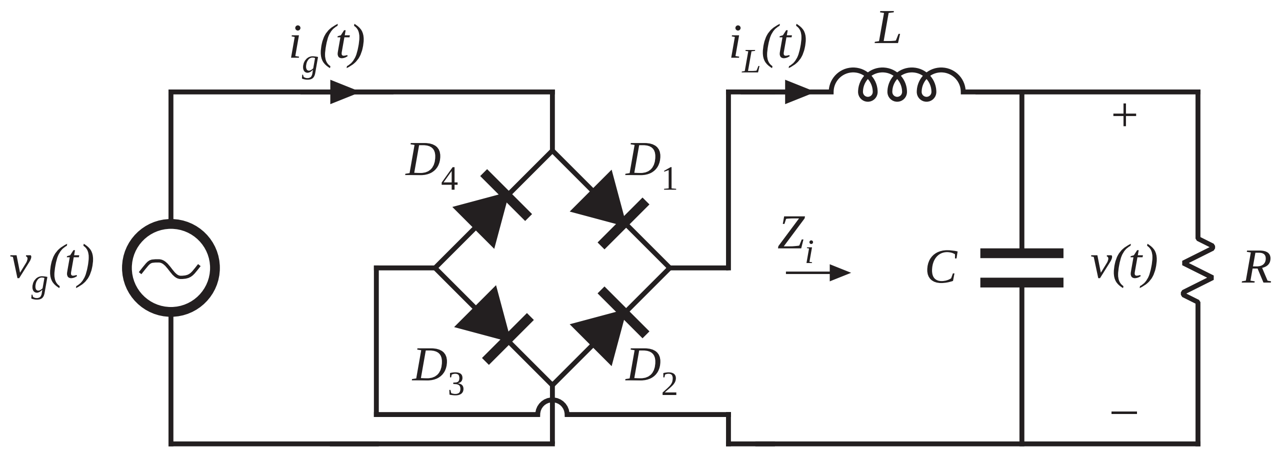
Chapter 16

Line-Commutated Rectifiers

16.1 The single-phase full-wave rectifier



Full-wave rectifier with dc-side L-C filter

Two common reasons for including the dc-side L-C filter:

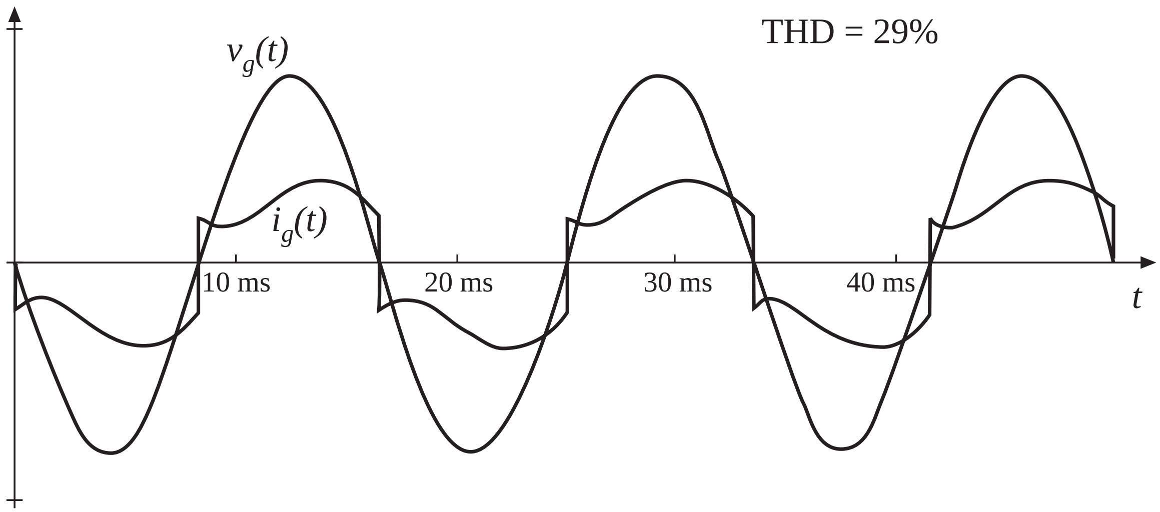
- Obtain good dc output voltage (large C) and acceptable ac line current waveform (large L)
- Filter conducted EMI generated by dc load (small L and C)

16.1.1 Continuous conduction mode

Large L

Typical ac line waveforms for CCM :

As $L \to \infty$, ac line current approaches a square wave



CCM results, for $L \to \infty$:

$$\text{distortion factor} = \frac{I_{1,\,rms}}{I_{rms}} = \frac{4}{\pi\sqrt{2}} = 90.0\%$$

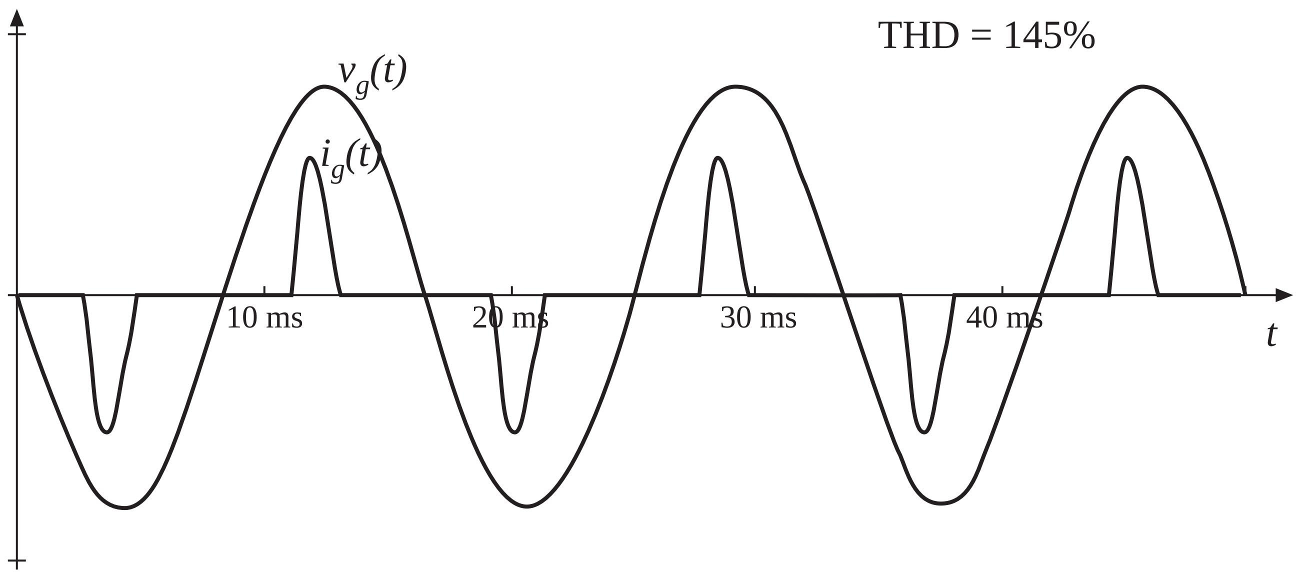
$$\text{THD} = \sqrt{\left(\frac{1}{\text{distortion factor}}\right)^2 - 1} = 48.3\%$$

16.1.2 Discontinuous conduction mode

Small L

Typical ac line waveforms for DCM :

As $L \to 0$, ac line current approaches impulse functions (peak detection)



As the inductance is reduced, the THD rapidly increases, and the distortion factor decreases.

Typical distortion factor of a full-wave rectifier with no inductor is in the range 55% to 65%, and is governed by ac system inductance.

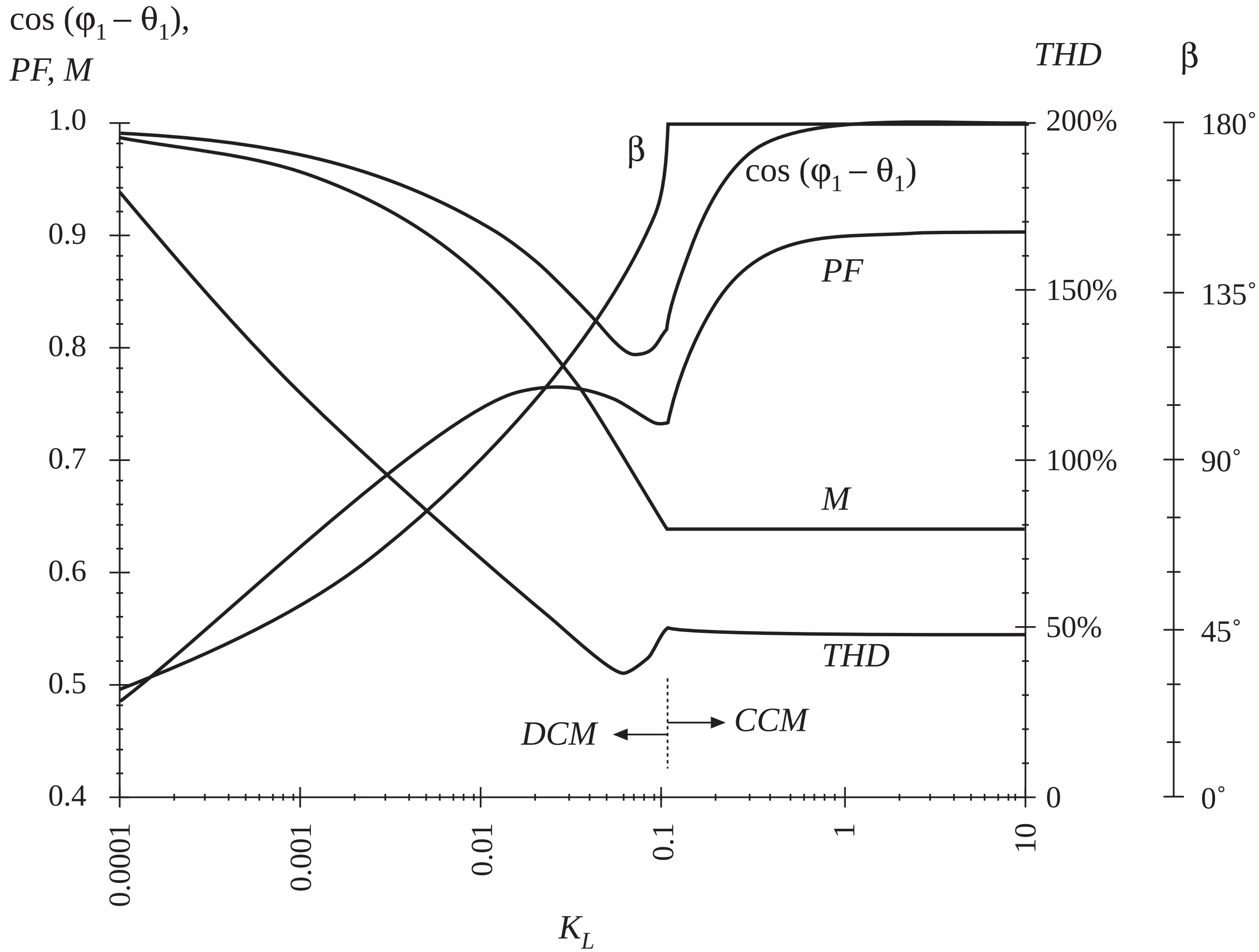
16.1.3 Behavior when C is large

Solution of the full-wave rectifier circuit for infinite C:

Define

$$K_L = \frac{2L}{RT_L}$$

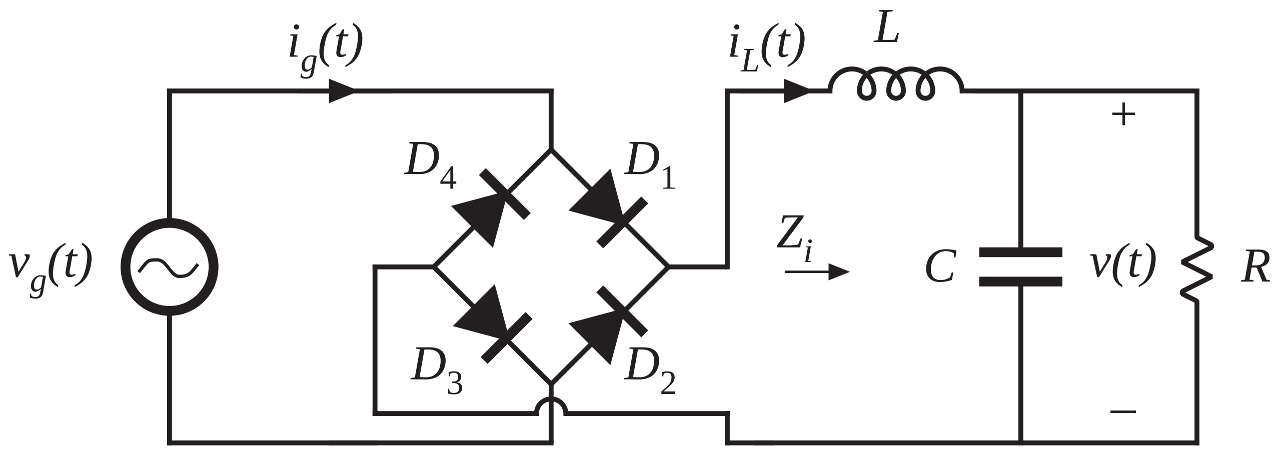
$$M = \frac{V}{V_m}$$



16.1.4 Minimizing *THD* when *C* is small

Sometimes the L-C filter is present only to remove high-frequency conducted EMI generated by the dc load, and is not intended to modify the ac line current waveform. If L and C are both zero, then the load resistor is connected directly to the output of the diode bridge, and the ac line current waveform is purely sinusoidal.

An approximate argument: the L-C filter has negligible effect on the ac line current waveform provided that the filter input impedance Z_i has zero phase shift at the second harmonic of the ac line frequency, $2\,f_L$.



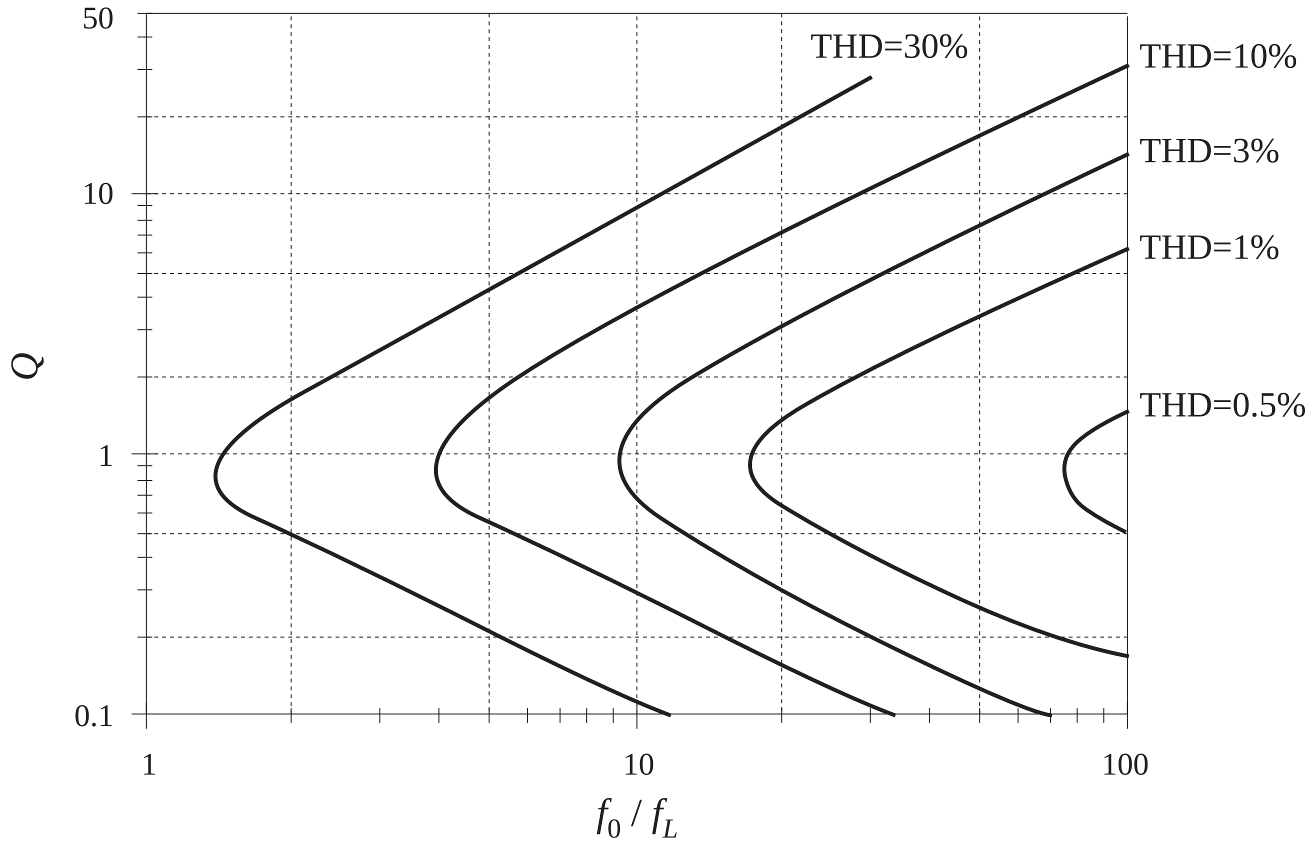
Approximate THD

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

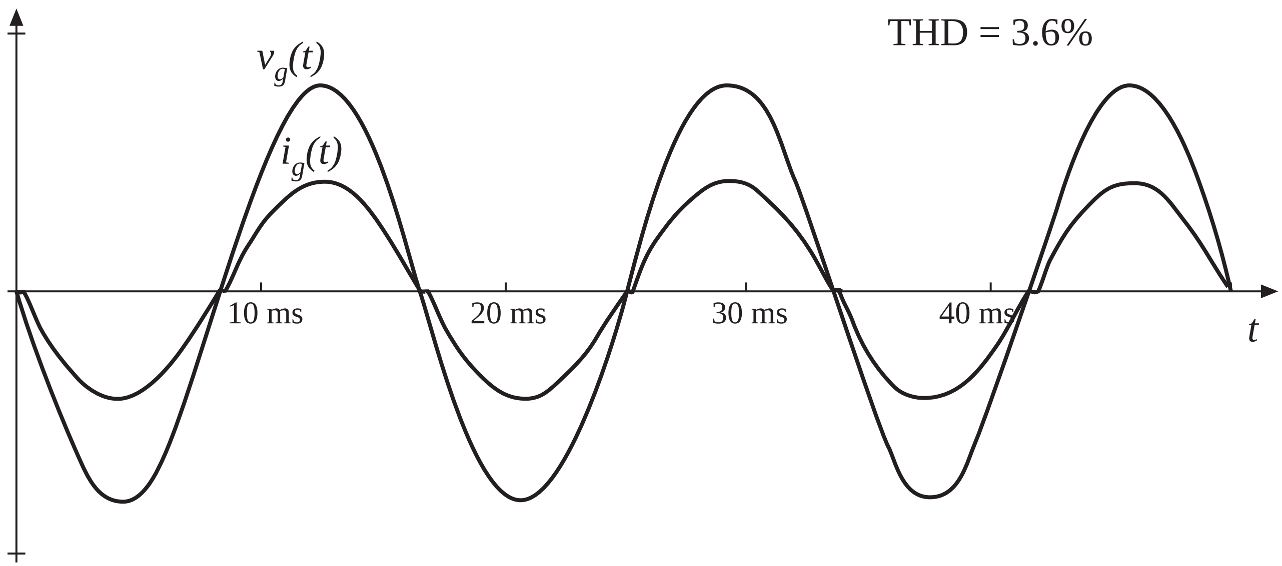
$$R_0 = \sqrt{\frac{L}{C}}$$

$$Q = \frac{R}{R_0}$$

$$f_p = \frac{1}{2\pi RC} = \frac{f_0}{Q}$$

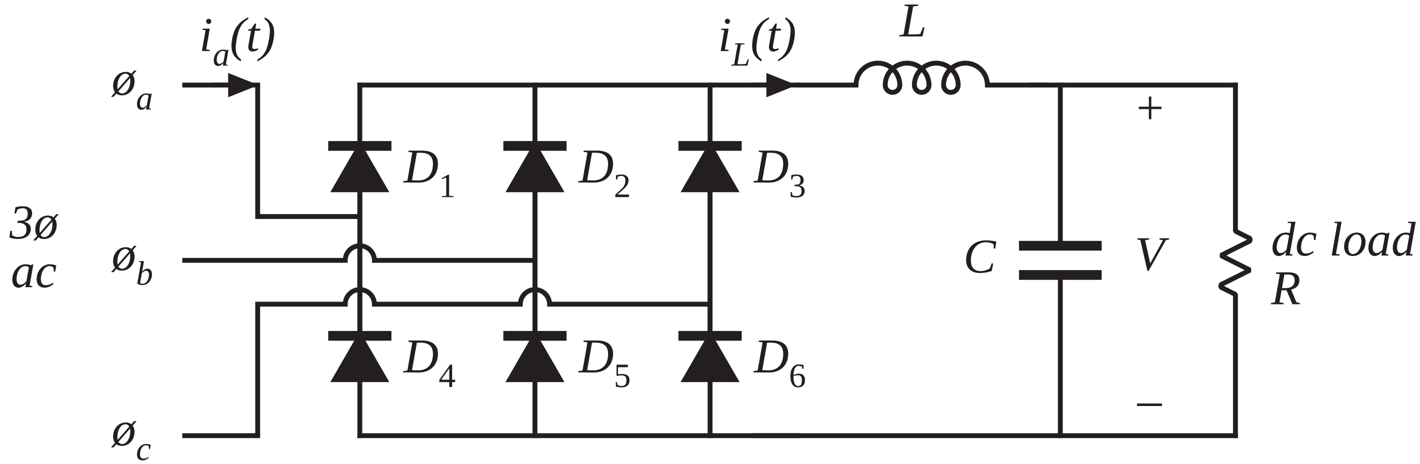


Example

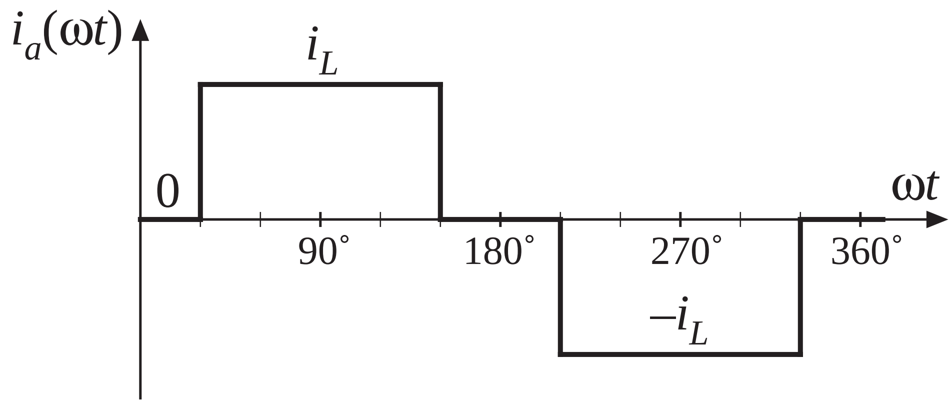


Typical ac line current and voltage waveforms, near the boundary between continuous and discontinuous modes and with small dc filter capacitor. $f_0/f_L = 10$, $Q = 1$

16.2 The Three-Phase Bridge Rectifier



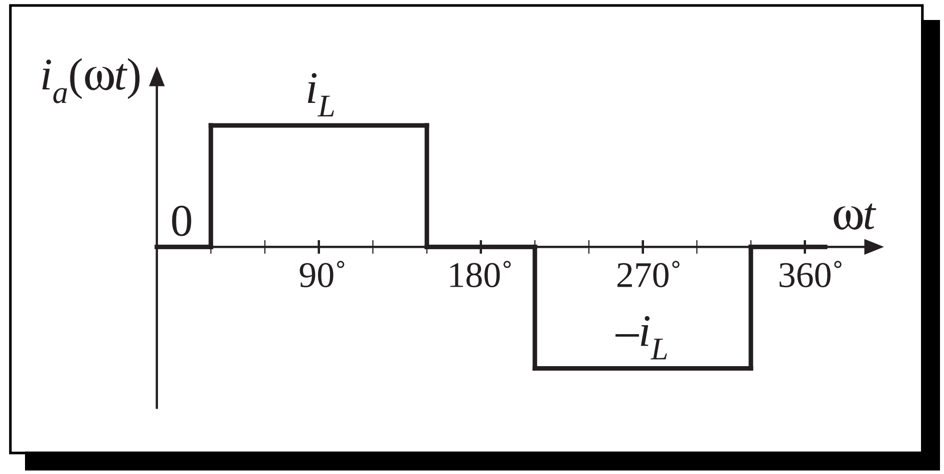
Line current waveform for infinite L



16.2.1 Continuous conduction mode

Fourier series:

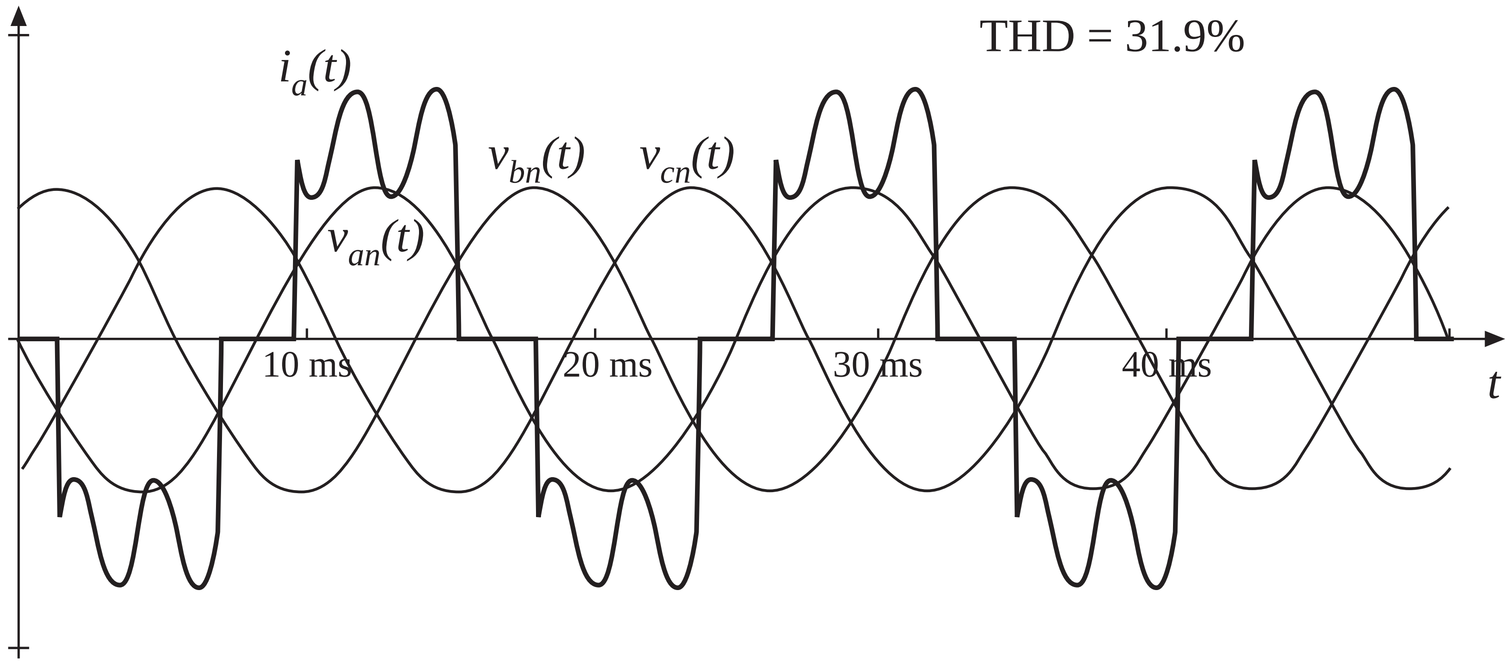
$$i_a(t) = \sum_{n=1,5,7,11,...}^{\infty} \frac{4}{n\pi} I_L \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{n\pi}{3}\right) \sin\left(n\omega t\right)$$



- Similar to square wave, but missing triplen harmonics
- THD = 31%
- Distortion factor = 3/π = 95.5%
- In comparison with single phase case:

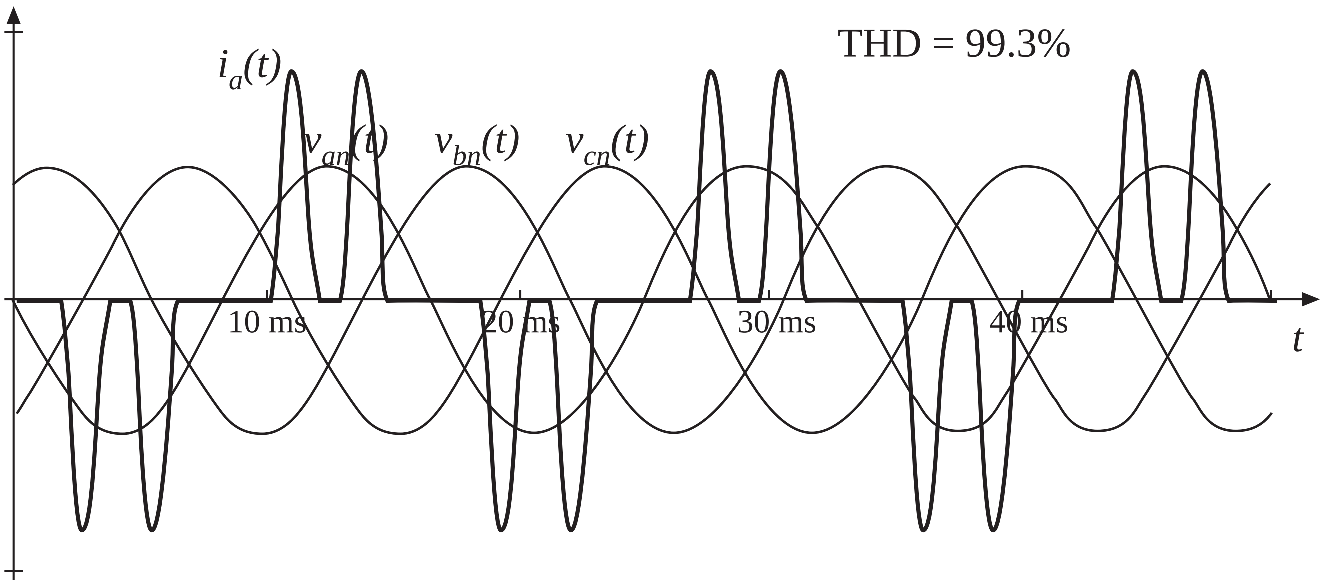
  the missing 60˚ of current improves the distortion factor from 90% to 95%, because the triplen harmonics are removed

A typical CCM waveform



Inductor current contains sixth harmonic ripple (360 Hz for a 60 Hz ac system). This ripple is superimposed on the ac line current waveform, and influences the fifth and seventh harmonic content of $i_a(t)$.

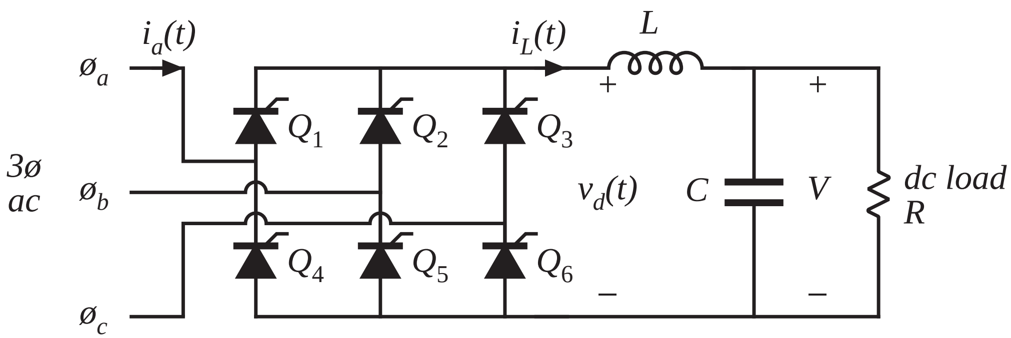
16.2.2 Discontinuous conduction mode



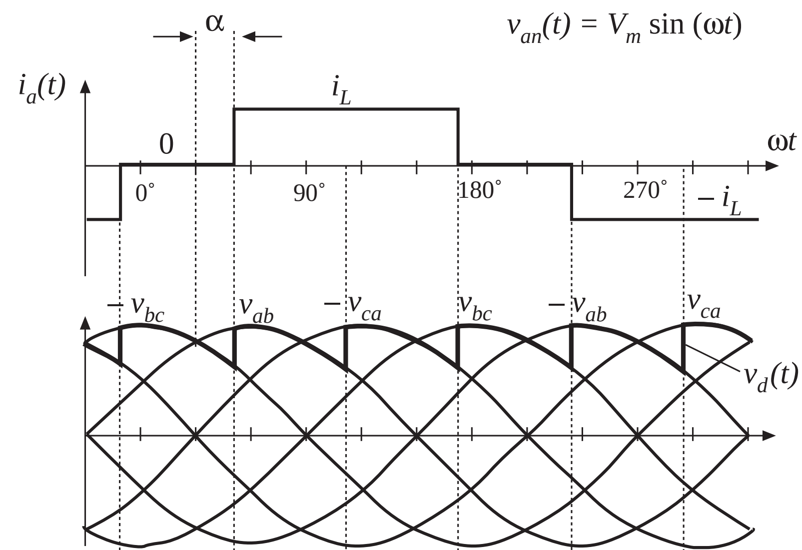
Phase *a* current contains pulses at the positive and negative peaks of the line-to-line voltages $v_{ab}(t)$ and $v_{ac}(t)$. Distortion factor and THD are increased. Distortion factor of the typical waveform illustrated above is 71%.

16.3 Phase control

Replace diodes with SCRs:



Phase control waveforms:



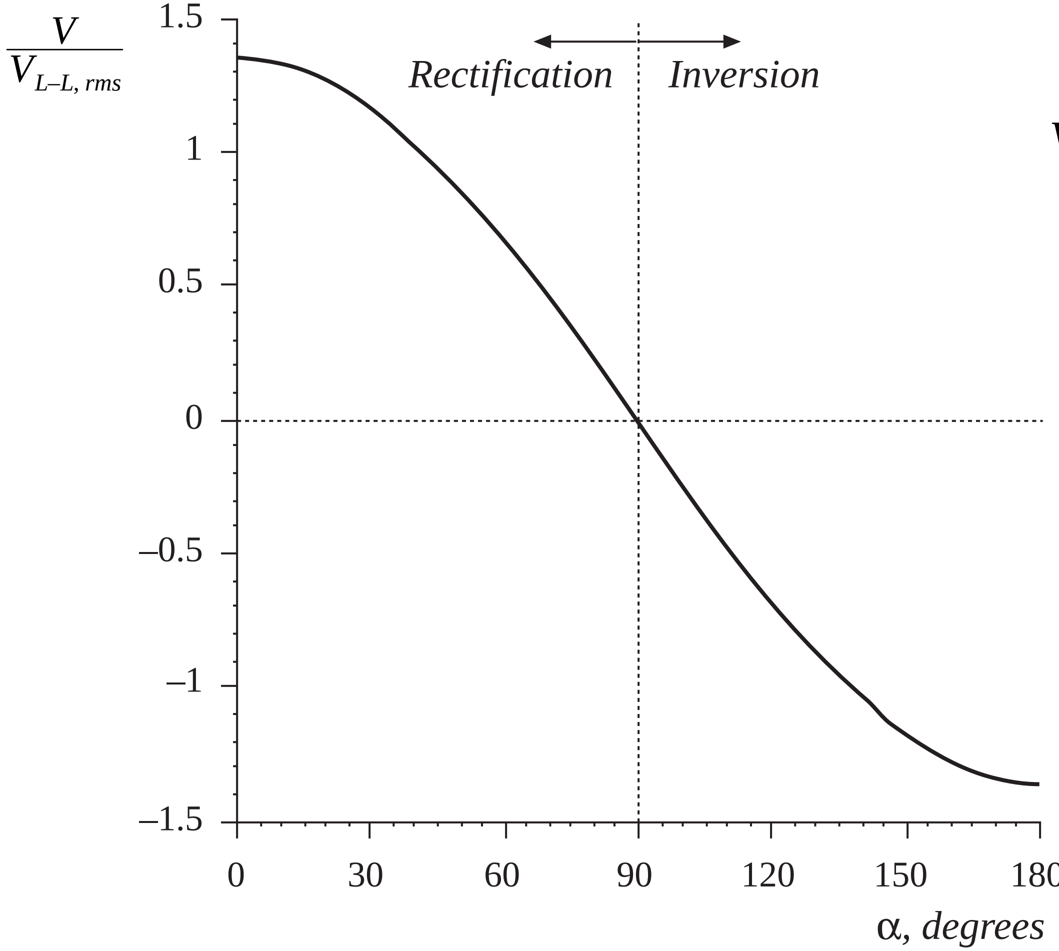
Upper thyristor:	Q_3	Q_1	Q_1	Q_2	Q_2	Q_3
Lower thyristor:	Q_5	Q_5	Q_6	Q_6	Q_4	Q_4

Average (dc) output voltage:

$$V = \frac{3}{\pi}\int_{30^\circ+\alpha}^{90^\circ+\alpha} \sqrt{3}\, V_m \sin(\theta + 30^\circ)\, d\theta$$

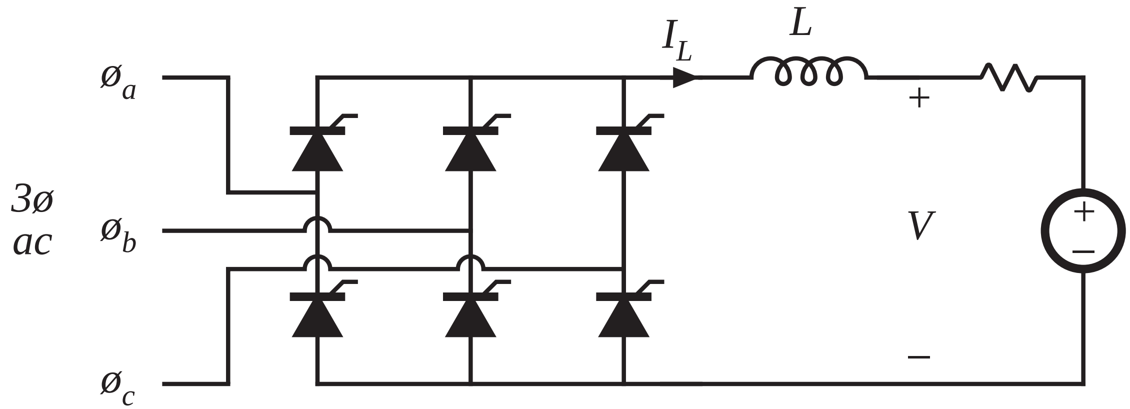
$$= \frac{3\sqrt{2}}{\pi} V_{L\text{-}L,\, rms} \cos\alpha$$

Dc output voltage vs. delay angle α



$$V = \frac{3}{\pi} \int_{30^\circ+\alpha}^{90^\circ+\alpha} \sqrt{3}\, V_m \sin(\theta + 30^\circ)\, d\theta$$
$$= \frac{3\sqrt{2}}{\pi} V_{L\text{-}L,\,rms} \cos\alpha$$

16.3.1 Inverter mode



If the load is capable of supplying power, then the direction of power flow can be reversed by reversal of the dc output voltage V. The delay angle α must be greater than 90˚. The current direction is unchanged.

16.3.2 Harmonics and power factor

Fourier series of ac line current waveform, for large dc-side inductance:

$$i_a(t) = \sum_{n=1,5,7,11,\ldots}^{\infty} \frac{4}{n\pi} I_L \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{n\pi}{3}\right) \sin(n\omega t - n\alpha)$$

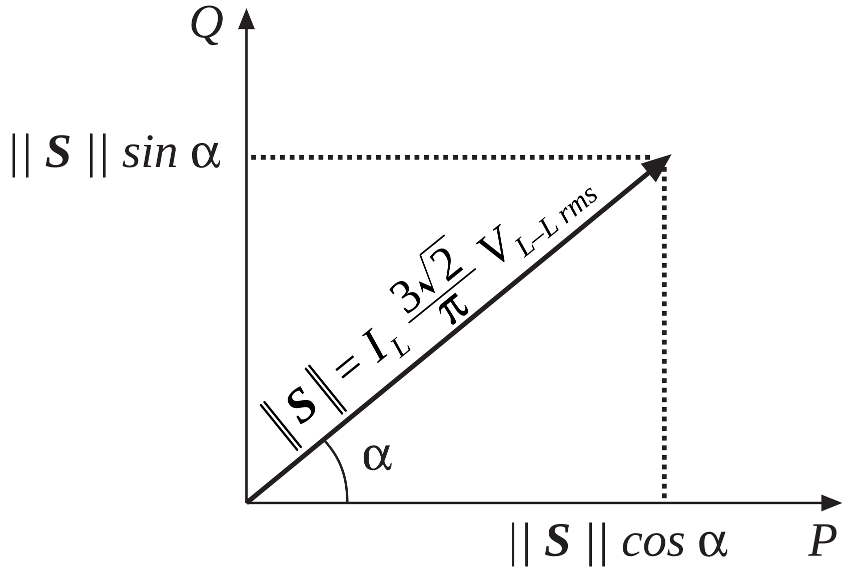
Same as uncontrolled rectifier case, except that waveform is delayed by the angle α. This causes the current to lag, and decreases the displacement factor. The power factor becomes:

$$\text{power factor} = 0.955 \left| \cos(\alpha) \right|$$

When the dc output voltage is small, then the delay angle α is close to 90˚ and the power factor becomes quite small. The rectifier apparently consumes reactive power, as follows:

$$Q = \sqrt{3}\, I_{a,\,rms} V_{L\text{-}L,\,rms} \sin\alpha = I_L \frac{3\sqrt{2}}{\pi} V_{L\text{-}L,\,rms} \sin\alpha$$

Real and reactive power in controlled rectifier
at fundamental frequency

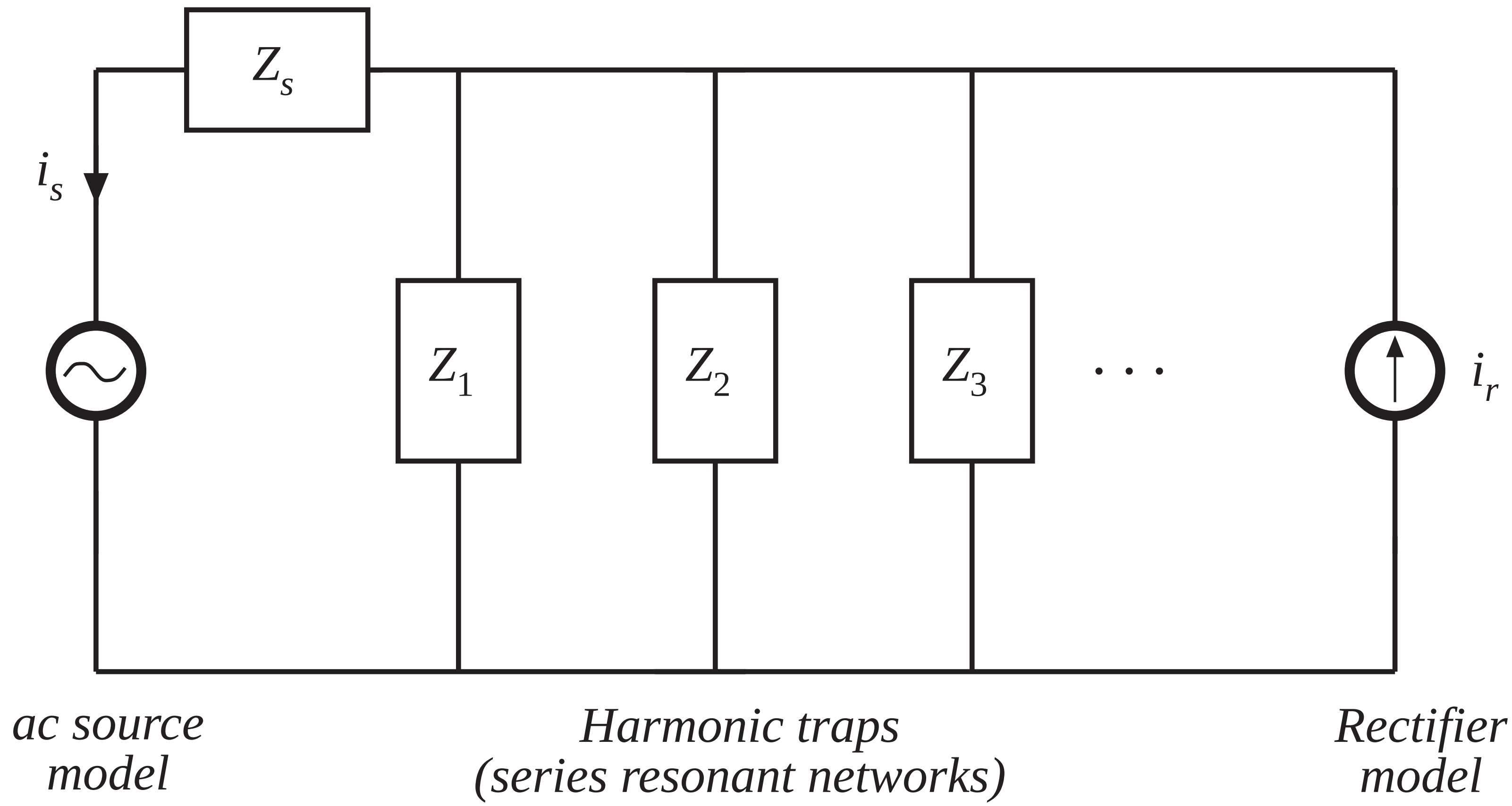


$$P = I_L \, \frac{3\sqrt{2}}{\pi} \, V_{L\text{-}L,\,rms} \cos \alpha$$

$$Q = \sqrt{3}\; I_{a,\,rms} V_{L\text{-}L,\,rms} \sin \alpha = I_L \, \frac{3\sqrt{2}}{\pi} \, V_{L\text{-}L,\,rms} \sin \alpha$$

16.4 Harmonic trap filters

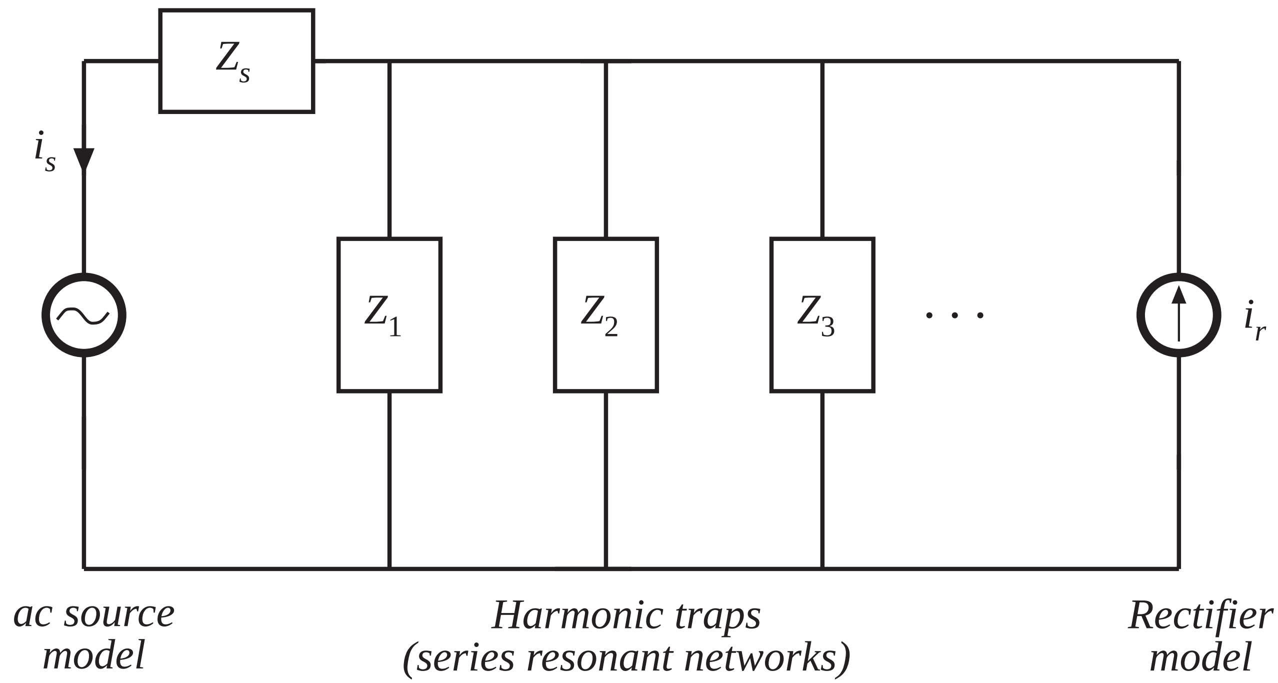
A passive filter, having resonant zeroes tuned to the harmonic frequencies



Harmonic trap

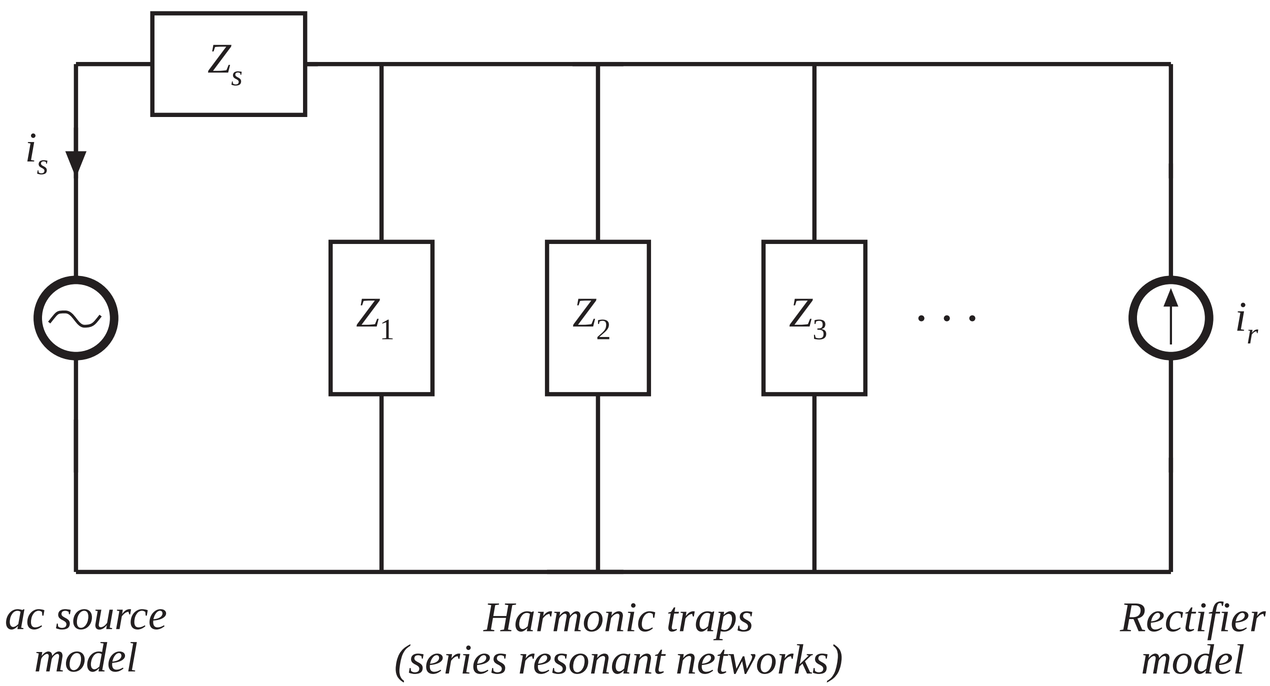
Ac source: model with Thevenin-equiv voltage source and impedance $Z_s'(s)$. Filter often contains series inductor sL_s'. Lump into effective impedance $Z_s(s)$:

$$Z_s(s) = Z_s'(s) + sL_s'$$

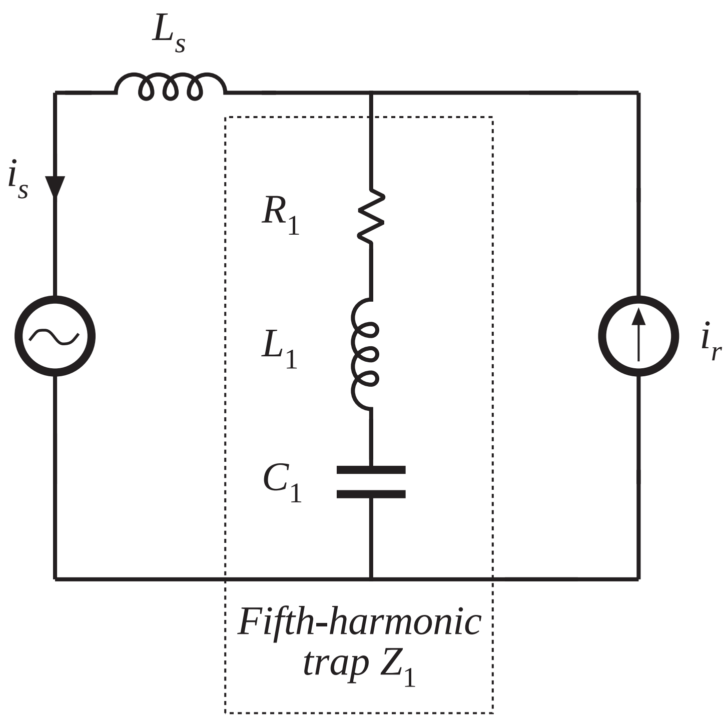


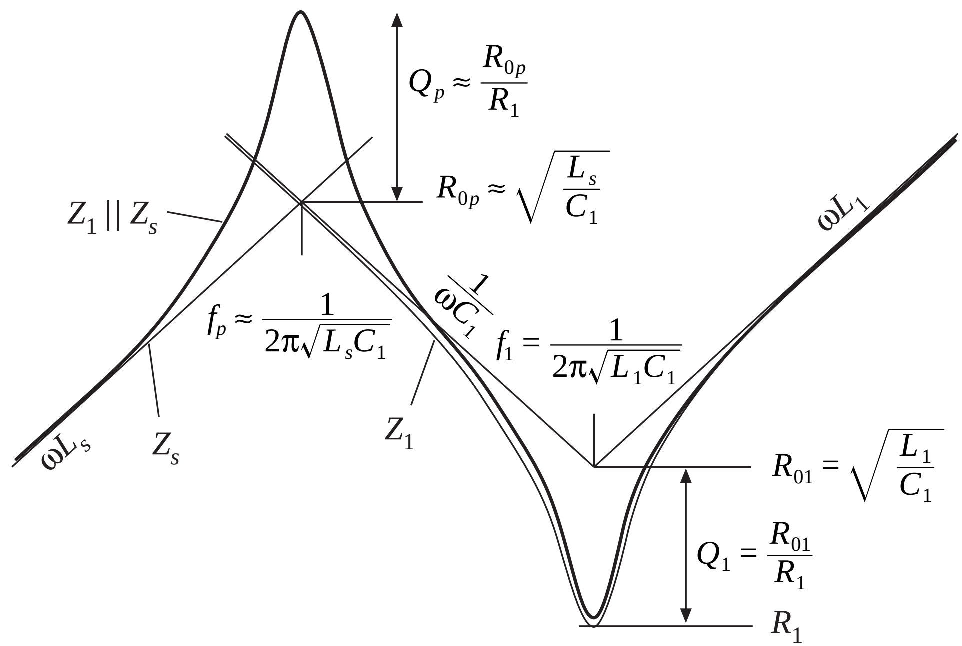
Filter transfer function

$$H(s) = \frac{i_s(s)}{i_R(s)} = \frac{Z_1 \parallel Z_2 \parallel \cdots}{Z_s + Z_1 \parallel Z_2 \parallel \cdots} \quad \text{or} \quad H(s) = \frac{i_s(s)}{i_R(s)} = \frac{Z_s \parallel Z_1 \parallel Z_2 \parallel \cdots}{Z_s}$$

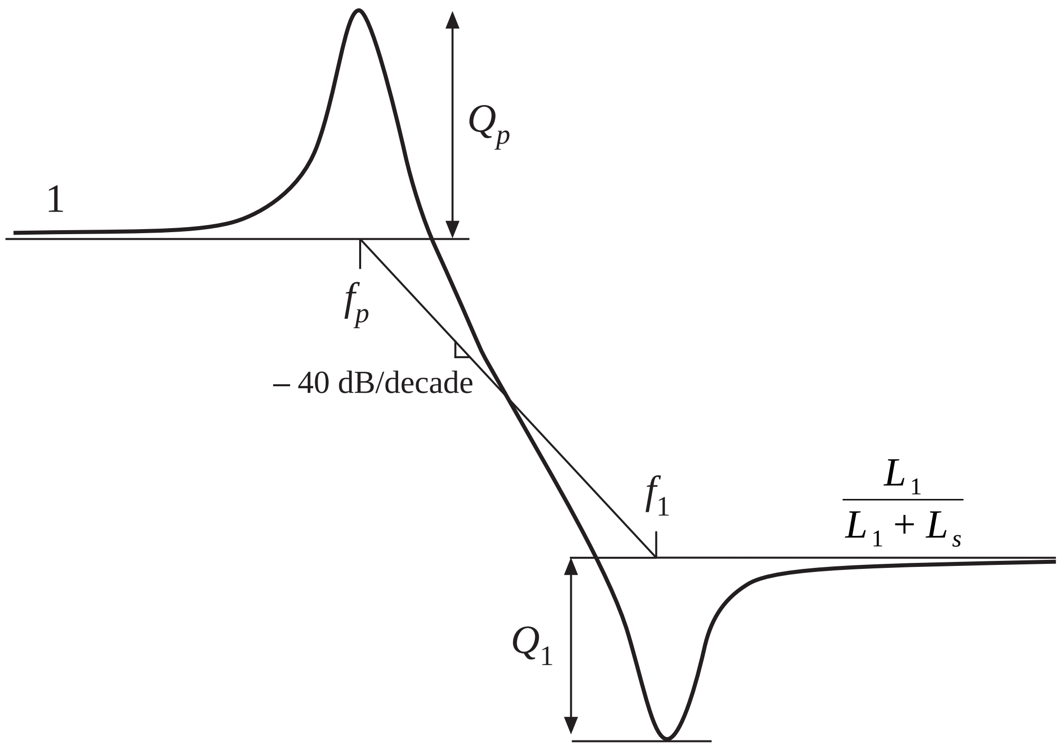


Simple example



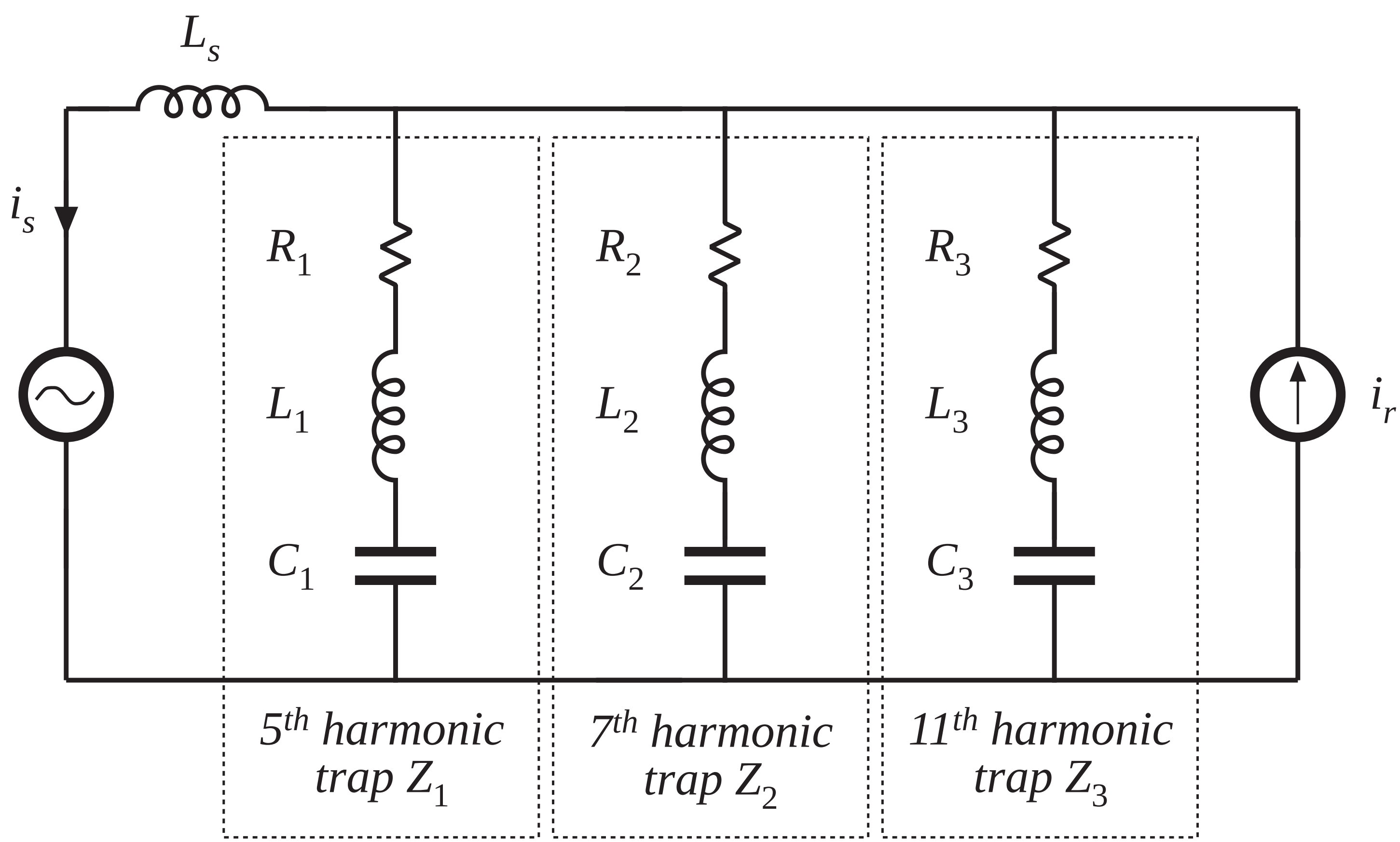


Simple example: transfer function



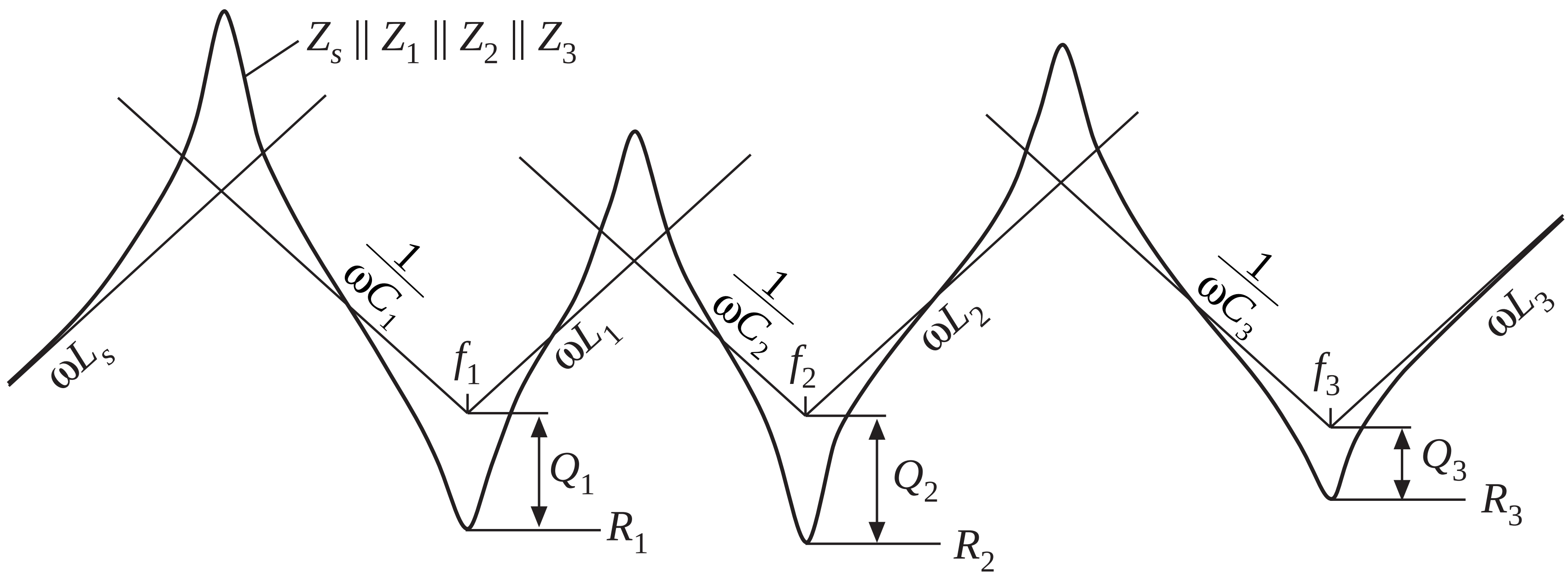
- Series resonance: fifth harmonic trap
- Parallel resonance: C_1 and L_s
- Parallel resonance tends to increase amplitude of third harmonic
- Q of parallel resonance is larger than Q of series resonance

Example 2

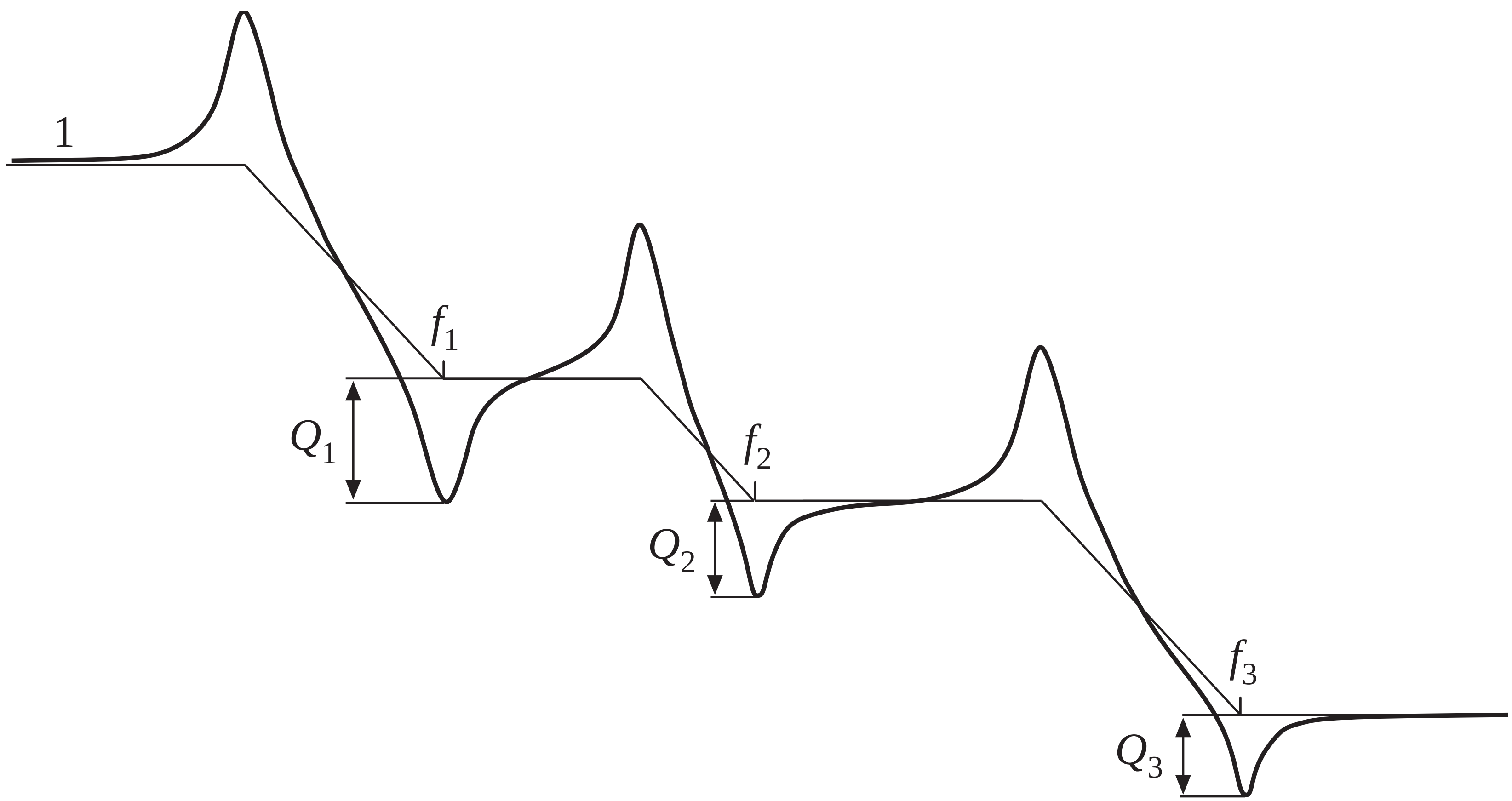


L_s
i_s
R_1
L_1
C_1
R_2
L_2
C_2
R_3
L_3
C_3
i_r
5^{th} harmonic trap Z_1
7^{th} harmonic trap Z_2
11^{th} harmonic trap Z_3

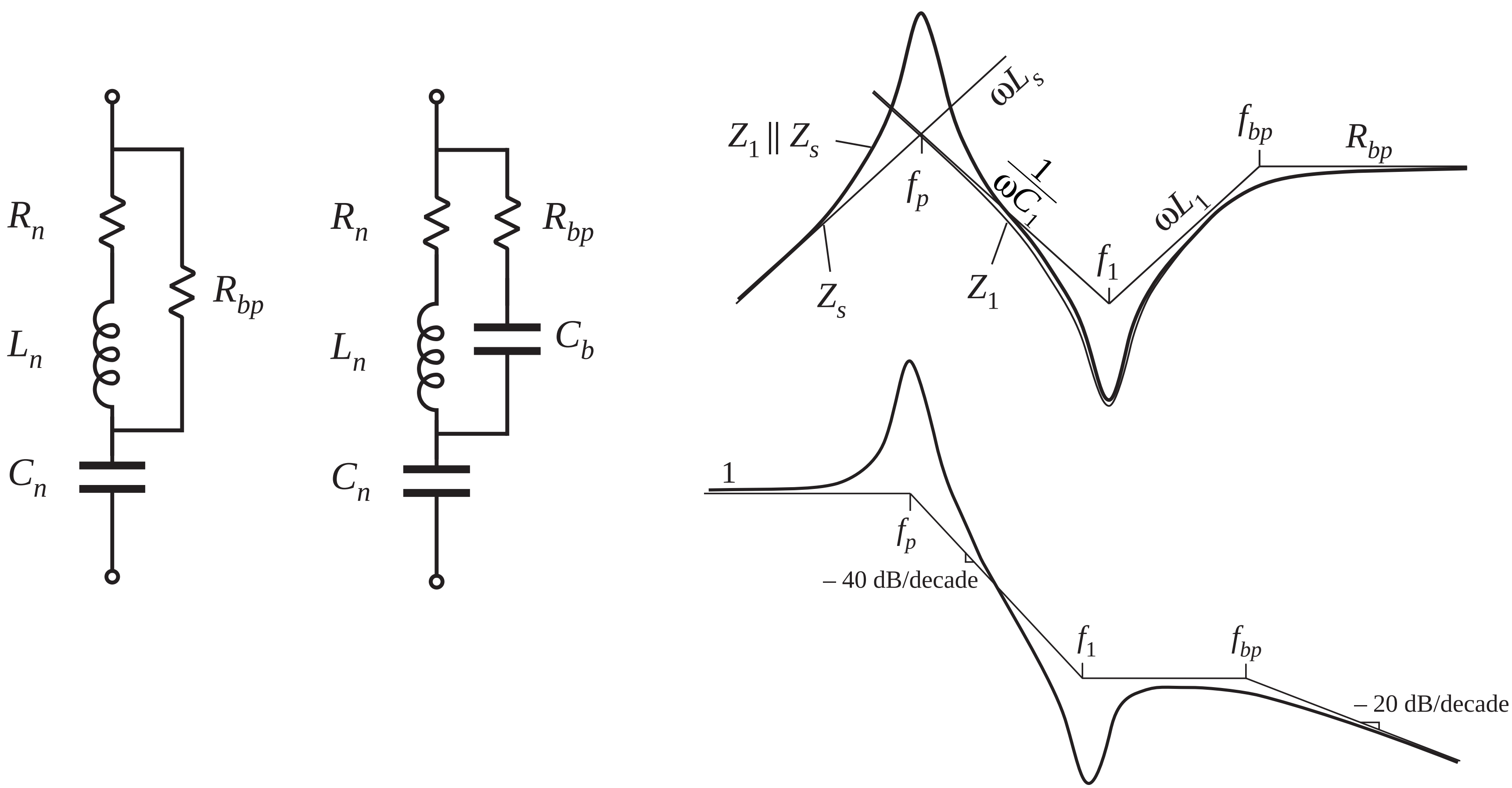
Approximate impedance asymptotes



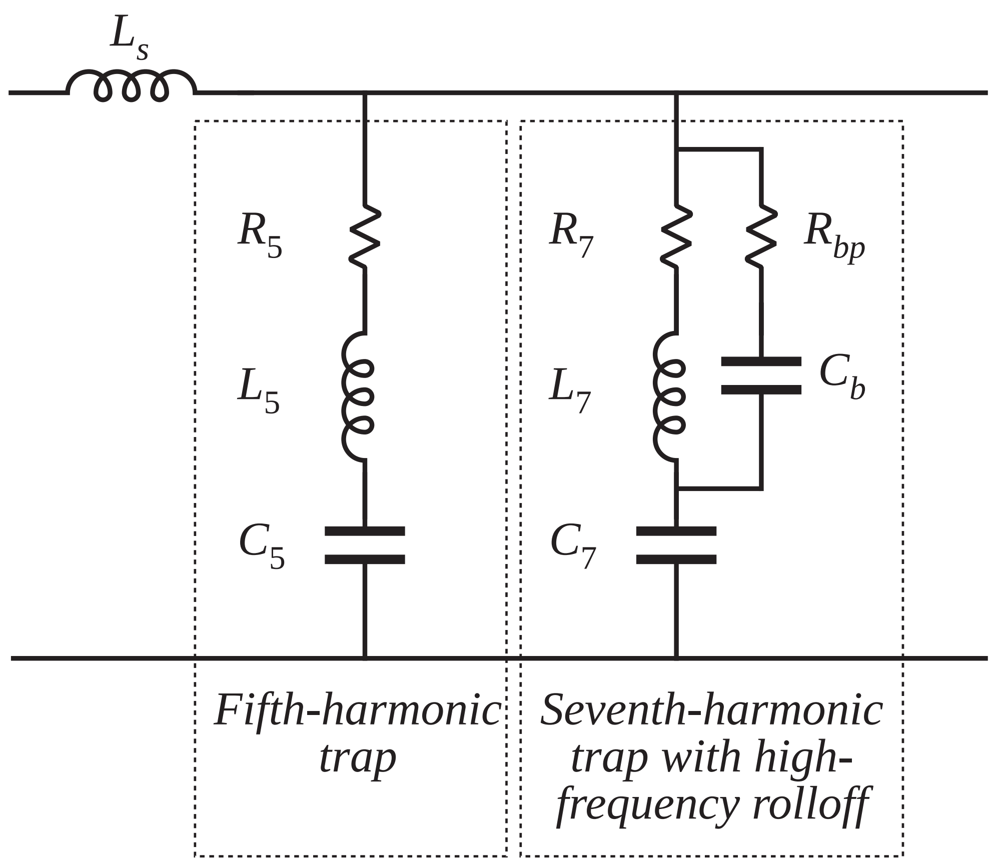
Transfer function asymptotes



Bypass resistor



Harmonic trap filter with high-frequency roll-off

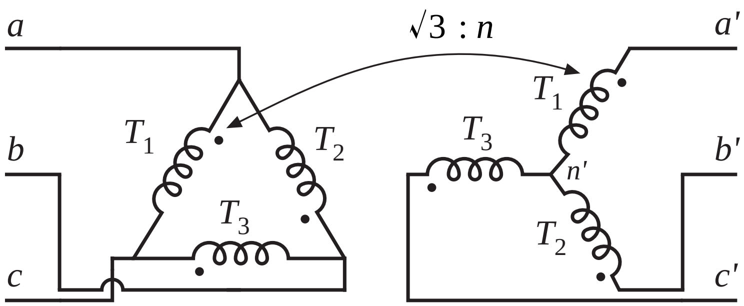


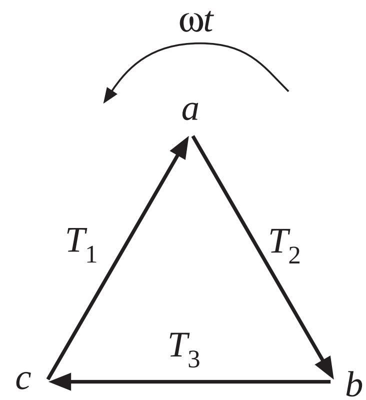
16.5 Transformer connections

Three-phase transformer connections can be used to shift the phase of the voltages and currents

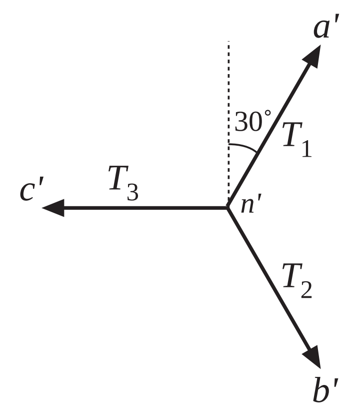
This shifted phase can be used to cancel out the low-order harmonics

Three-phase delta-wye transformer connection shifts phase by 30˚:



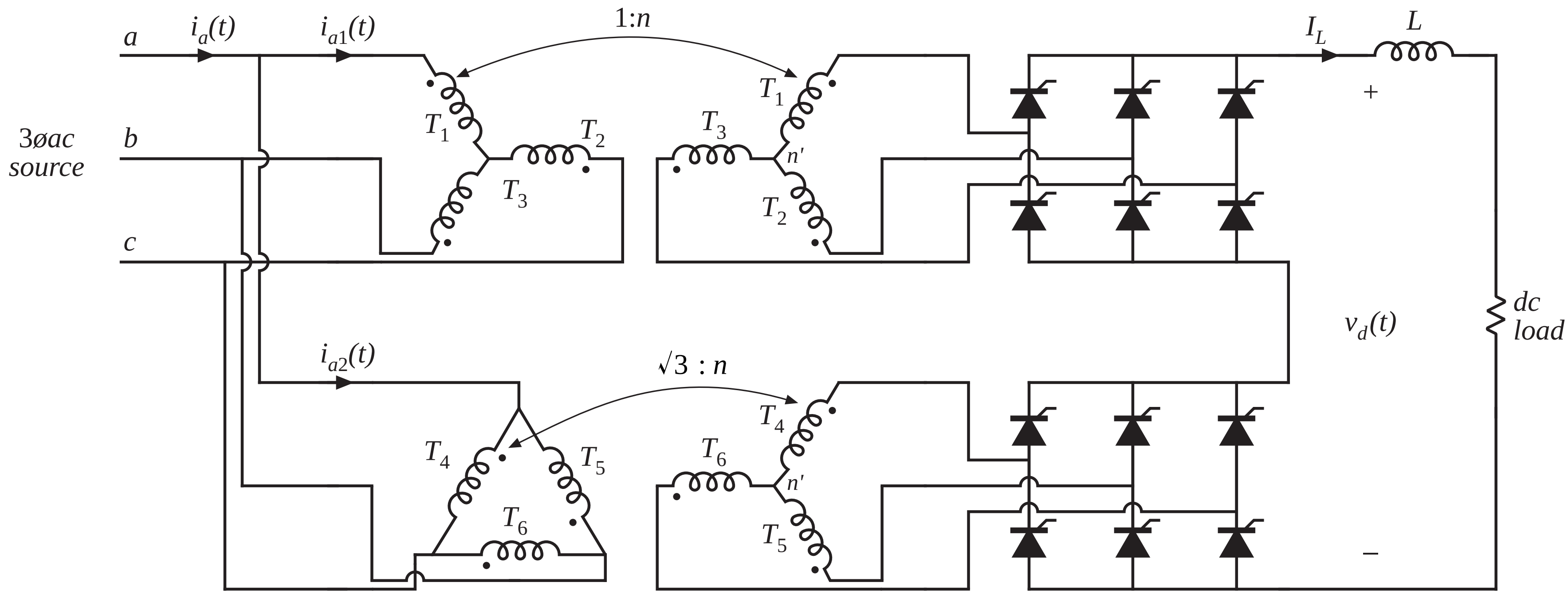


Primary voltages

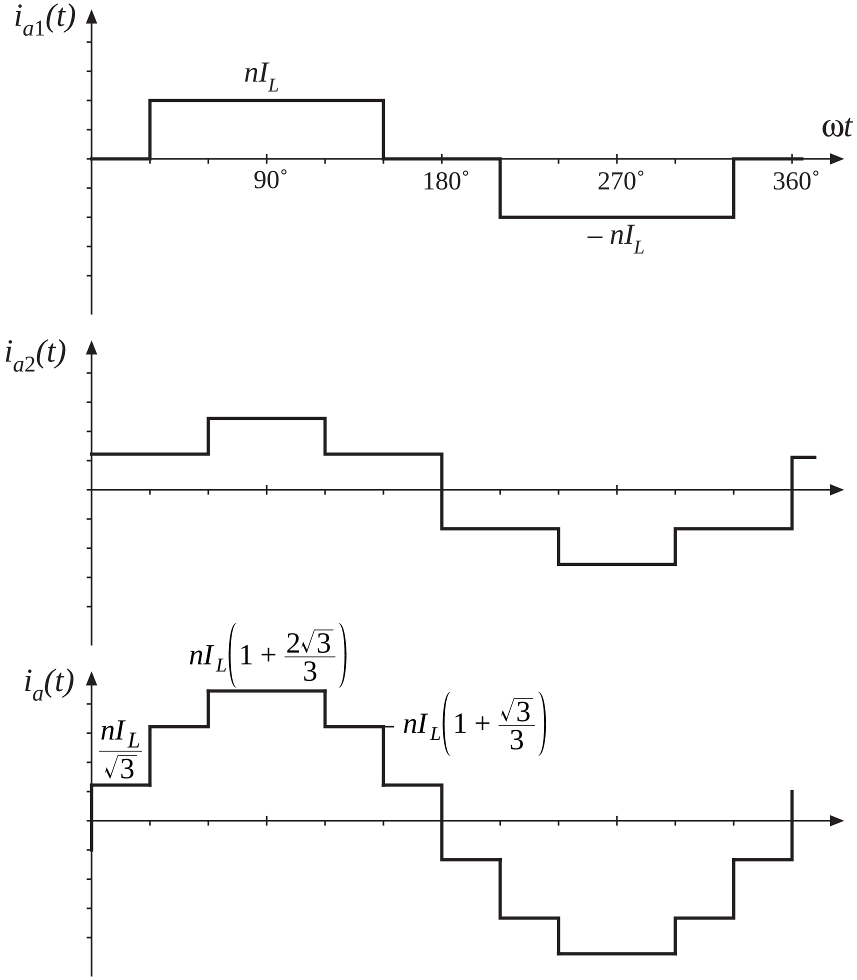


Secondary voltages

Twelve-pulse rectifier



Waveforms of 12 pulse rectifier



- Ac line current contains 1st, 11th, 13th, 23rd, 25th, etc. These harmonic amplitudes vary as $1/n$
- 5th, 7th, 17th, 19th, etc. harmonics are eliminated

Rectifiers with high pulse number

Eighteen-pulse rectifier:

- Use three six-pulse rectifiers
- Transformer connections shift phase by 0˚, +20˚, and –20˚
- No 5th, 7th, 11th, 13th harmonics

Twenty-four-pulse rectifier

- Use four six-pulse rectifiers
- Transformer connections shift phase by 0˚, 15˚, –15˚, and 30˚
- No 5th, 7th, 11th, 13th, 17th, or 19th harmonics

If p is pulse number, then rectifier produces line current harmonics of number $n = pk \pm 1$, with $k = 0, 1, 2, ...$