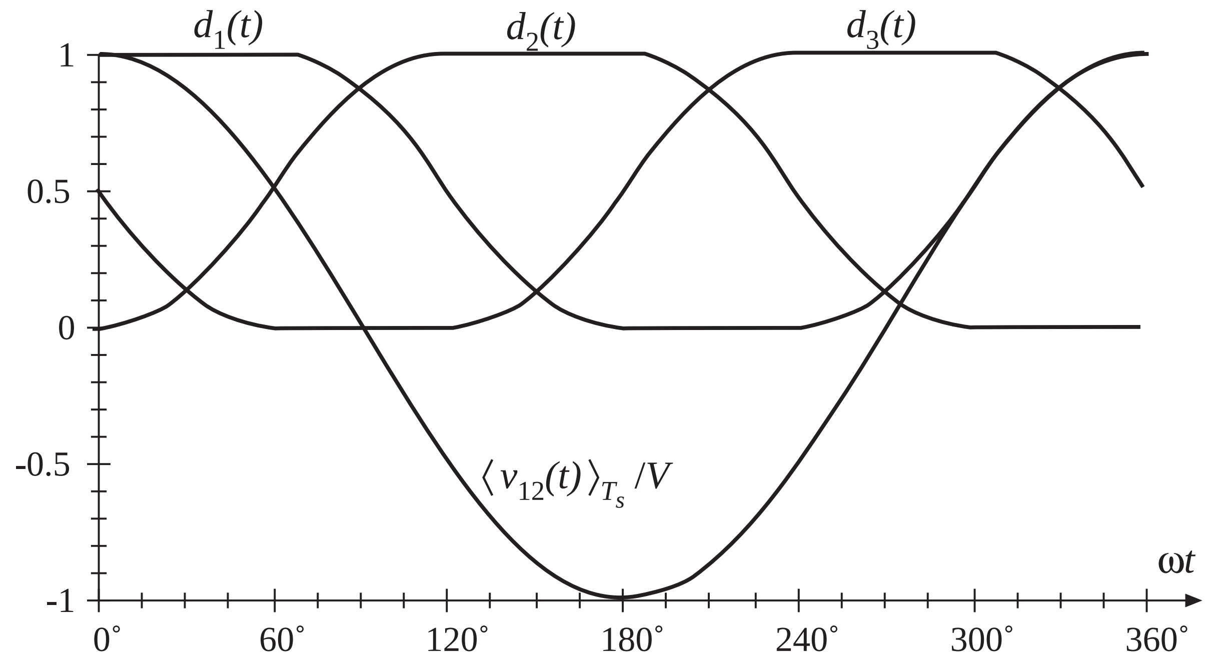
Boost rectifier with sinusoidal PWM

$$V = \frac{2}{\sqrt{3}} \frac{V_{L,pk}}{D_m} = 1.15 \frac{V_{L,pk}}{D_m}$$

With sinusoidal PWM, the dc output voltage must be greater than 1.15 times the peak line-line input voltage. Hence, the boost rectifier increases the voltage magnitude.

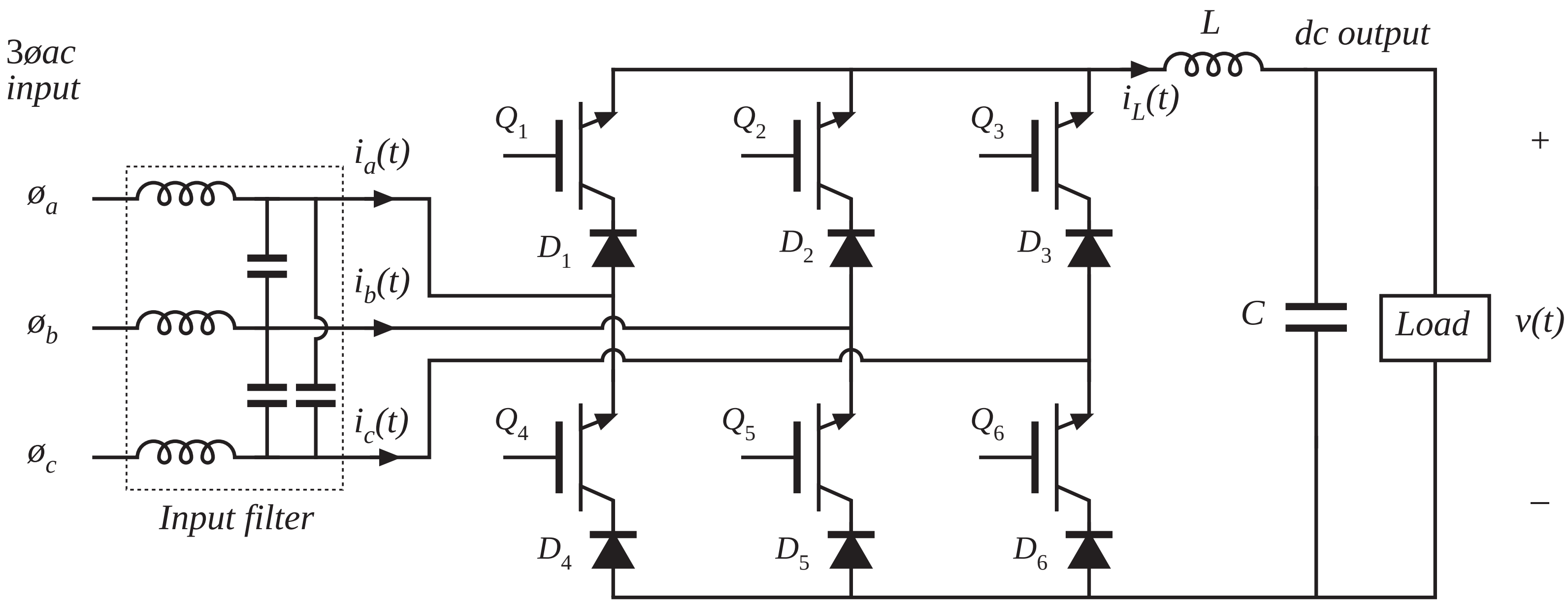
Nonlinear modulation

- Triplen harmonics can be added to the duty ratio modulation, without appearing in the line-line voltages.
- *Overmodulation*, in which the modulation index D_m is increased beyond 1, also



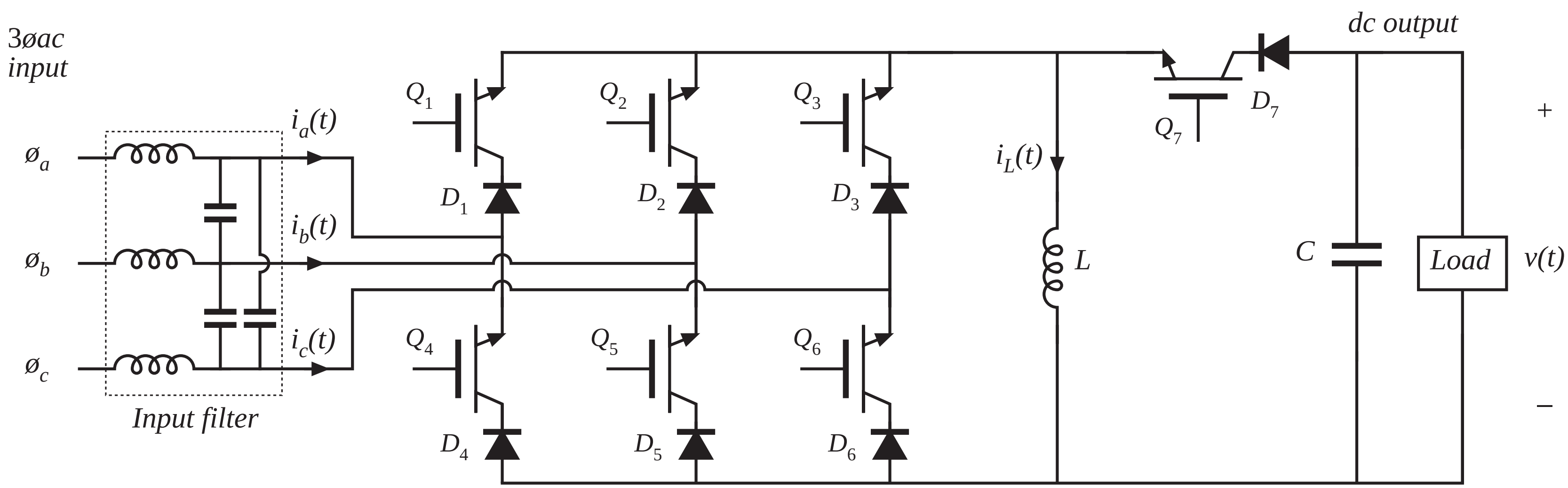
leads to undistorted line-line voltages provided that $D_m \le 1.15$. The pulse width modulator saturates, but the duty ratio variations contain only triplen harmonics. $V = V_{L,pk}$ is obtained at $D_m = 1.15$. Further increases in D_m cause distorted ac line waveforms.

Buck-type 3øac–dc rectifier

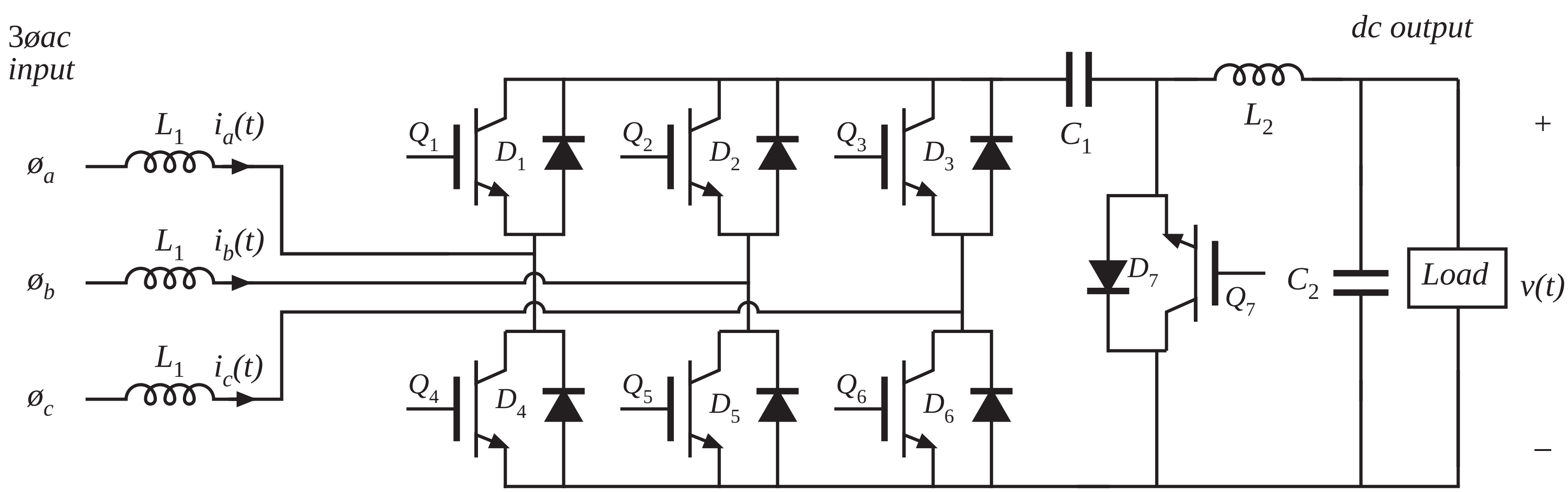


- Can produce controlled dc output voltages in the range $0 \le V \le V_{L,pk}$
- Requires two-quadrant voltage-bidirectional switches
- Exhibits greater active semiconductor stress than boost topology
- Can operate in inverter mode by reversal of output voltage polarity

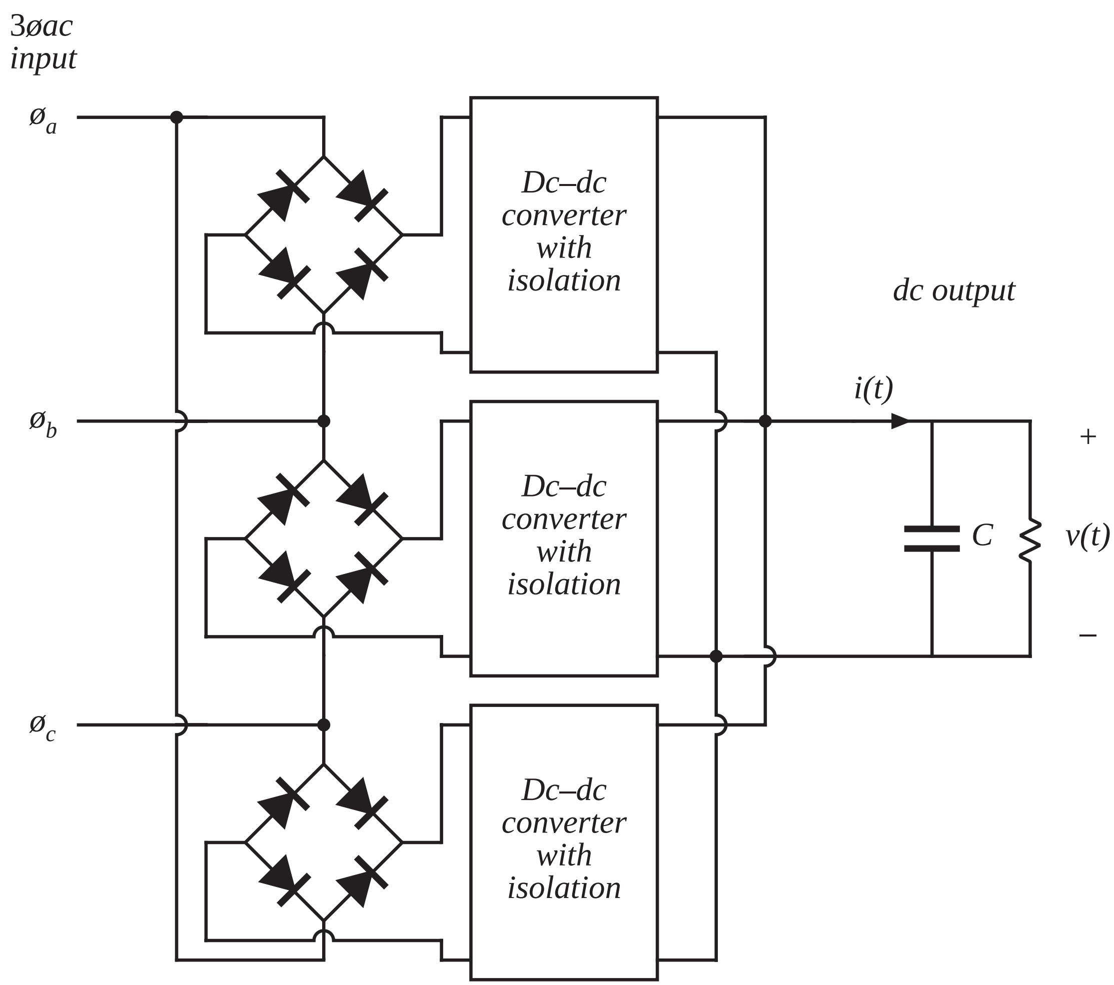
Buck–boost topology



Cuk topology



Use of three single-phase rectifiers



- Each rectifier must include isolation between input and output
- Isolation transformers must be rated to carry the pulsating single-phase ac power $p_{ac}(t)$
- Outputs can be connected in series or parallel
- Because of the isolation requirement, semiconductor stresses are greater than in 3ø boost rectifier

17.5.2 Some other approaches to three-phase rectification

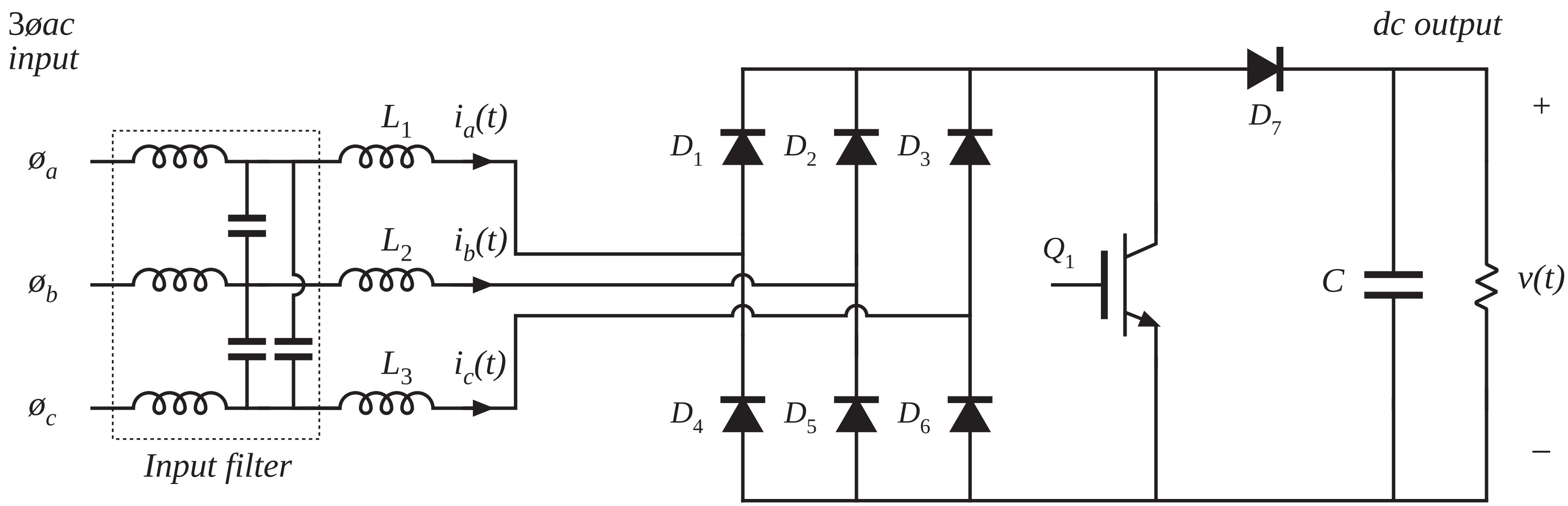
Low-harmonic rectification requires active semiconductor devices that are much more expensive than simple peak-detection diode rectifiers.

What is the minimum active silicon required to perform the function of 3ø low-harmonic rectification?

- No active devices are needed: diodes and harmonic traps will do the job, but these require low-frequency reactive elements
- When control of the output voltage is needed, then there must be at least one active device
- To avoid low-frequency reactive elements, at least one high-frequency switch is needed

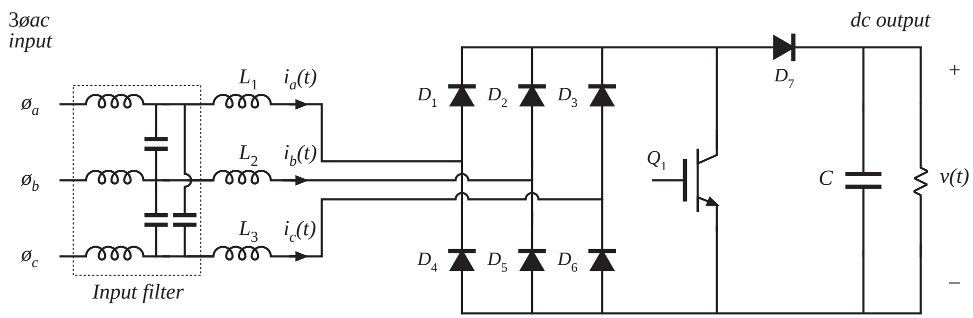
So let's search for approaches that use just one active switch, and only high-frequency reactive elements

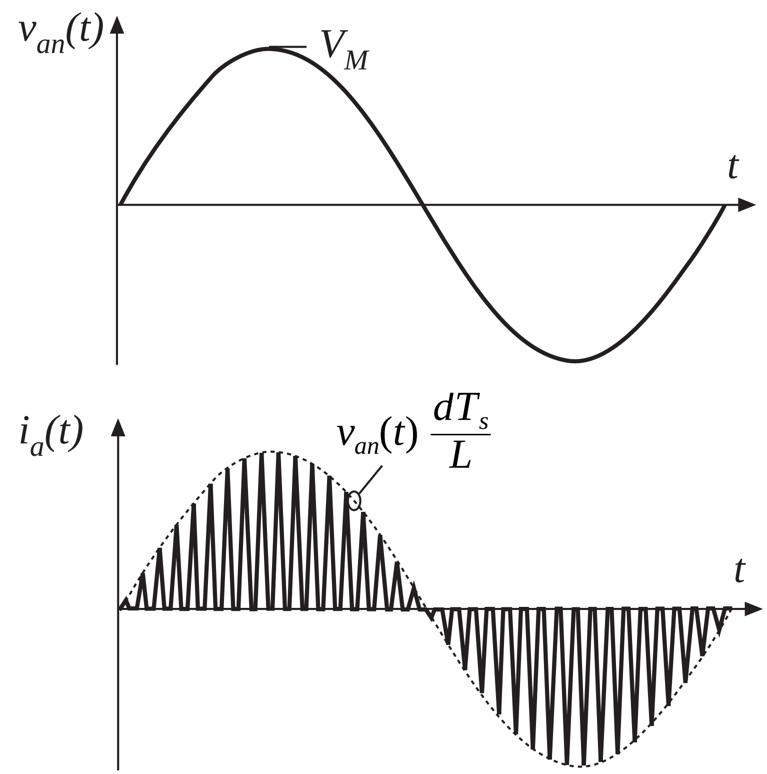
The single-switch DCM boost 3ø rectifier



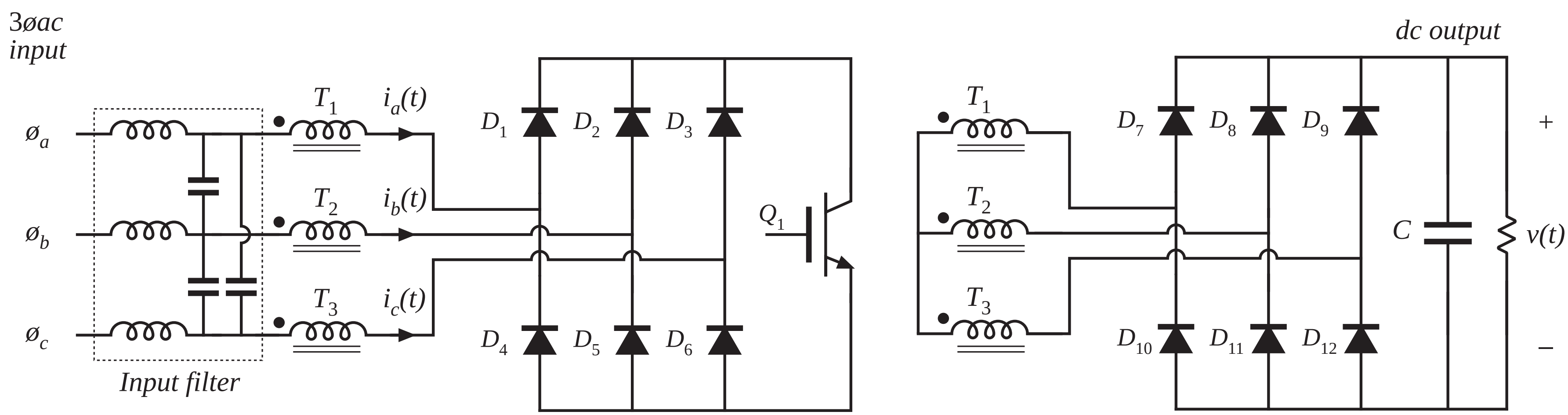
Inductors L_1 to L_3 operate in discontinuous conduction mode, in conjunction with diodes D_1 to D_6. Average input currents $\langle i_a(t) \rangle_{T_s}$, $\langle i_b(t) \rangle_{T_s}$, and $\langle i_c(t) \rangle_{T_s}$ are approximately proportional to the instantaneous input line-neutral voltages. Transistor is operated with constant duty cycle; slow variation of the duty cycle allows control of output power.

The single-switch DCM boost 3ø rectifier





The single-switch 3ø DCM flyback rectifier



The single-switch 3ø DCM flyback rectifier

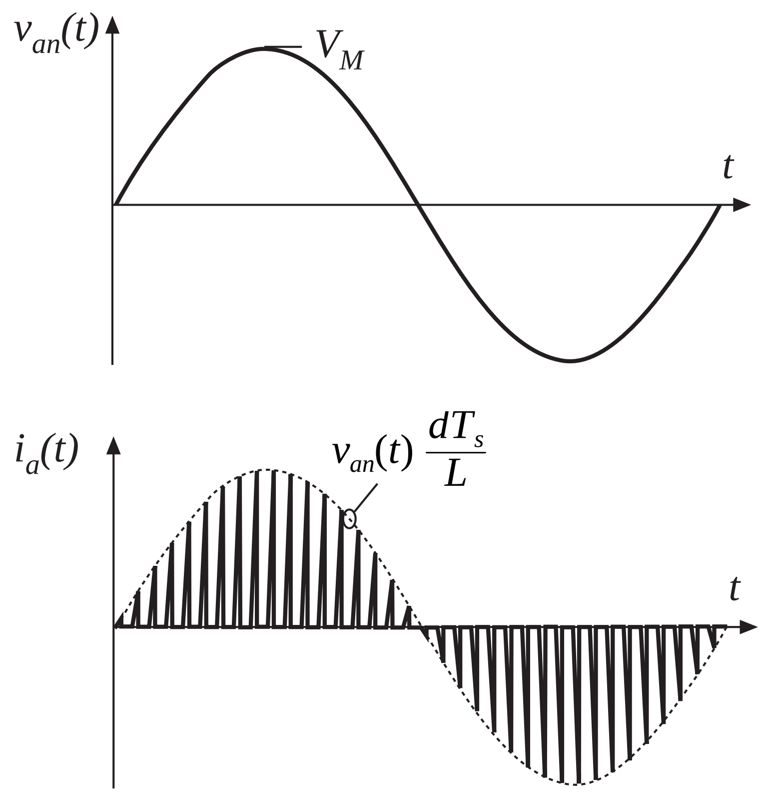
This converter is effectively three independent single-phase DCM flyback converters that share a common switch.

Since the open-loop DCM flyback converter can be modeled as a Loss-Free Resistor, three-phase low-harmonic rectification is obtained naturally.

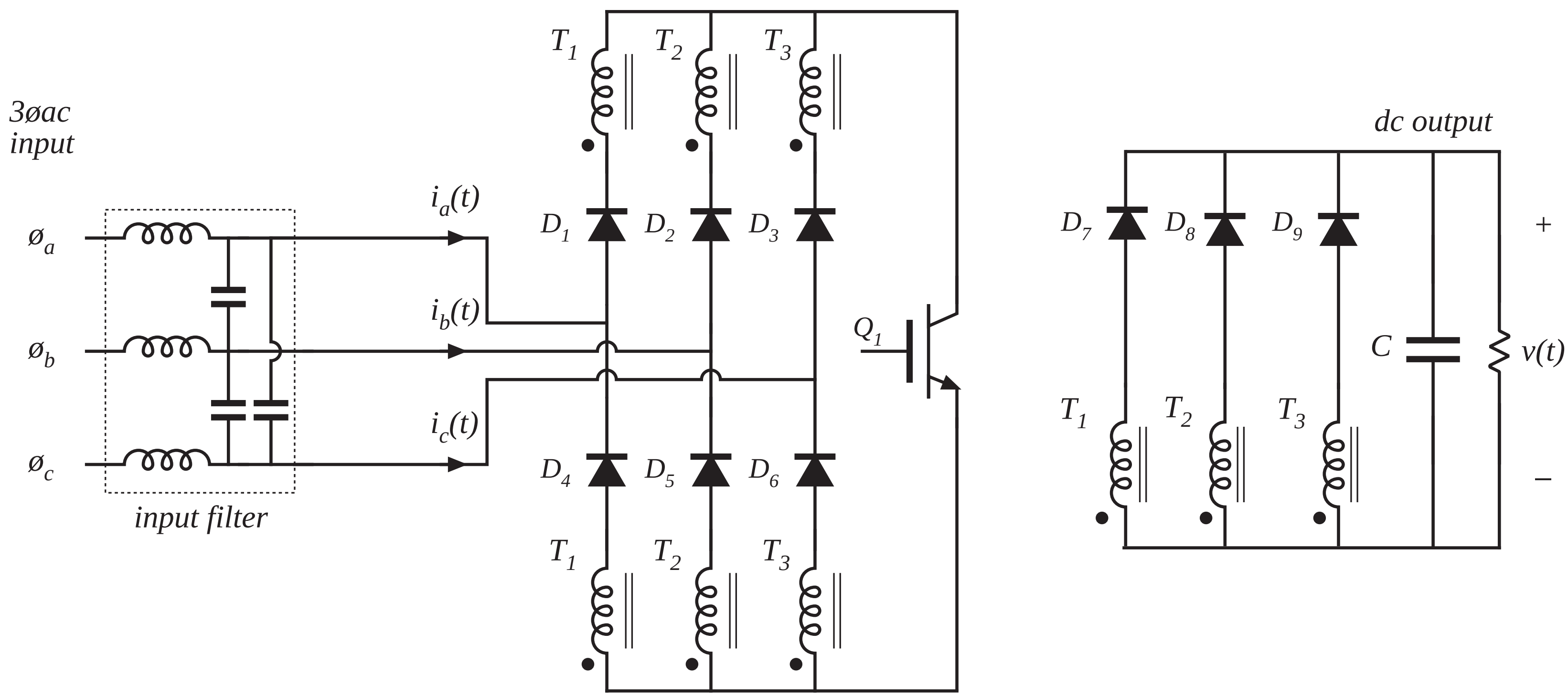
Basic converter has a boost characteristic, but buck-boost characteristic is possible (next slide).

Inrush current limiting and isolation are obtained easily.

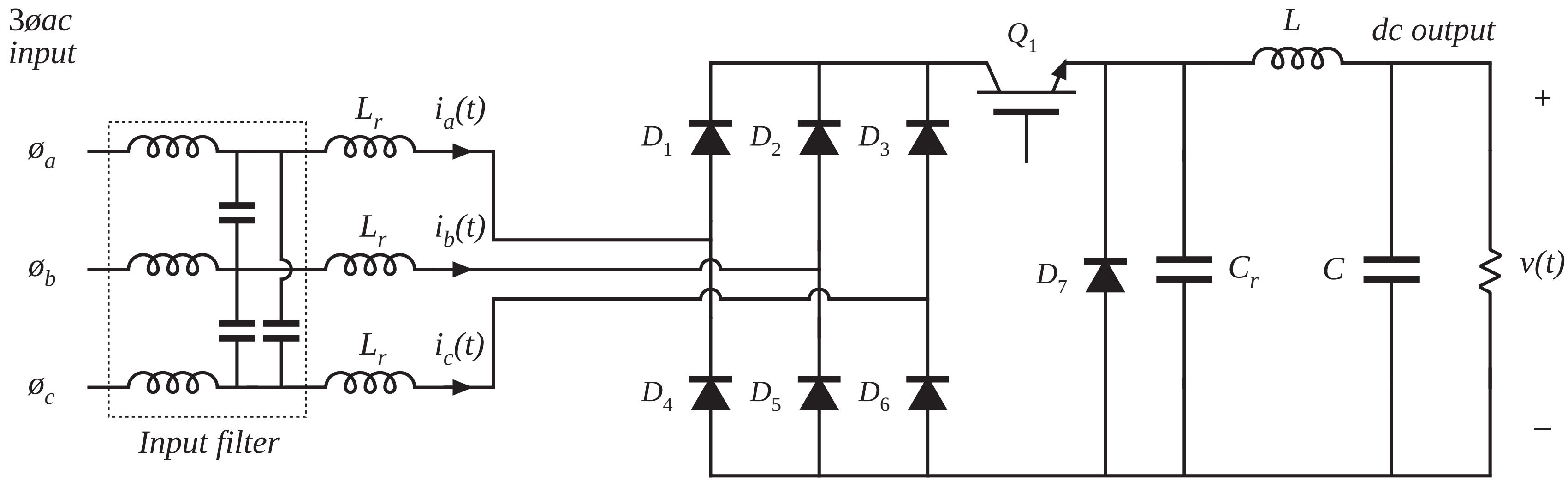
High peak currents, needs an input EMI filter



3ø Flyback rectifier with buck-boost conversion ratio



Single-switch three-phase zero-current-switching quasi-resonant buck rectifier

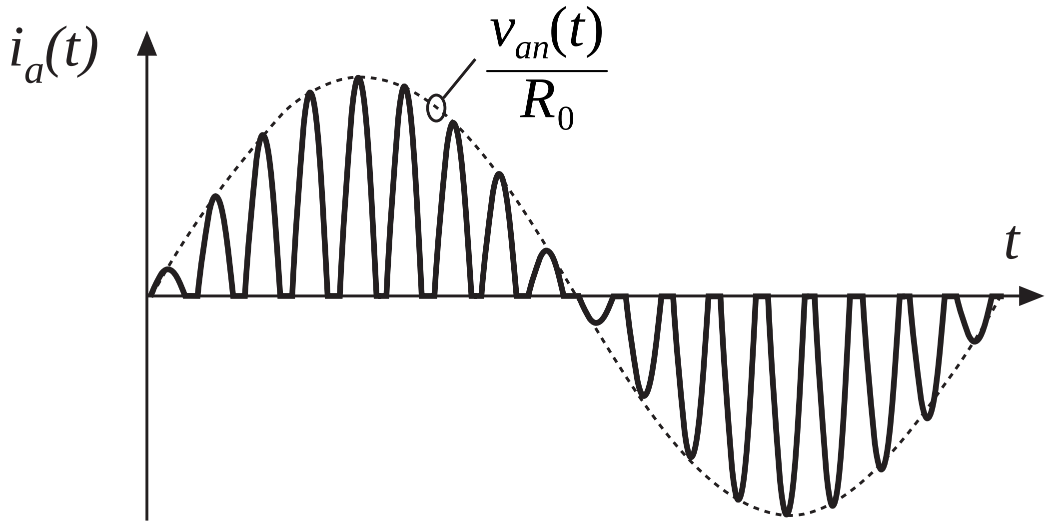


Inductors L_r and capacitor C_r form resonant tank circuits, having resonant frequency slightly greater than the switching frequency.

Turning on Q_1 initiates resonant current pulses, whose amplitudes depend on the instantaneous input line-neutral voltages.

When the resonant current pulses return to zero, diodes D_1 to D_6 are reverse-biased. Transistor Q_1 can then be turned off at zero current.

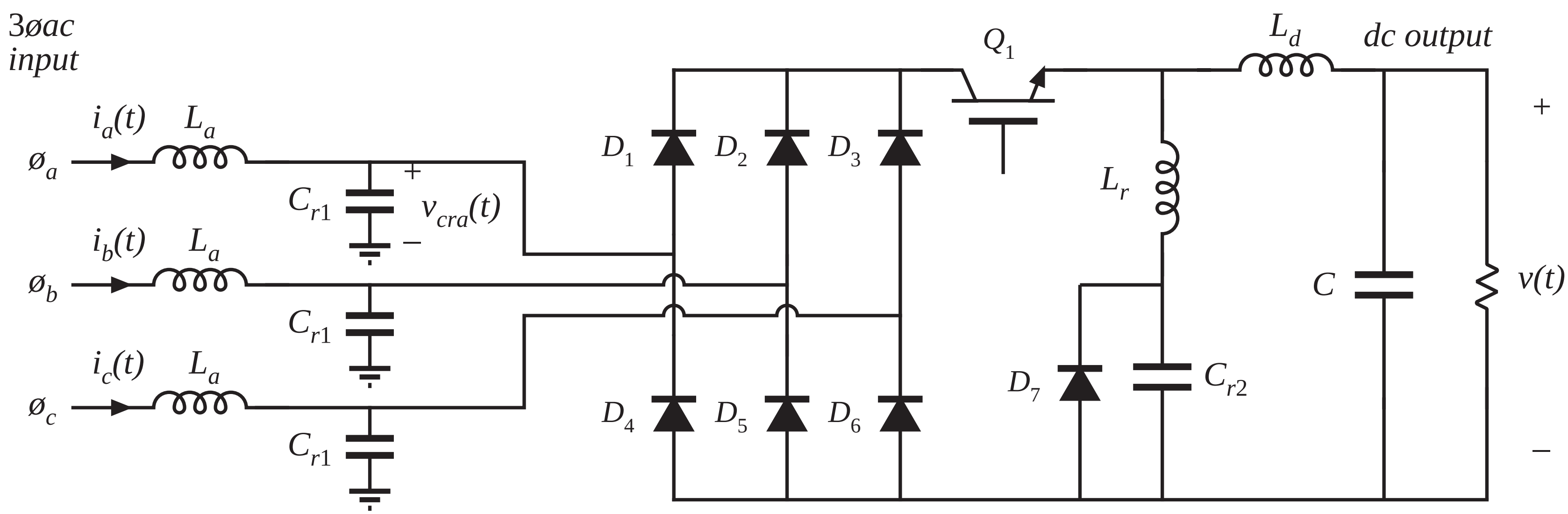
Single-switch three-phase zero-current-switching quasi-resonant buck rectifier



Input line currents are approximately sinusoidal pulses, whose amplitudes follow the input line-neutral voltages.

Lowest total active semiconductor stress of all buck-type 3ø low harmonic rectifiers

Multiresonant single-switch zero-current switching 3ø buck rectifier

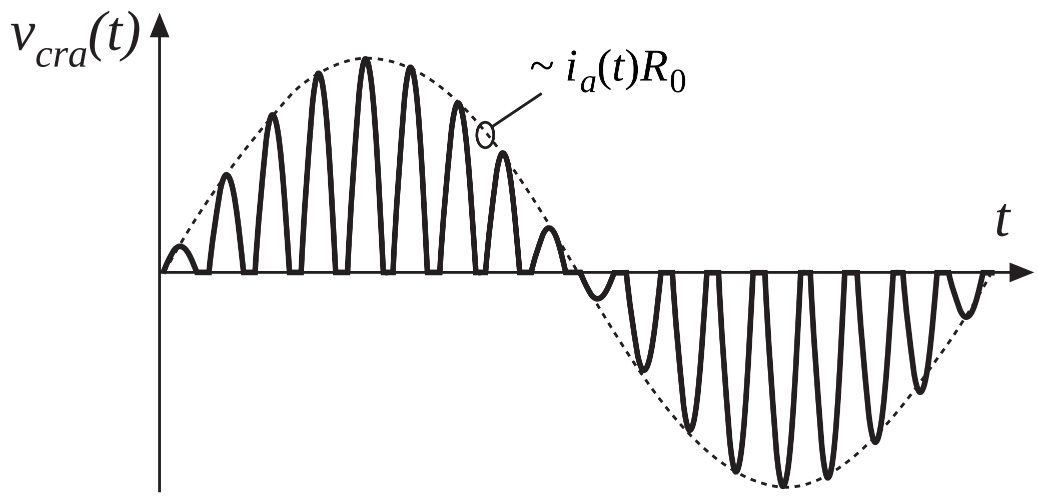


Inductors L_r and capacitors C_{r1} and C_{r2} form resonant tank circuits, having resonant frequency slightly greater than the switching frequency.

Turning on Q_1 initiates resonant voltage pulses in $v_{cra}(t)$, whose amplitudes depend on the instantaneous input line-neutral currents $i_a(t)$ to $i_c(t)$.

All diodes switch off when their respective tank voltages reach zero. Transistor Q_1 is turned off at zero current.

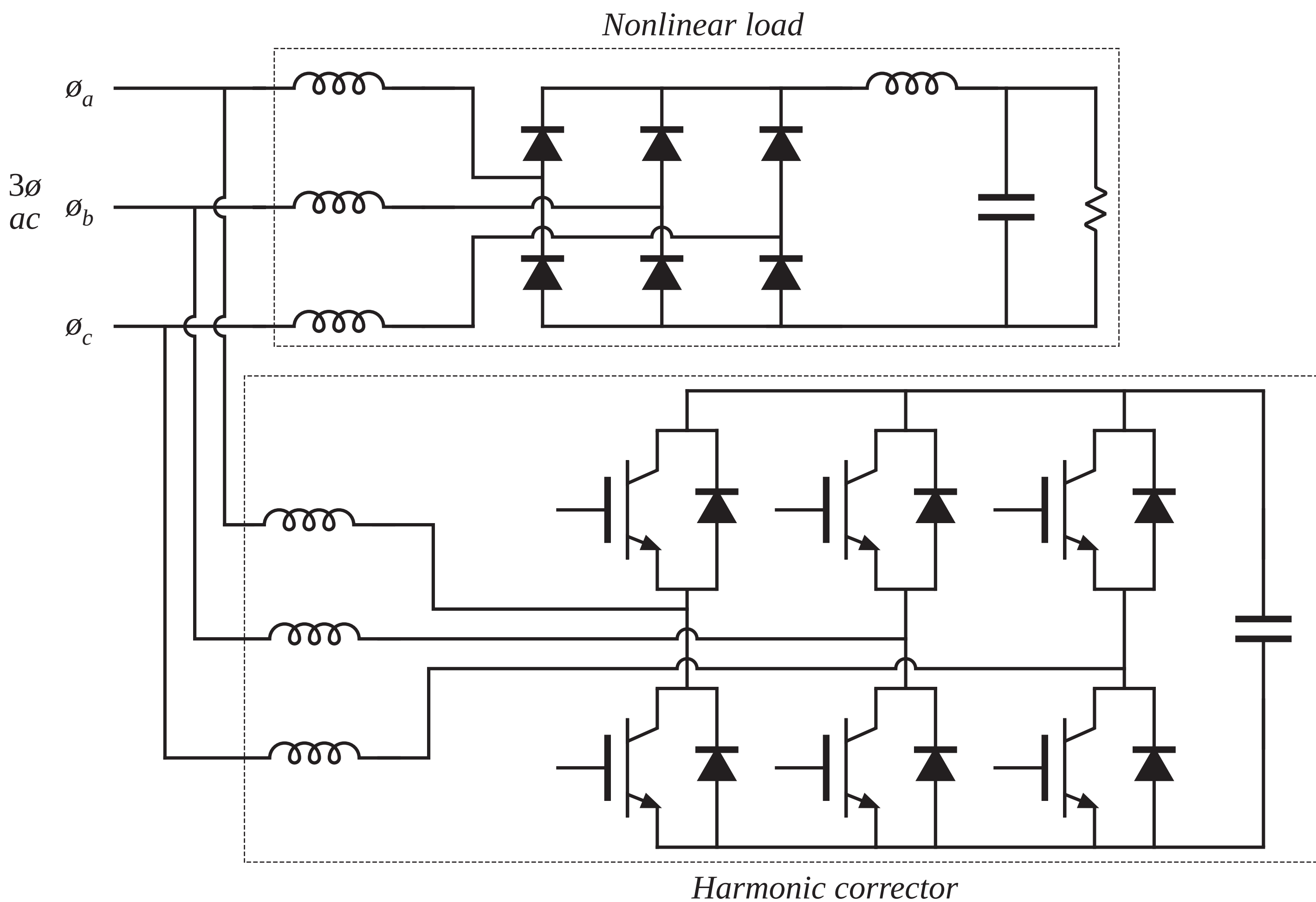
Multiresonant single-switch zero-current switching 3ø buck rectifier



Input-side resonant voltages are approximately sinusoidal pulses, whose amplitudes follow the input currents. Input filter inductors operate in CCM.

Higher total active semiconductor stress than previous approach, but less EMI filtering is needed. Low THD: < 4% THD can be obtained.

Harmonic correction



Harmonic correction

- An active filter that is controlled to cancel the harmonic currents created by a nonlinear load.
- Does not need to conduct the average load power.
- Total active semiconductor stress is high when the nonlinear load generates large harmonic currents having high THD.
- In the majority of applications, this approach exhibits greater total active semiconductor stress than the simple 3ø CCM boost rectifier.

17.6 Summary of key points

1. The ideal rectifier presents an effective resistive load, the emulated resistance R_e, to the ac power system. The power apparently "consumed" by R_e is transferred to the dc output port. In a three-phase ideal rectifier, input resistor emulation is obtained in each phase. In both the single-phase and three-phase cases, the output port follows a power source characteristic, dependent on the instantaneous ac input power. Ideal rectifiers can perform the function of low-harmonic rectification, without need for low-frequency reactive elements.

2. The dc-dc boost converter, as well as other converters capable of increasing the voltage, can be adapted to the ideal rectifier application. A control system causes the input current to be proportional to the input voltage. The converter may operate in CCM, DCM, or in both modes. The mode boundary is expressed as a function of R_e , $2L/T_s$, and the instantaneous voltage ratio $v_g(t)/V$. A well-designed average current controller leads to resistor emulation regardless of the operating mode; however, other schemes discussed in the next chapter may lead to distorted current waveforms when the mode boundary is crossed.

Summary of key points

3. In a single-phase system, the instantaneous ac input power is pulsating, while the dc load power is constant. Whenever the instantaneous input and output powers are not equal, the ideal rectifier system must contain energy storage. A large capacitor is commonly employed; the voltage of this capacitor must be allowed to vary independently, as necessary to store and release energy. A slow feedback loop regulates the dc component of the capacitor voltage, to ensure that the average ac input power and dc load power are balanced.

4. RMS values of rectifiers waveforms can be computed by double integration. In the case of the boost converter, the rms transistor current can be as low as 39% of the rms ac input current, when V is close in value to V_M. Other converter topologies such as the buck-boost, SEPIC, and Cuk converters exhibit significantly higher rms transistor currents but are capable of limiting the converter inrush current.

Summary of key points

5. In the three-phase case, a boost-type rectifier based on the PWM voltage-source inverter also exhibits low rms transistor currents. This approach requires six active switching elements, and its dc output voltage must be greater than the peak input line-to-line voltage. Average current control can be used to obtain input resistor emulation. An equivalent circuit can be derived by averaging the switch waveforms. The converter operation can be understood by assuming that the switch duty cycles vary sinusoidally; expressions for the average converter waveforms can then be derived.

6. Other three-phase rectifier topologies are known, including six-switch rectifiers having buck and buck-boost characteristics. In addition, three-phase low-harmonic rectifiers having a reduced number of active switches, as few as one, are discussed here.