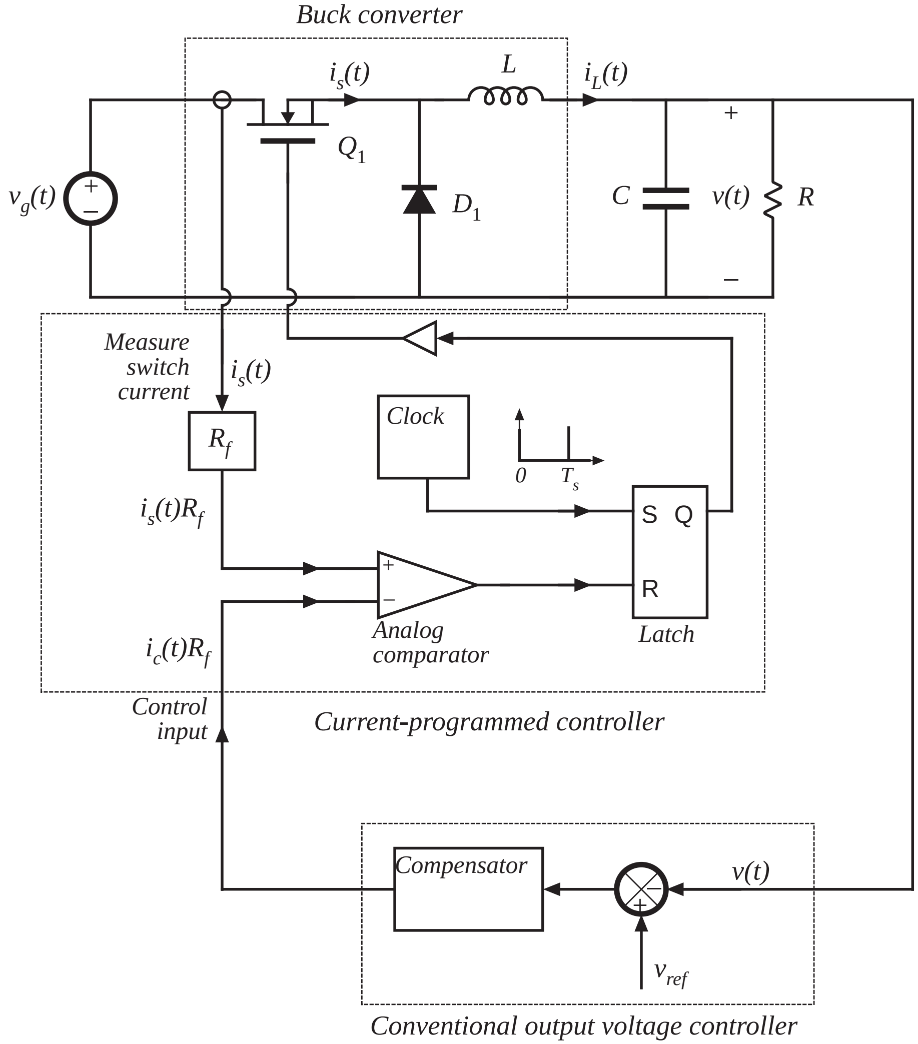
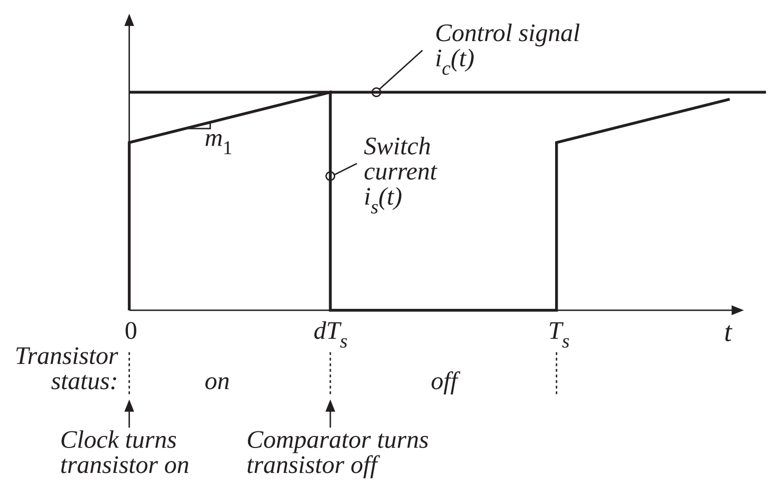
Chapter 11
Current Programmed Control



The peak transistor current replaces the duty cycle as the converter control input.



Current programmed control vs. duty cycle control

Advantages of current programmed control:

- Simpler dynamics —inductor pole is moved to high frequency
- Simple robust output voltage control, with large phase margin, can be obtained without use of compensator lead networks
- It is always necessary to sense the transistor current, to protect against overcurrent failures. We may as well use the information during normal operation, to obtain better control
- Transistor failures due to excessive current can be prevented simply by limiting $i_c(t)$
- Transformer saturation problems in bridge or push-pull converters can be mitigated

A disadvantage: susceptibility to noise

Chapter 11: Outline

11.1 Oscillation for $D > 0.5$

11.2 A simple first-order model

- Simple model via algebraic approach
- Averaged switch modeling

11.3 A more accurate model

- Current programmed controller model: block diagram
- CPM buck converter example

11.4 Discontinuous conduction mode

11.5 Summary

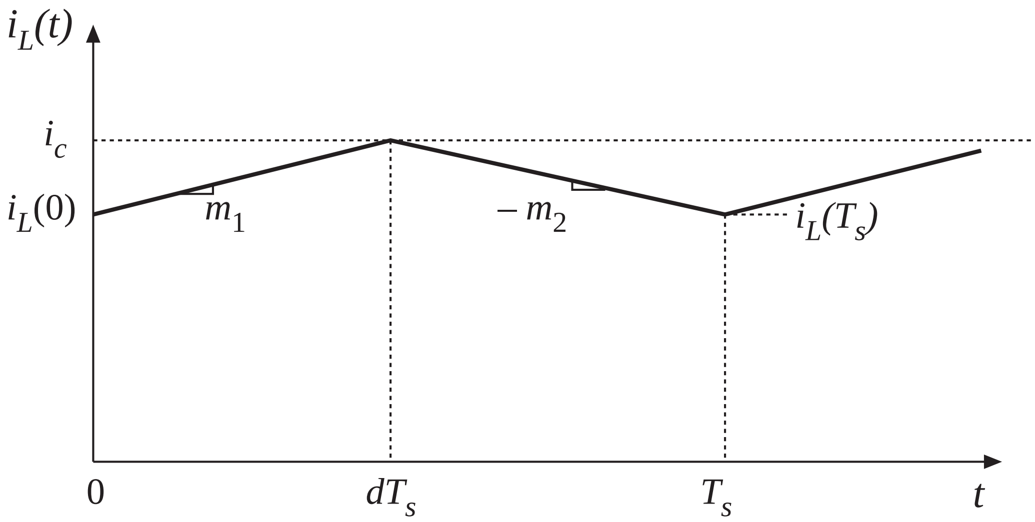
11.1 Oscillation for $D > 0.5$

- The current programmed controller is inherently unstable for $D > 0.5$, regardless of the converter topology
- Controller can be stabilized by addition of an artificial ramp

Objectives of this section:

- Stability analysis
- Describe artificial ramp scheme

Inductor current waveform, CCM



Inductor current slopes m_1 *and* $-m_2$

buck converter

$$m_1 = \frac{v_g - v}{L} \qquad -m_2 = -\frac{v}{L}$$

boost converter

$$m_1 = \frac{v_g}{L} \qquad -m_2 = \frac{v_g - v}{L}$$

buck–boost converter

$$m_1 = \frac{v_g}{L} \qquad -m_2 = \frac{v}{L}$$

Steady-state inductor current waveform, CPM

First interval:

$$i_L(dT_s) = i_c = i_L(0) + m_1 dT_s$$

Solve for d:

$$d = \frac{i_c - i_L(0)}{m_1 T_s}$$

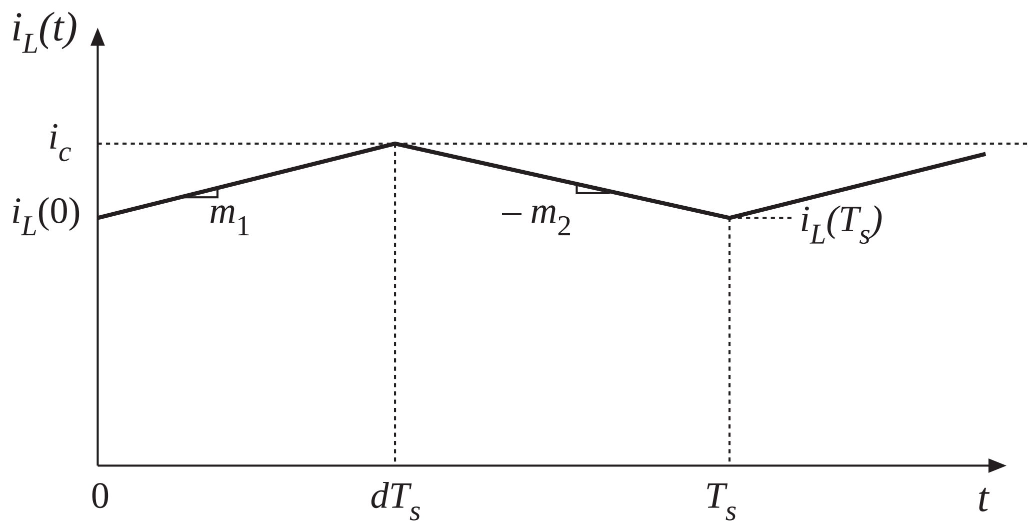
Second interval:

$$\begin{aligned} i_L(T_s) &= i_L(dT_s) - m_2 d'T_s \\ &= i_L(0) + m_1 dT_s - m_2 d'T_s \end{aligned}$$

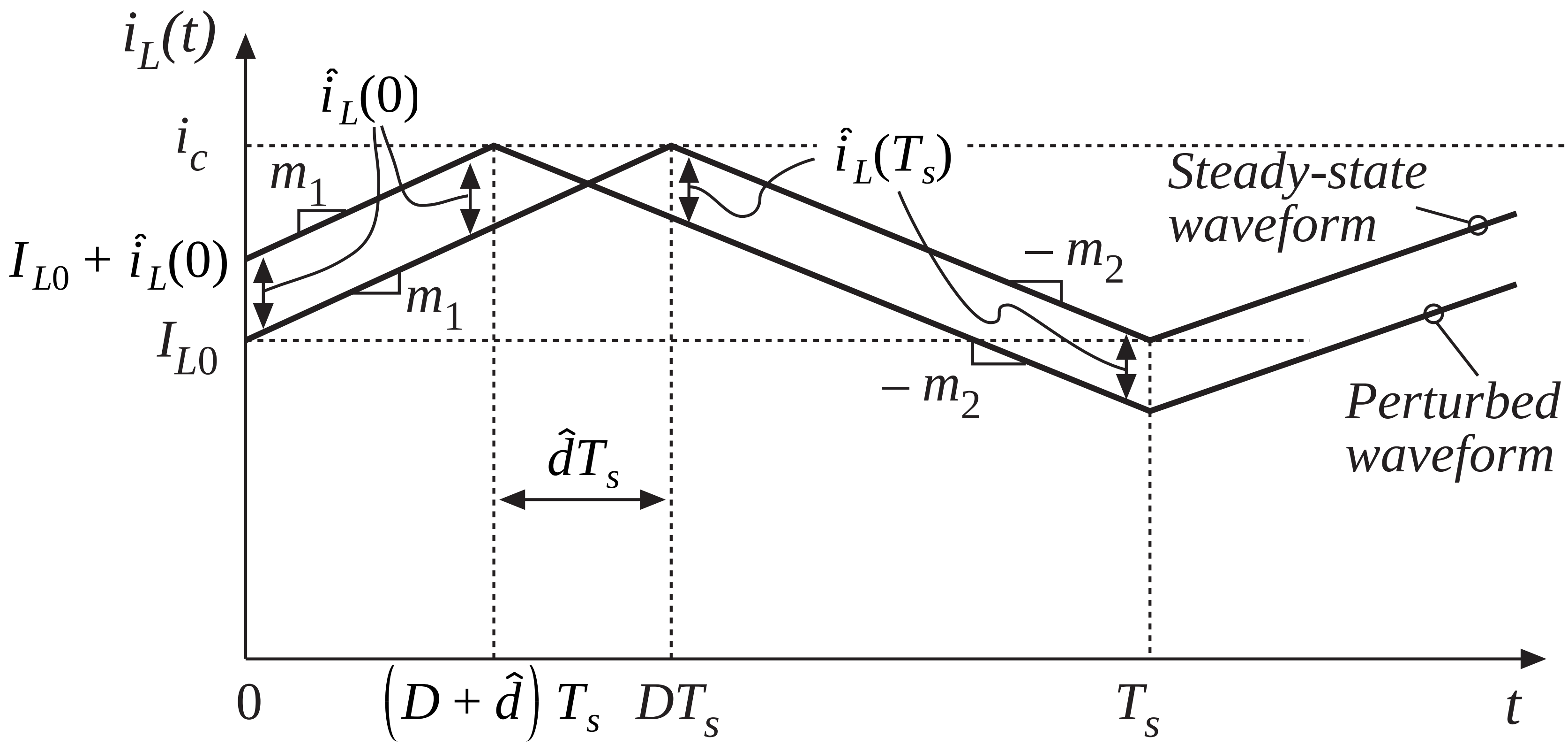
In steady state:

$$0 = M_1 DT_s - M_2 D'T_s$$

$$\frac{M_2}{M_1} = \frac{D}{D'}$$

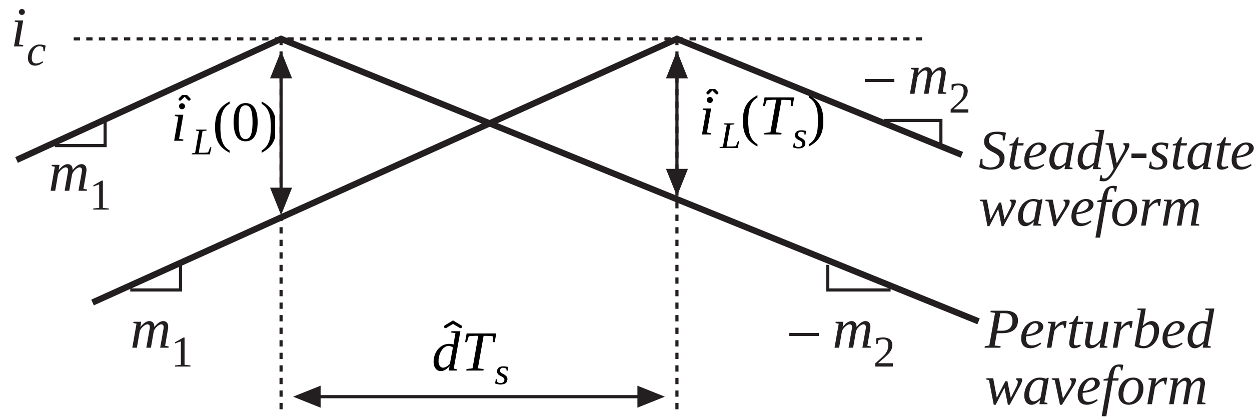


Perturbed inductor current waveform



Change in inductor current perturbation over one switching period

magnified view



$$\hat{i}_L(0) = -m_1 \hat{d} T_s$$

$$\hat{i}_L(T_s) = m_2 \hat{d} T_s$$

$$\hat{i}_L(T_s) = \hat{i}_L(0)\left(-\frac{m_2}{m_1}\right)$$

$$\hat{i}_L(T_s) = \hat{i}_L(0)\left(-\frac{D}{D'}\right)$$

Change in inductor current perturbation over many switching periods

$$\hat{i}_L(T_s) = \hat{i}_L(0)\left(-\frac{D}{D'}\right)$$

$$\hat{i}_L(2T_s) = \hat{i}_L(T_s)\left(-\frac{D}{D'}\right) = \hat{i}_L(0)\left(-\frac{D}{D'}\right)^2$$

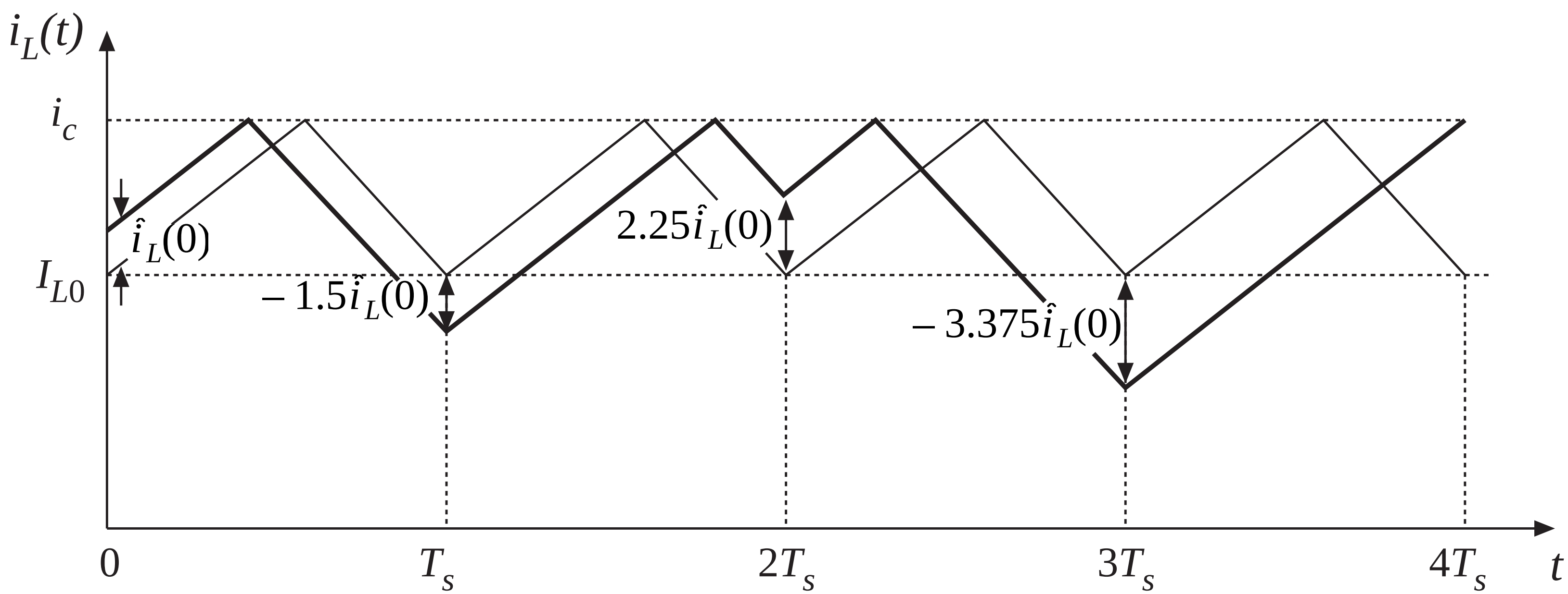
$$\hat{i}_L(nT_s) = \hat{i}_L((n-1)T_s)\left(-\frac{D}{D'}\right) = \hat{i}_L(0)\left(-\frac{D}{D'}\right)^n$$

$$\left|\hat{i}_L(nT_s)\right| \rightarrow \begin{cases} 0 & \text{when } \left|-\frac{D}{D'}\right| < 1 \\ \infty & \text{when } \left|-\frac{D}{D'}\right| > 1 \end{cases}$$

For stability: $D < 0.5$

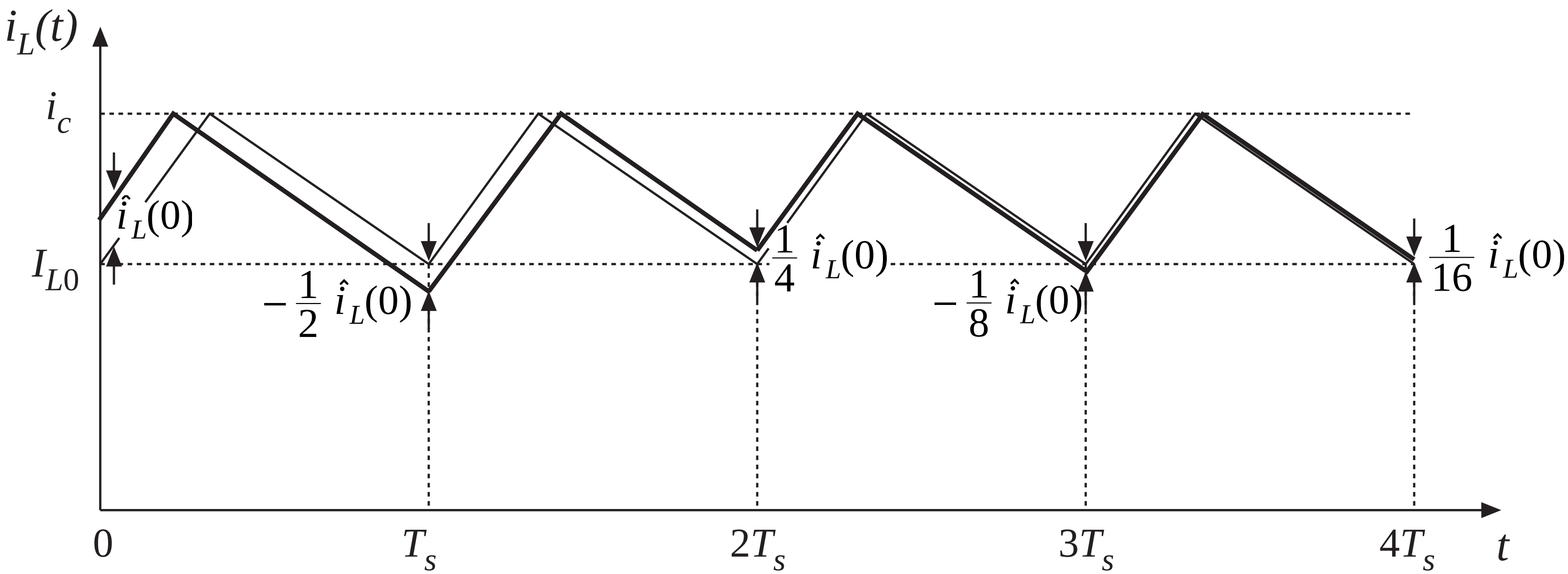
Example: unstable operation for $D = 0.6$

$$\alpha = -\frac{D}{D'} = \left(-\frac{0.6}{0.4}\right) = -1.5$$

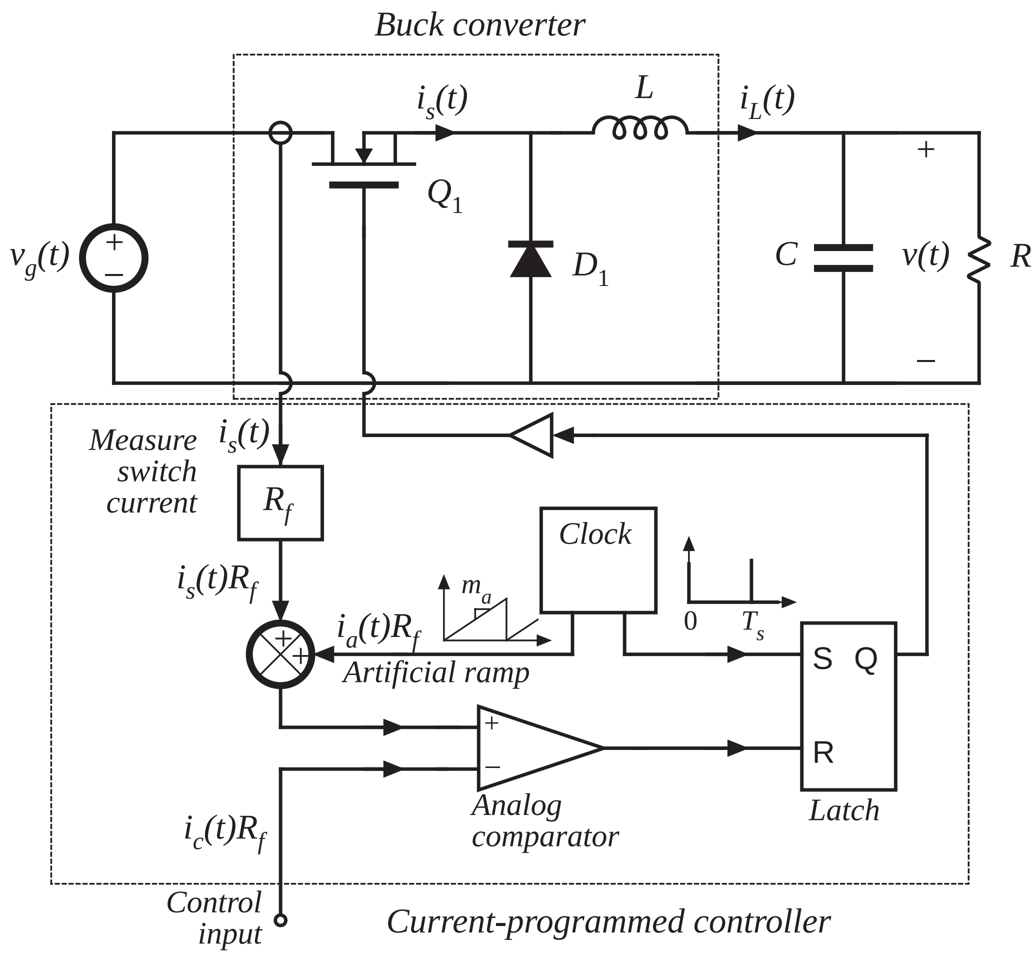


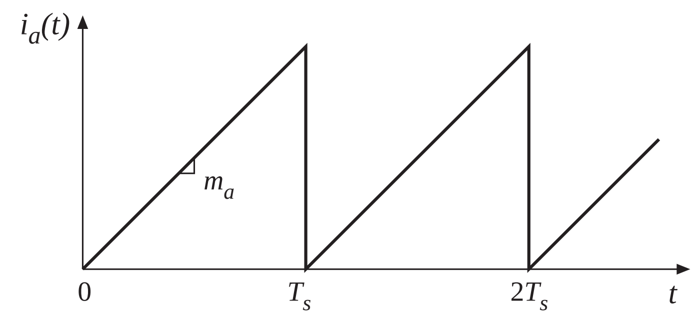
Example: stable operation for $D = 1/3$

$$\alpha = -\frac{D}{D'} = \left(-\frac{1/3}{2/3}\right) = -0.5$$



Stabilization via addition of an artificial ramp to the measured switch current waveform





Now, transistor switches off when

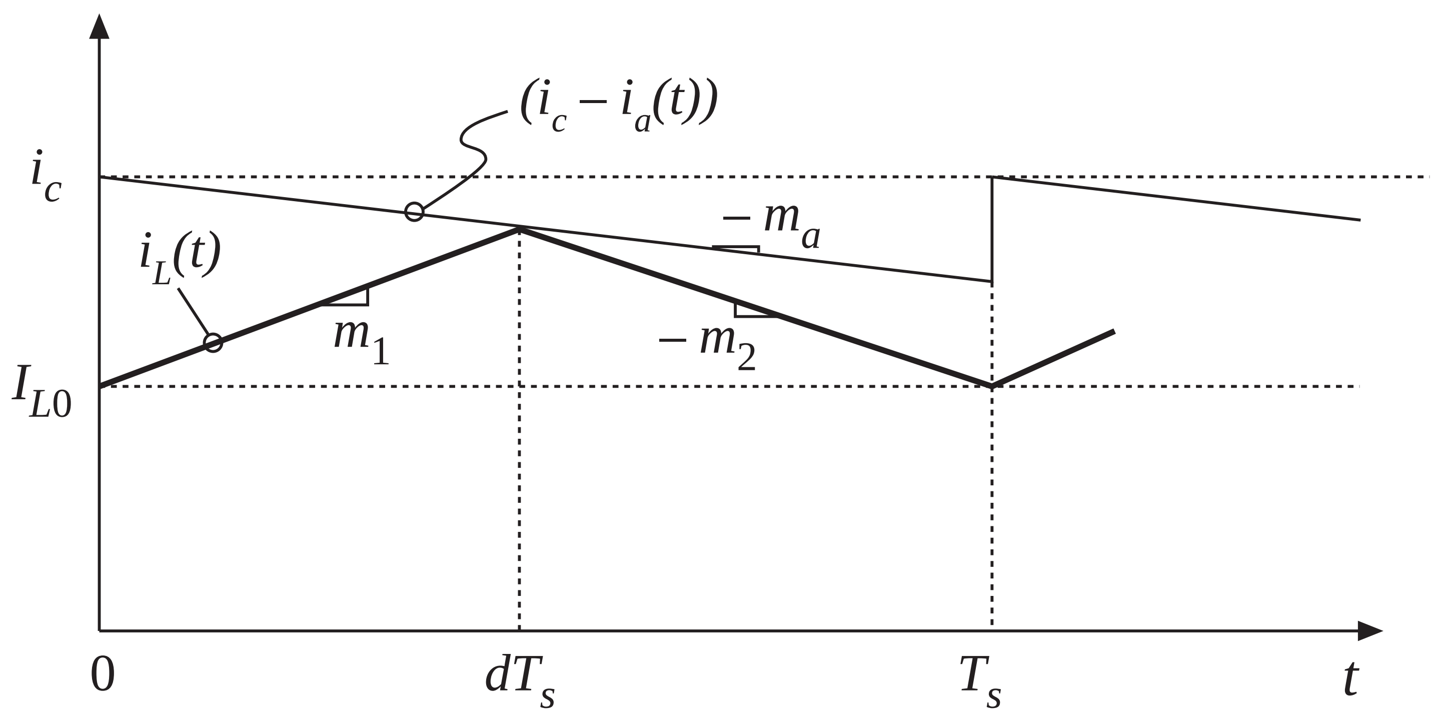
$$i_a(dT_s) + i_L(dT_s) = i_c$$

or,

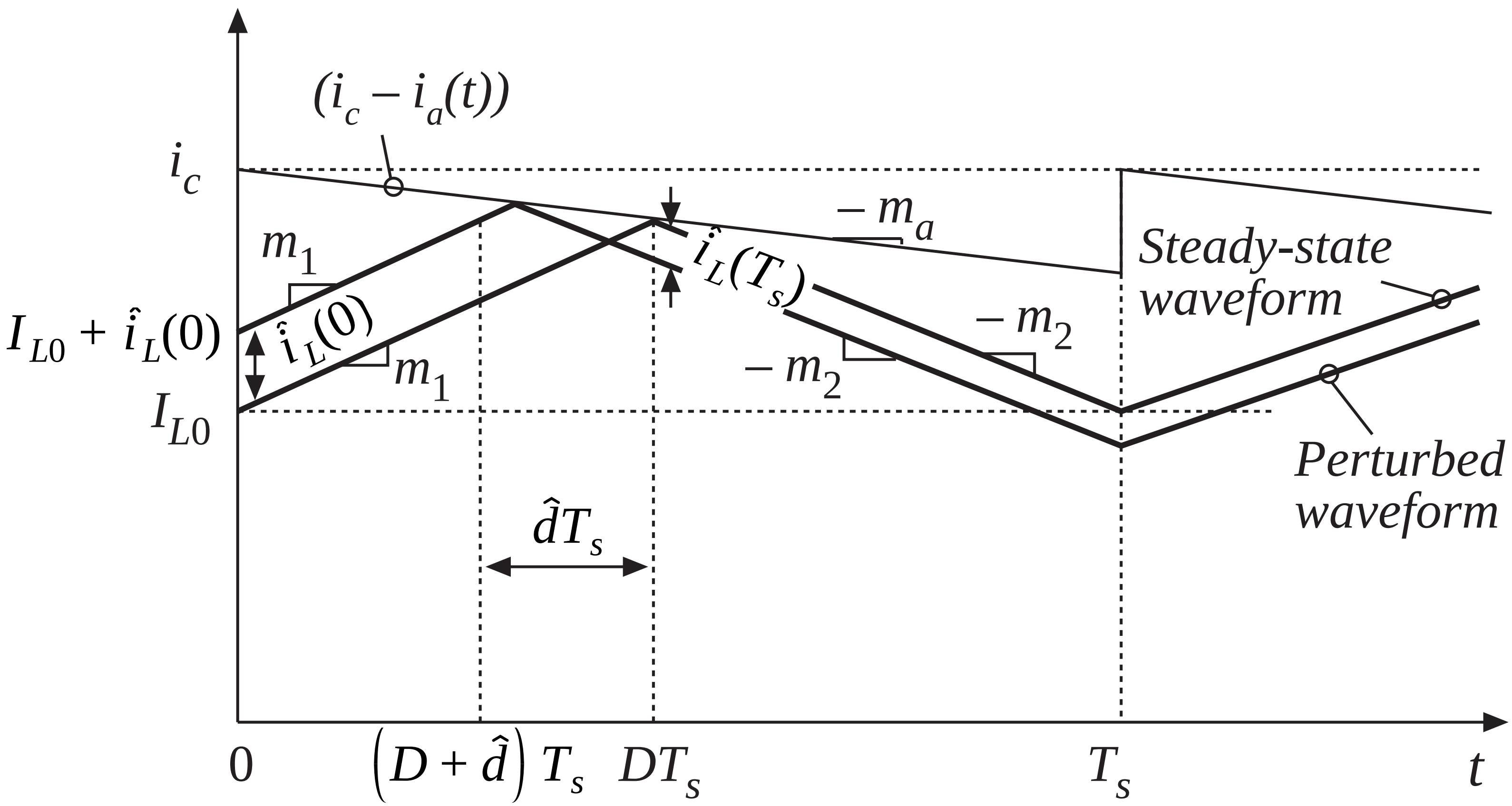
$$i_L(dT_s) = i_c - i_a(dT_s)$$

Steady state waveforms with artificial ramp

$$i_L(dT_s) = i_c - i_a(dT_s)$$



Stability analysis: perturbed waveform



Stability analysis: change in perturbation over complete switching periods

First subinterval:

$$\hat{i}_L(0) = -\,\hat{d}T_s\left(m_1 + m_a\right)$$

Second subinterval:

$$\hat{i}_L(T_s) = -\,\hat{d}T_s\left(m_a - m_2\right)$$

Net change over one switching period:

$$\hat{i}_L(T_s) = \hat{i}_L(0)\left(-\frac{m_2 - m_a}{m_1 + m_a}\right)$$

After n switching periods:

$$\hat{i}_L(nT_s) = \hat{i}_L((n-1)T_s)\left(-\frac{m_2 - m_a}{m_1 + m_a}\right) = \hat{i}_L(0)\left(-\frac{m_2 - m_a}{m_1 + m_a}\right)^n = \hat{i}_L(0)\,\alpha^n$$

Characteristic value:

$$\alpha = -\frac{m_2 - m_a}{m_1 + m_a}$$

$$\left|\,\hat{i}_L(nT_s)\right| \rightarrow \begin{cases} 0 & \text{when } \left|\alpha\right| < 1 \\ \infty & \text{when } \left|\alpha\right| > 1 \end{cases}$$

The characteristic value α

$$\alpha = -\frac{1 - \frac{m_a}{m_2}}{\frac{D'}{D} + \frac{m_a}{m_2}}$$

- For stability, require $|\alpha| < 1$
- Buck and buck-boost converters: $m_2 = -v/L$

  So if v is well-regulated, then m_2 is also well-regulated
- A common choice: $m_a = 0.5\, m_2$

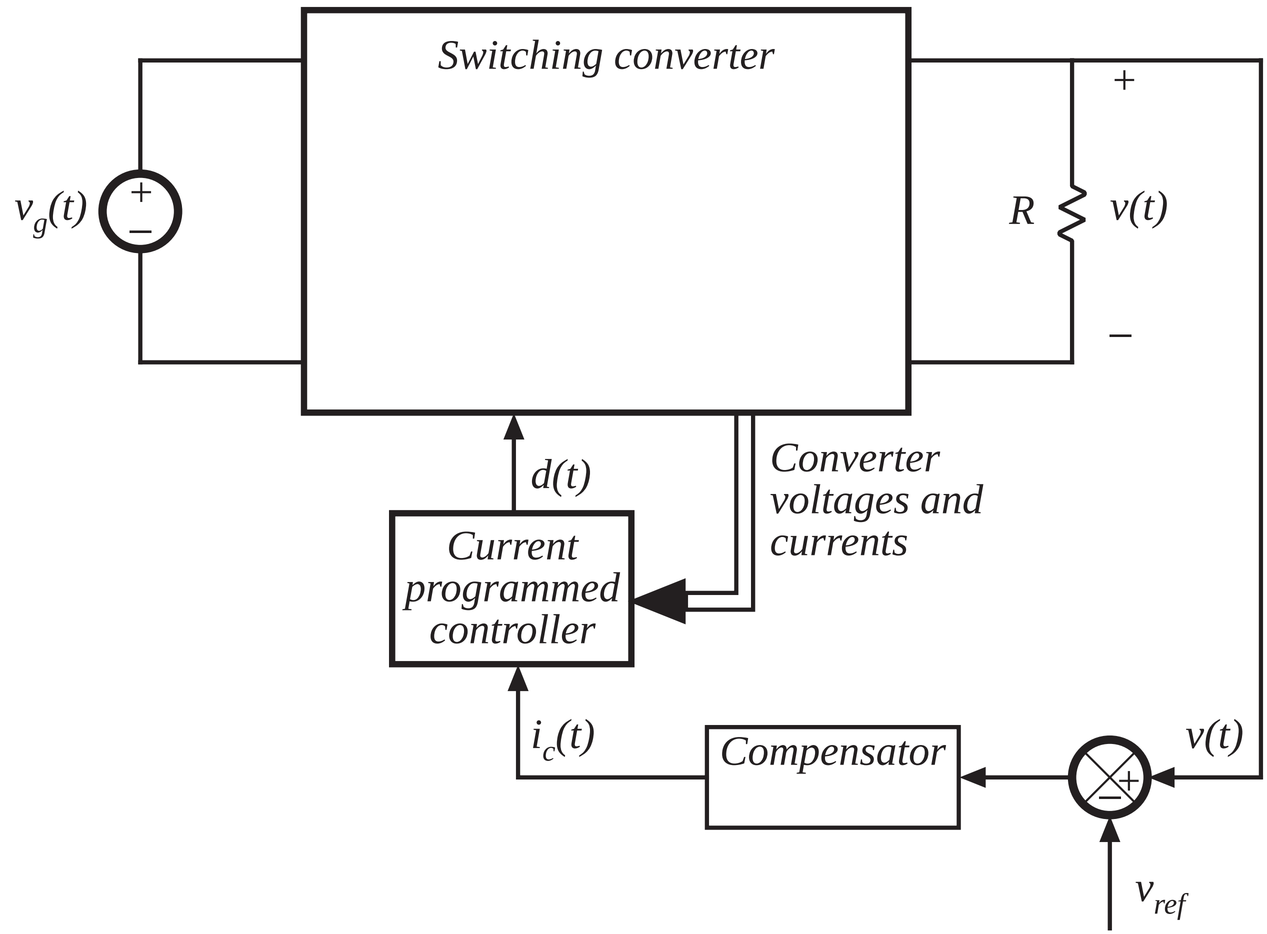
  This leads to $\alpha = -1$ at $D = 1$, and $|\alpha| < 1$ for $0 \le D < 1$.

  The minimum α that leads to stability for all D.
- Another common choice: $m_a = m_2$

  This leads to $\alpha = 0$ for $0 \le D < 1$.

  Deadbeat control, finite settling time

11.2 A Simple First-Order Model



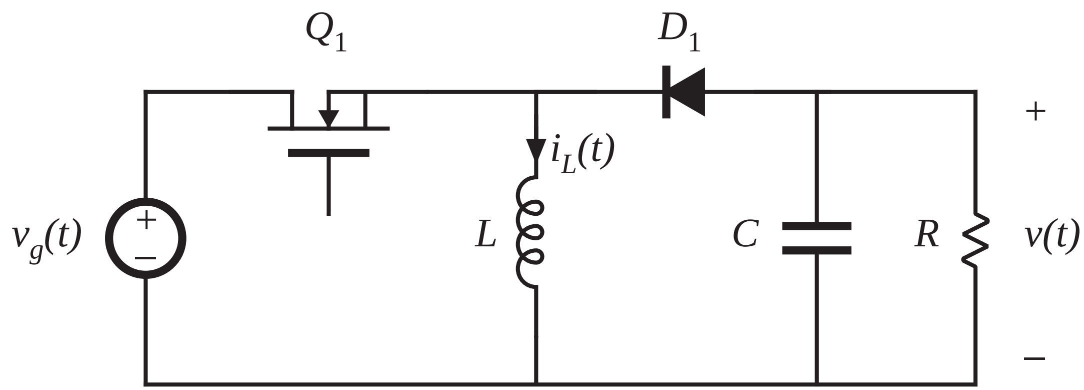
The first-order approximation

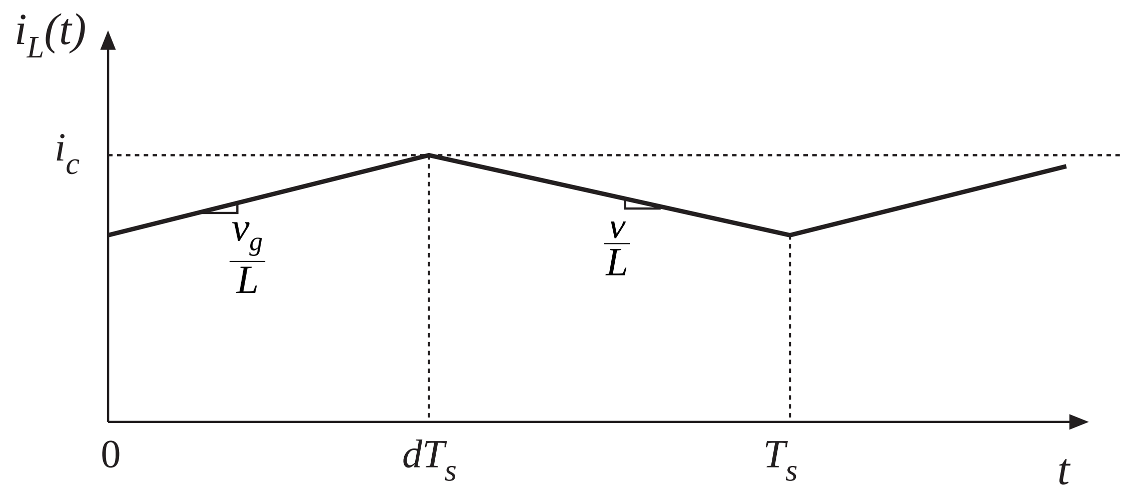
$$\left\langle i_L(t) \right\rangle_{T_s} = i_c(t)$$

- Neglects switching ripple and artificial ramp
- Yields physical insight and simple first-order model
- Accurate when converter operates well into CCM (so that switching ripple is small) and when the magnitude of the artificial ramp is not too large
- Resulting small-signal relation:

$$\hat{i}_L(s) \approx \hat{i}_c(s)$$

11.2.1 Simple model via algebraic approach: CCM buck-boost example





Small-signal equations of CCM buck–boost, duty cycle control

$$L\frac{d\hat{i}_L(t)}{dt} = D\hat{v}_g(t) + D'\hat{v}(t) + \left(V_g - V\right)\hat{d}(t)$$

$$C\frac{d\hat{v}(t)}{dt} = -D'\hat{i}_L - \frac{\hat{v}(t)}{R} + I_L\hat{d}(t)$$

$$\hat{i}_g(t) = D\hat{i}_L + I_L\hat{d}(t)$$

Derived in Chapter 7

Transformed equations

Take Laplace transform, letting initial conditions be zero:

$$sL\hat{i}_L(s) = D\hat{v}_g(s) + D'\hat{v}(s) + \left(V_g - V\right)\hat{d}(s)$$

$$sC\hat{v}(s) = -\,D'\hat{i}_L(s) - \frac{\hat{v}(s)}{R} + I_L\hat{d}(s)$$

$$\hat{i}_g(s) = D\hat{i}_L(s) + I_L\hat{d}(s)$$

The simple approximation

Now let

$$\hat{i}_L(s) \approx \hat{i}_c(s)$$

Eliminate the duty cycle (now an intermediate variable), to express the equations using the new control input i_L. The inductor equation becomes:

$$sL\hat{i}_c(s) \approx D\hat{v}_g(s) + D'\hat{v}(s) + \left(V_g - V\right)\hat{d}(s)$$

Solve for the duty cycle variations:

$$\hat{d}(s) = \frac{sL\hat{i}_c(s) - D\hat{v}_g(s) - D'\hat{v}(s)}{\left(V_g - V\right)}$$

The simple approximation, continued

Substitute this expression to eliminate the duty cycle from the remaining equations:

$$sC\hat{v}(s) = -D'\hat{i}_c(s) - \frac{\hat{v}(s)}{R} + I_L\,\frac{sL\hat{i}_c(s) - D\hat{v}_g(s) - D'\hat{v}(s)}{\left(V_g - V\right)}$$

$$\hat{i}_g(s) = D\hat{i}_c(s) + I_L\,\frac{sL\hat{i}_c(s) - D\hat{v}_g(s) - D'\hat{v}(s)}{\left(V_g - V\right)}$$

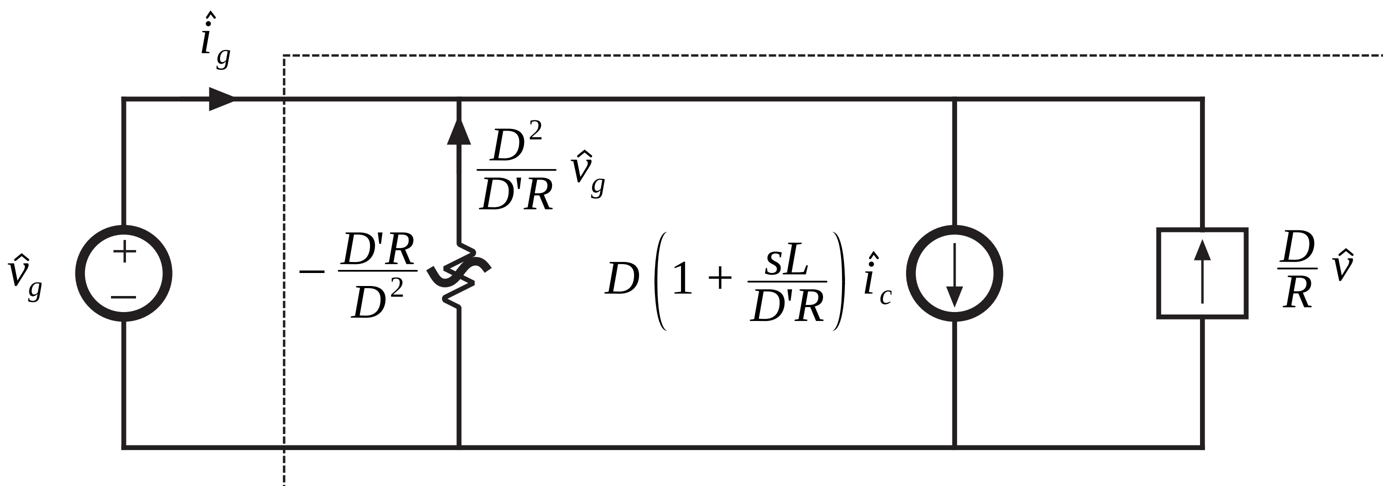
Collect terms, simplify using steady-state relations:

$$sC\hat{v}(s) = \left(\frac{sLD}{D'R} - D'\right)\hat{i}_c(s) - \left(\frac{D}{R} + \frac{1}{R}\right)\hat{v}(s) - \left(\frac{D^2}{D'R}\right)\hat{v}_g(s)$$

$$\hat{i}_g(s) = \left(\frac{sLD}{D'R} + D\right)\hat{i}_c(s) - \left(\frac{D}{R}\right)\hat{v}(s) - \left(\frac{D^2}{D'R}\right)\hat{v}_g(s)$$

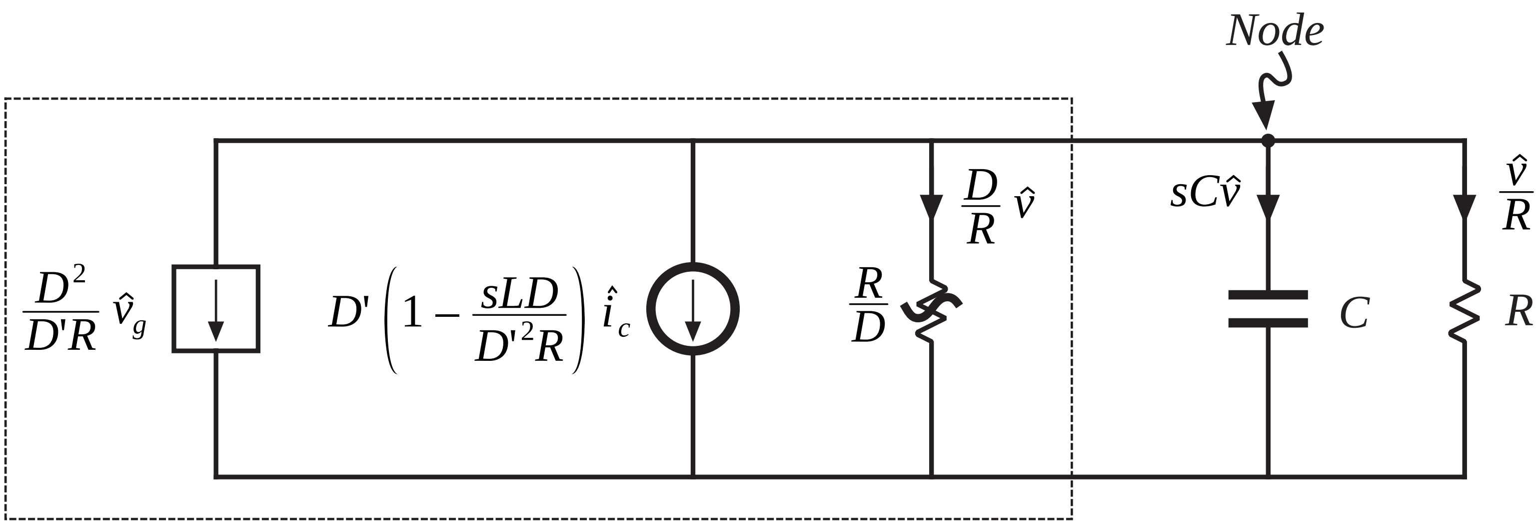
Construct equivalent circuit: input port

$$\hat{i}_g(s) = \left(\frac{sLD}{D'R} + D\right)\hat{i}_c(s) - \left(\frac{D}{R}\right)\hat{v}(s) - \left(\frac{D^2}{D'R}\right)\hat{v}_g(s)$$



Construct equivalent circuit: output port

$$sC\hat{v}(s) = \left(\frac{sLD}{D'R} - D'\right) \hat{i}_c(s) - \left(\frac{D}{R} + \frac{1}{R}\right) \hat{v}(s) - \left(\frac{D^2}{D'R}\right) \hat{v}_g(s)$$



CPM Canonical Model, Simple Approximation

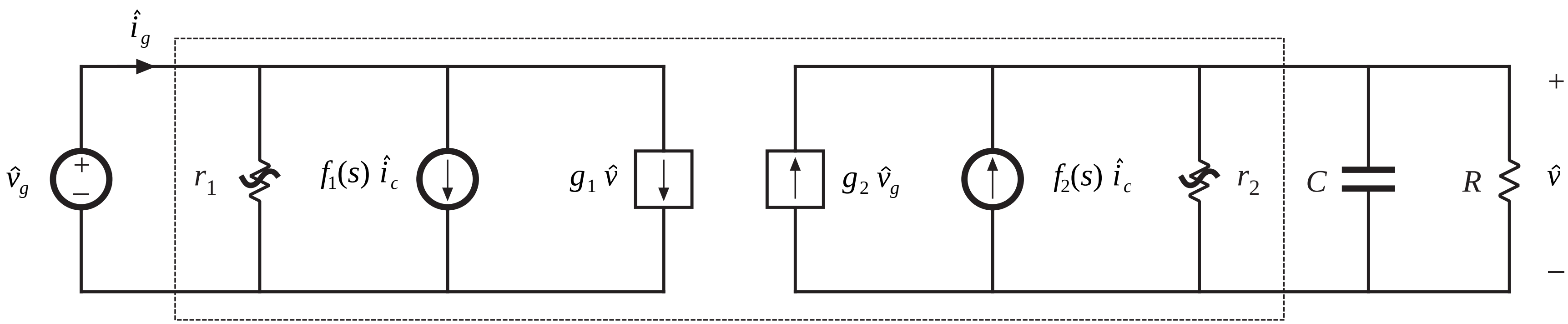
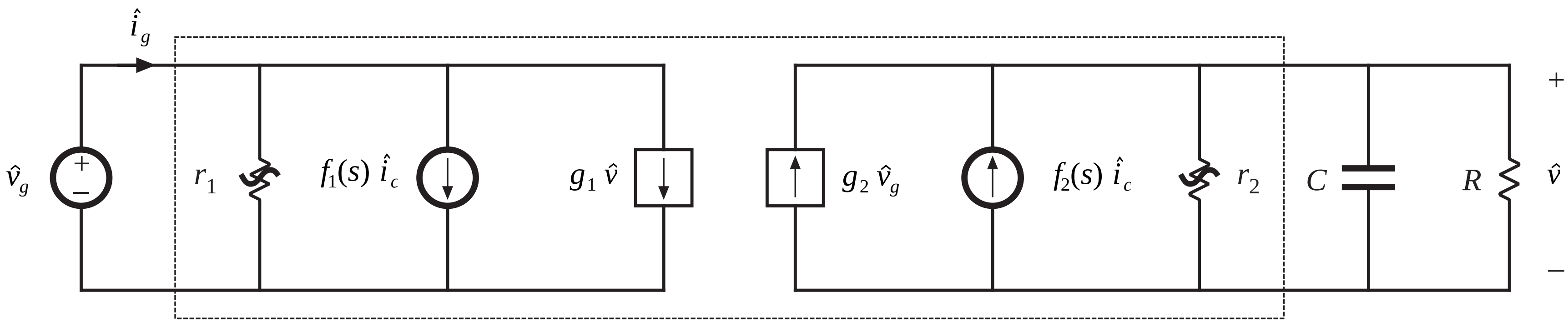


Table of results for basic converters

Table 11.1 Current programmed mode small-signal equivalent circuit parameters, simple model

Converter	g_1	f_1	r_1	g_2	f_2	r_2
Buck	$\frac{D}{R}$	$D\left(1+\frac{sL}{R}\right)$	$-\frac{R}{D^2}$	0	1	∞
Boost	0	1	∞	$\frac{1}{D'R}$	$D'\left(1-\frac{sL}{D'^2R}\right)$	R
Buck-boost	$-\frac{D}{R}$	$D\left(1+\frac{sL}{D'R}\right)$	$-\frac{D'R}{D^2}$	$-\frac{D^2}{D'R}$	$-D'\left(1-\frac{sDL}{D'^2R}\right)$	$\frac{R}{D}$

Transfer functions predicted by simple model



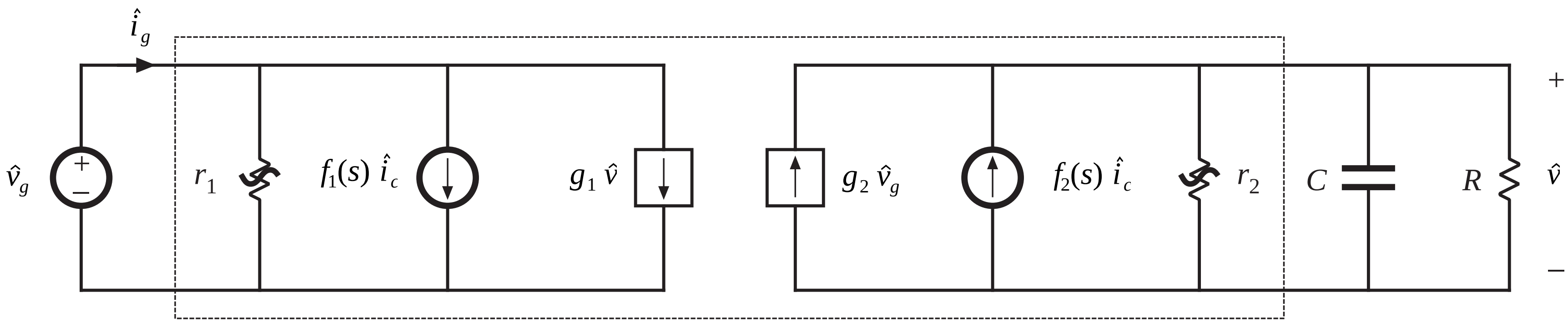
Control-to-output transfer function

$$G_{vc}(s) = \left.\frac{\hat{v}(s)}{\hat{i}_c(s)}\right|_{\hat{v}_g = 0} = f_2 \left(r_2 \parallel R \parallel \frac{1}{sC} \right)$$

Result for buck-boost example

$$G_{vc}(s) = -R\, \frac{D'}{1+D}\, \frac{\left(1 - s\, \frac{DL}{D'^2 R}\right)}{\left(1 + s\, \frac{RC}{1+D}\right)}$$

Transfer functions predicted by simple model



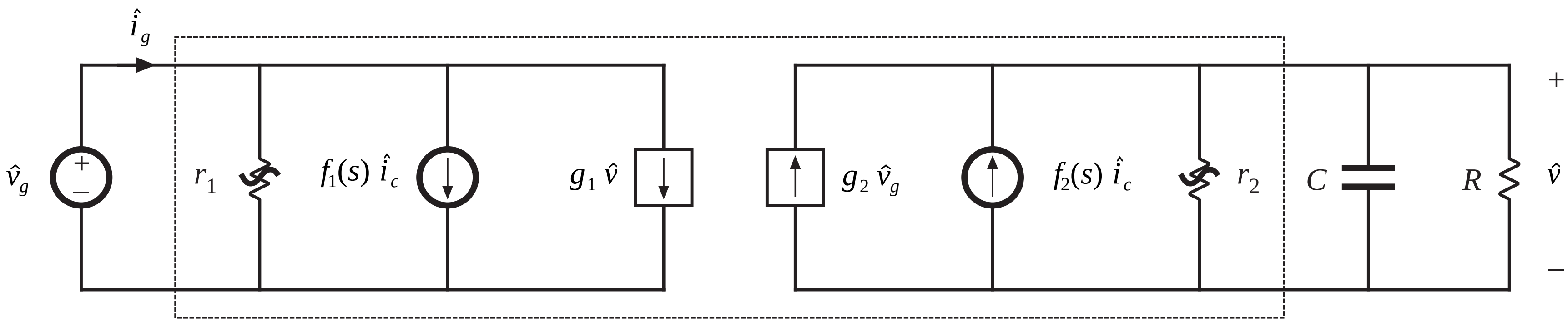
Line-to-output transfer function

$$G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\hat{i}_c = 0} = g_2\left(r_2 \,\|\, R \,\|\, \frac{1}{sC}\right)$$

Result for buck-boost example

$$G_{vg}(s) = -\frac{D^2}{1-D^2}\,\frac{1}{\left(1 + s\,\frac{RC}{1+D}\right)}$$

Transfer functions predicted by simple model



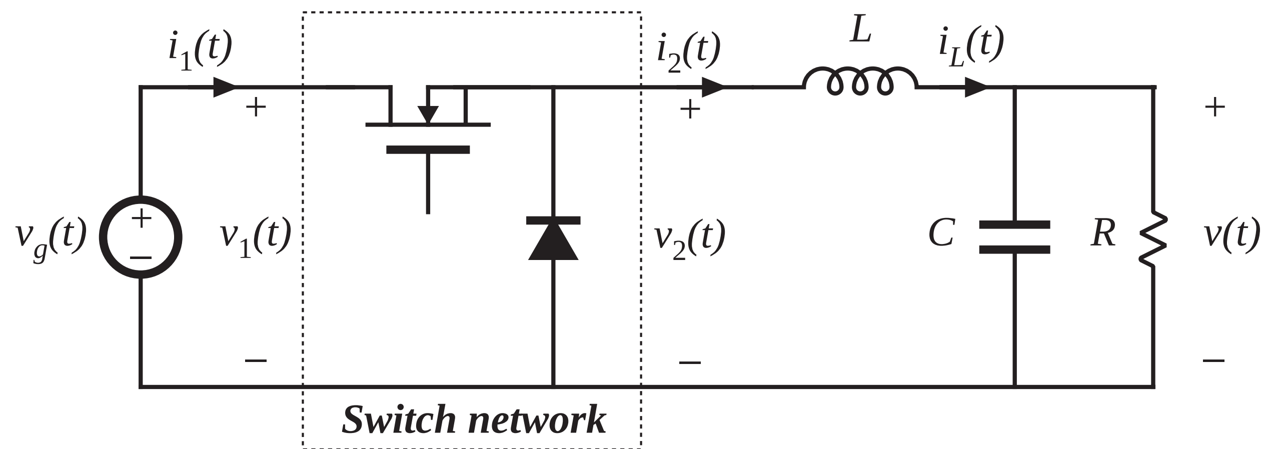
Output impedance

$$Z_{out}(s) = r_2 \parallel R \parallel \frac{1}{sC}$$

Result for buck-boost example

$$Z_{out}(s) = \frac{R}{1+D} \frac{1}{\left(1 + s\,\frac{RC}{1+D}\right)}$$

11.2.2 Averaged switch modeling
with the simple approximation



Averaged terminal waveforms, CCM:

$$\left\langle v_2(t) \right\rangle_{T_s} = d(t) \left\langle v_1(t) \right\rangle_{T_s}$$

$$\left\langle i_1(t) \right\rangle_{T_s} = d(t) \left\langle i_2(t) \right\rangle_{T_s}$$

The simple approximation:

$$\left\langle i_2(t) \right\rangle_{T_s} \approx \left\langle i_c(t) \right\rangle_{T_s}$$

CPM averaged switch equations

$$\left\langle v_2(t) \right\rangle_{T_s} = d(t) \left\langle v_1(t) \right\rangle_{T_s}$$

$$\left\langle i_1(t) \right\rangle_{T_s} = d(t) \left\langle i_2(t) \right\rangle_{T_s}$$

$$\left\langle i_2(t) \right\rangle_{T_s} \approx \left\langle i_c(t) \right\rangle_{T_s}$$

Eliminate duty cycle:

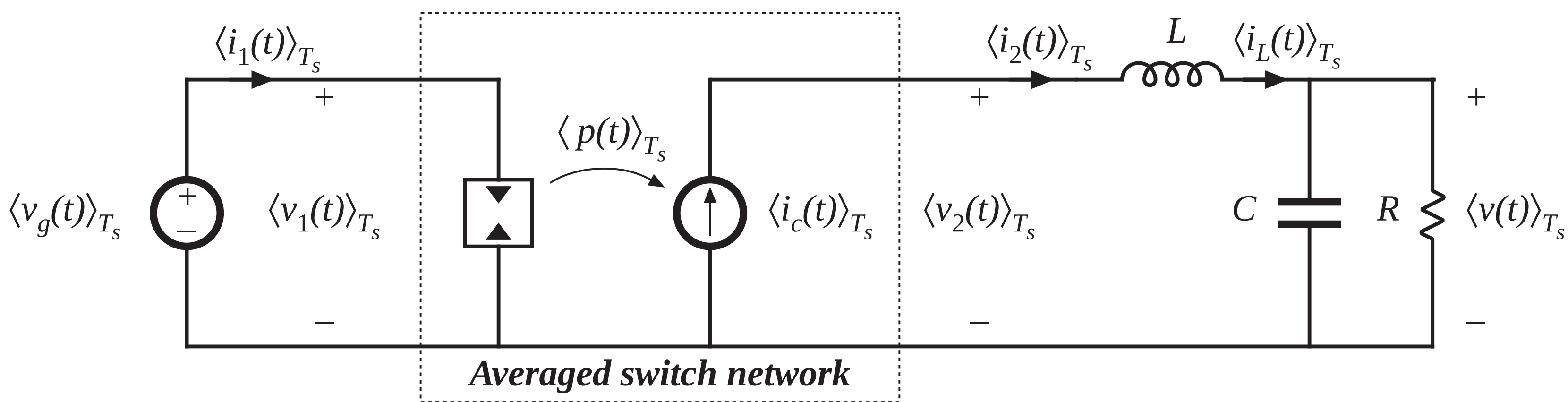
$$\left\langle i_1(t) \right\rangle_{T_s} = d(t) \left\langle i_c(t) \right\rangle_{T_s} = \frac{\left\langle v_2(t) \right\rangle_{T_s}}{\left\langle v_1(t) \right\rangle_{T_s}} \left\langle i_c(t) \right\rangle_{T_s}$$

$$\left\langle i_1(t) \right\rangle_{T_s} \left\langle v_1(t) \right\rangle_{T_s} = \left\langle i_c(t) \right\rangle_{T_s} \left\langle v_2(t) \right\rangle_{T_s} = \left\langle p(t) \right\rangle_{T_s}$$

So:

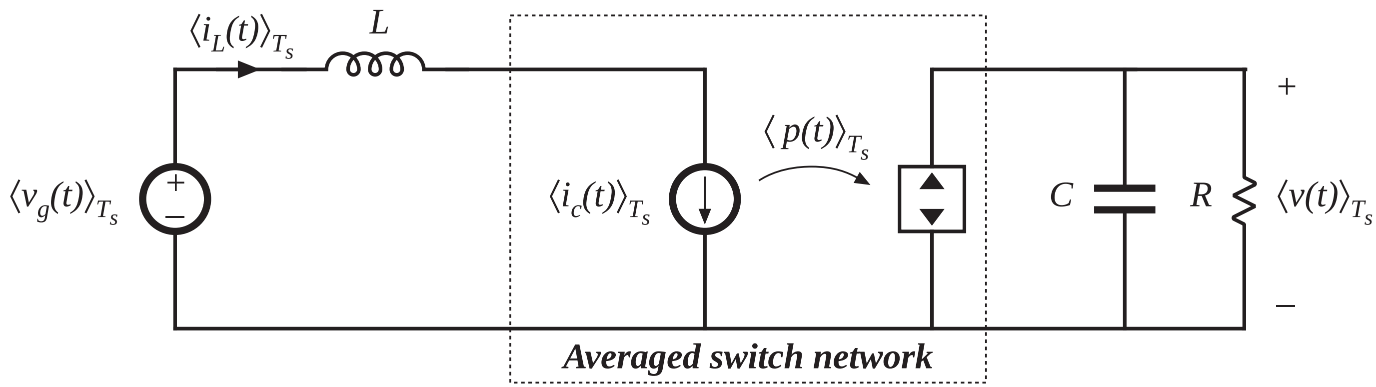
- Output port is a current source
- Input port is a dependent current sink

CPM averaged switch model

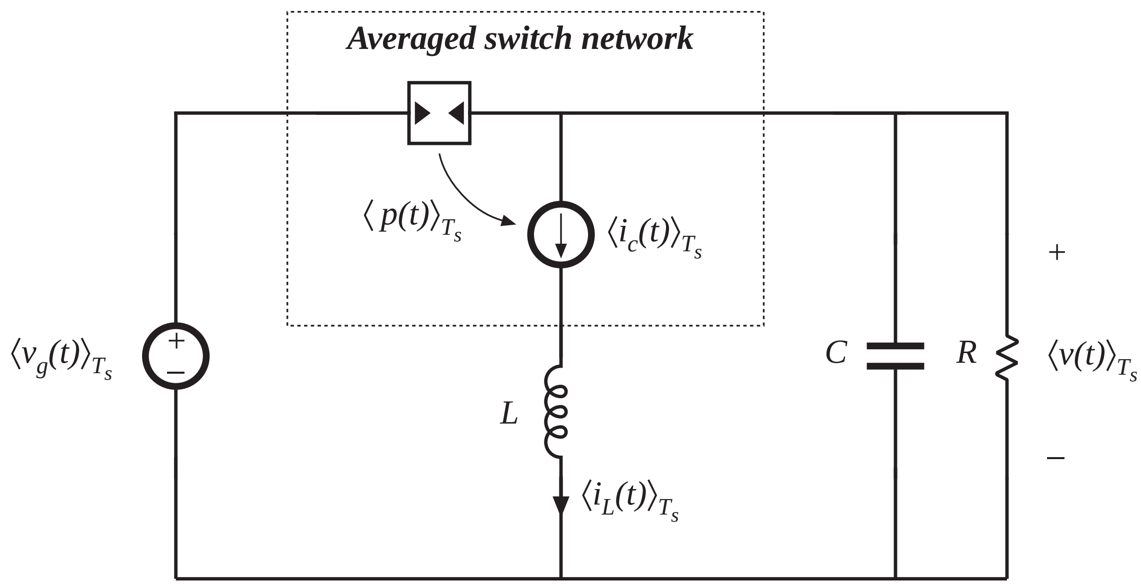


Results for other converters

Boost



Buck-boost



Perturbation and linearization to construct small-signal model

Let

$$\left\langle v_1(t) \right\rangle_{T_s} = V_1 + \hat{v}_1(t)$$

$$\left\langle i_1(t) \right\rangle_{T_s} = I_1 + \hat{i}_1(t)$$

$$\left\langle v_2(t) \right\rangle_{T_s} = V_2 + \hat{v}_2(t)$$

$$\left\langle i_2(t) \right\rangle_{T_s} = I_2 + \hat{i}_2(t)$$

$$\left\langle i_c(t) \right\rangle_{T_s} = I_c + \hat{i}_c(t)$$

Resulting input port equation:

$$\left(V_1 + \hat{v}_1(t)\right)\left(I_1 + \hat{i}_1(t)\right) = \left(I_c + \hat{i}_c(t)\right)\left(V_2 + \hat{v}_2(t)\right)$$

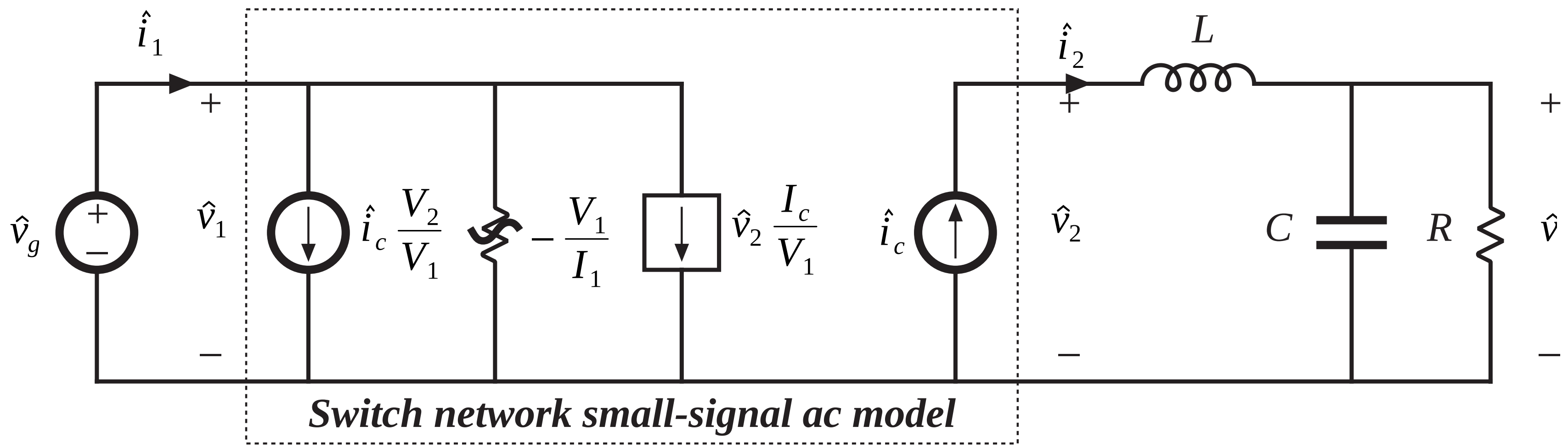
Small-signal result:

$$\hat{i}_1(t) = \hat{i}_c(t)\,\frac{V_2}{V_1} + \hat{v}_2(t)\,\frac{I_c}{V_1} - \hat{v}_1(t)\,\frac{I_1}{V_1}$$

Output port equation:

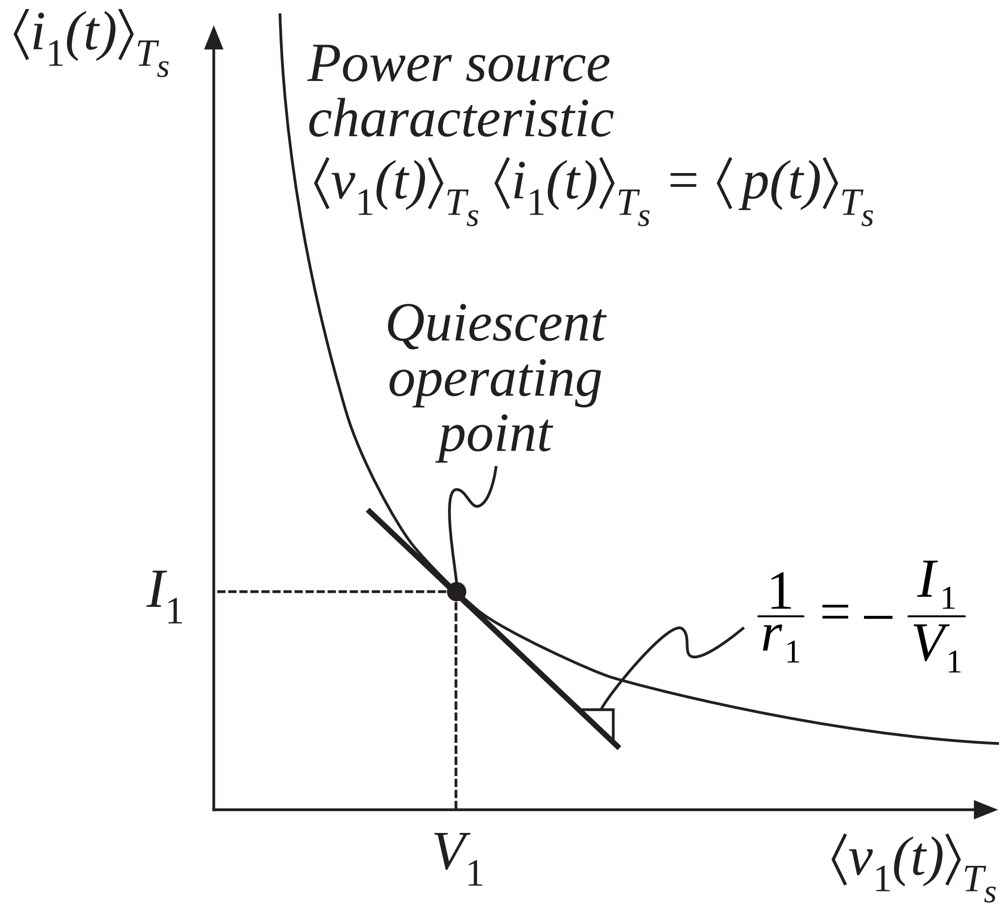
$$\hat{\imath}_2 = \hat{\imath}_\mathrm{c}$$

Resulting small-signal model
Buck example

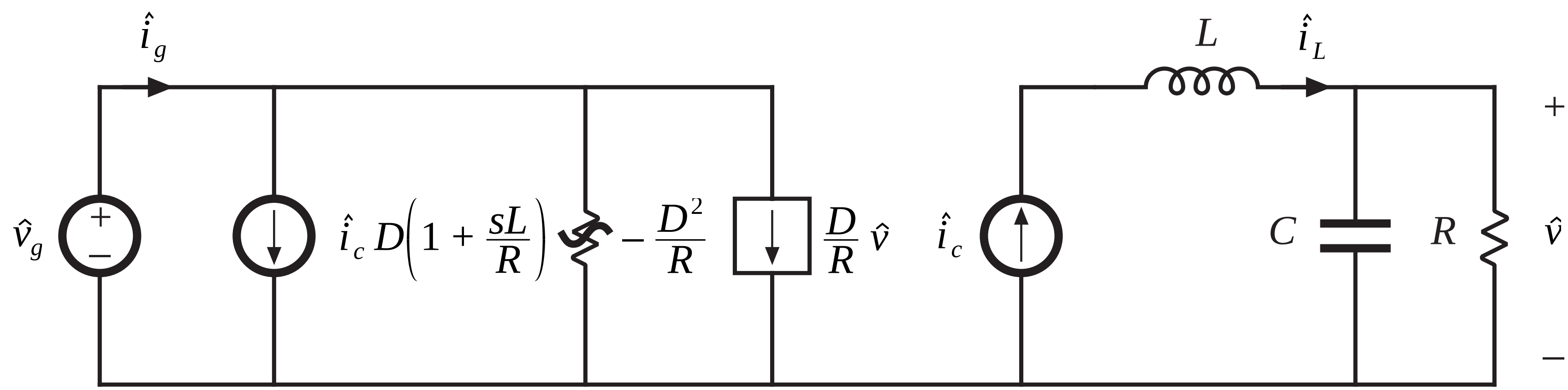


$$\hat{i}_1(t) = \hat{i}_c(t)\,\frac{V_2}{V_1} + \hat{v}_2(t)\,\frac{I_c}{V_1} - \hat{v}_1(t)\,\frac{I_1}{V_1}$$

Origin of input port negative incremental resistance

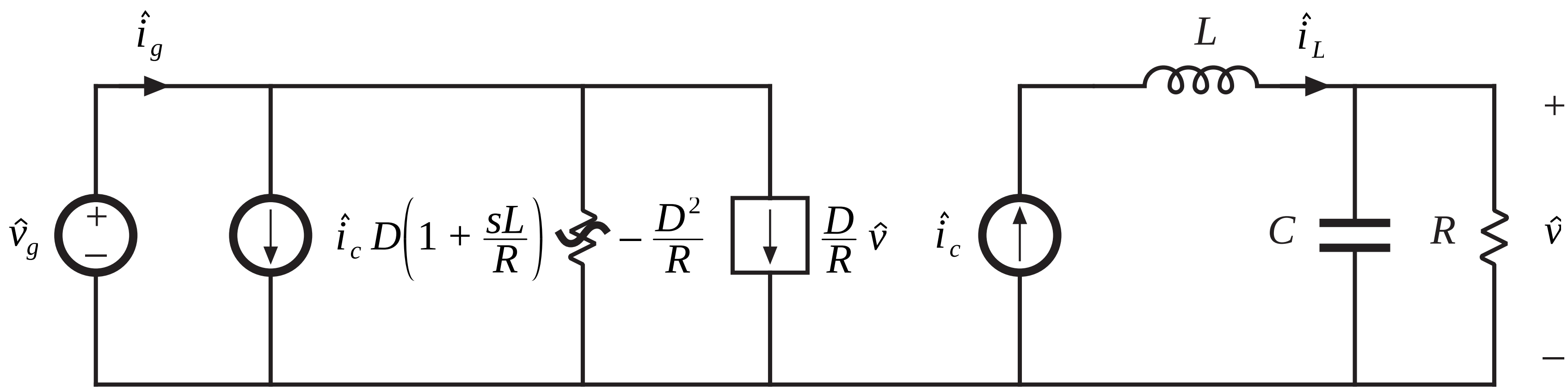


Expressing the equivalent circuit in terms of the converter input and output voltages



$$\hat{i}_1(s) = D\left(1 + s\,\frac{L}{R}\right)\hat{i}_c(s) + \frac{D}{R}\,\hat{v}(s) - \frac{D^2}{R}\,\hat{v}_g(s)$$

Predicted transfer functions of the CPM buck converter



$$G_{vc}(s) = \left.\frac{\hat{v}(s)}{\hat{i}_c(s)}\right|_{\hat{v}_g = 0} = \left(R \parallel \frac{1}{sC}\right)$$

$$G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\hat{i}_c = 0} = 0$$

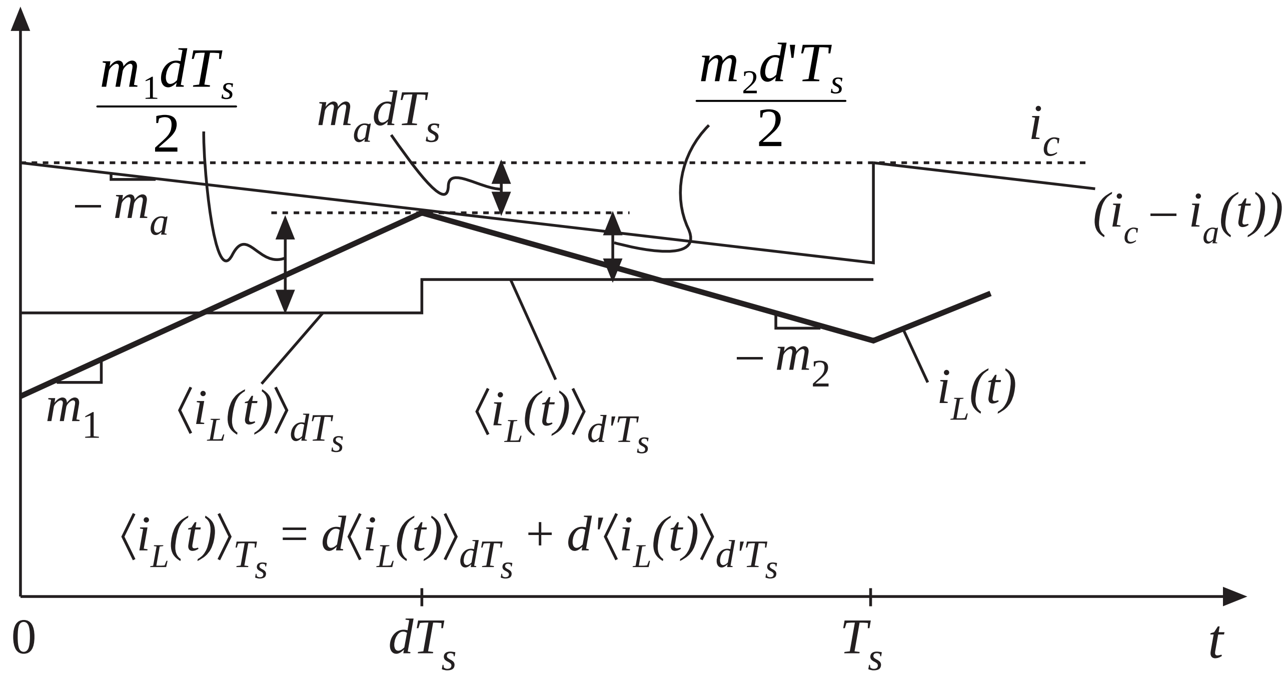
11.3 A More Accurate Model

- The simple models of the previous section yield insight into the low-frequency behavior of CPM converters
- Unfortunately, they do not always predict everything that we need to know:

  Line-to-output transfer function of the buck converter

  Dynamics at frequencies approaching f_s
- More accurate model accounts for nonideal operation of current mode controller built-in feedback loop
- Converter duty-cycle-controlled model, plus block diagram that accurately models equations of current mode controller

11.3.1 Current programmed controller model



$$\left\langle i_L(t) \right\rangle_{T_s} = \left\langle i_c(t) \right\rangle_{T_s} - m_a dT_s - d\,\frac{m_1 dT_s}{2} - d'\,\frac{m_2 d'T_s}{2}$$

$$= \left\langle i_c(t) \right\rangle_{T_s} - m_a dT_s - m_1\,\frac{d^2 T_s}{2} - m_2\,\frac{d'^2 T_s}{2}$$

Perturb

Let

$$\left\langle i_L(t) \right\rangle_{T_s} = I_L + \hat{i}_L(t)$$

$$\left\langle i_c(t) \right\rangle_{T_s} = I_c + \hat{i}_c(t)$$

$$d(t) = D + \hat{d}(t)$$

$$m_1(t) = M_1 + \hat{m}_1(t)$$

$$m_2(t) = M_2 + \hat{m}_2(t)$$

It is assumed that the artificial ramp slope does not vary.

Note that it is necessary to perturb the slopes, since these depend on the applied inductor voltage. For basic converters,

buck converter

$$\hat{m}_1 = \frac{\hat{v}_g - \hat{v}}{L} \qquad \hat{m}_2 = \frac{\hat{v}}{L}$$

boost converter

$$\hat{m}_1 = \frac{\hat{v}_g}{L} \qquad \hat{m}_2 = \frac{\hat{v} - \hat{v}_g}{L}$$

buck-boost converter

$$\hat{m}_1 = \frac{\hat{v}_g}{L} \qquad \hat{m}_2 = -\frac{\hat{v}}{L}$$

Linearize

$$\left(I_L + \hat{i}_L(t)\right) = \left(I_c + \hat{i}_c(t)\right) - M_a T_s \left(D + \hat{d}(t)\right) - \left(M_1 + \hat{m}_1(t)\right)\left(D + \hat{d}(t)\right)^2 \frac{T_s}{2}$$

$$- \left(M_2 + \hat{m}_2(t)\right)\left(D' - \hat{d}(t)\right)^2 \frac{T_s}{2}$$

The first-order ac terms are

$$\hat{i}_L(t) = \hat{i}_c(t) - \left(M_a T_s + D M_1 T_s - D' M_2 T_s\right) \hat{d}(t) - \frac{D^2 T_s}{2}\, \hat{m}_1(t) - \frac{D'^2 T_s}{2}\, \hat{m}_2(t)$$

Simplify using dc relationships:

$$\hat{i}_L(t) = \hat{i}_c(t) - M_a T_s \hat{d}(t) - \frac{D^2 T_s}{2}\, \hat{m}_1(t) - \frac{D'^2 T_s}{2}\, \hat{m}_2(t)$$

Solve for duty cycle variations:

$$\hat{d}(t) = \frac{1}{M_a T_s} \left[\hat{i}_c(t) - \hat{i}_L(t) - \frac{D^2 T_s}{2}\, \hat{m}_1(t) - \frac{D'^2 T_s}{2}\, \hat{m}_2(t) \right]$$

Equation of the current programmed controller

The expression for the duty cycle is of the general form

$$\hat{d}(t) = F_m \left[\hat{i}_c(t) - \hat{i}_L(t) - F_g \hat{v}_g(t) - F_v \hat{v}(t) \right]$$

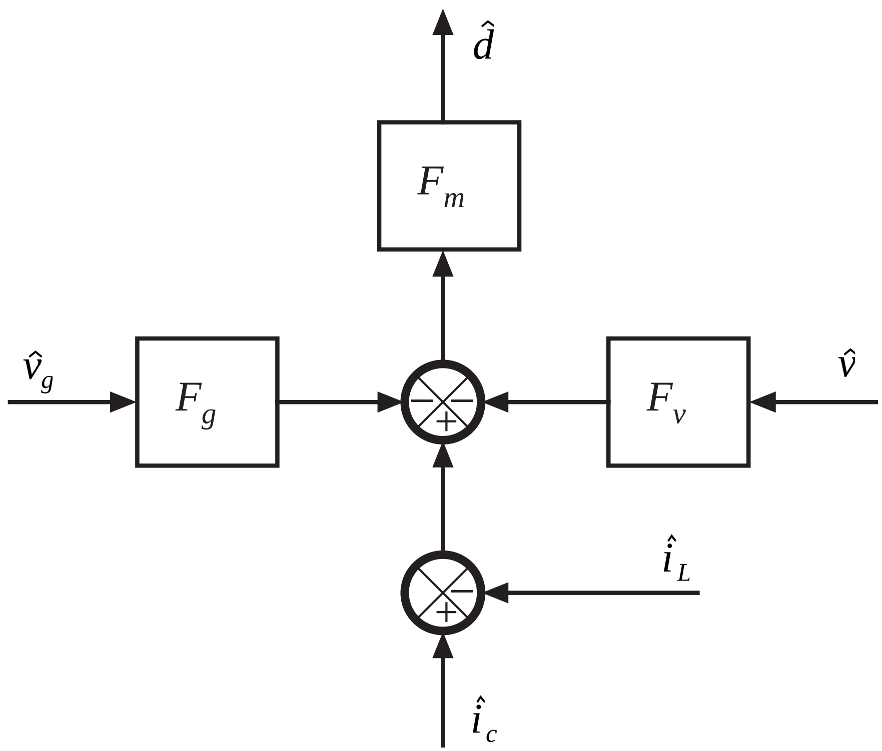
$$F_m = 1/M_a T_s$$

Table 11.2. Current programmed controller gains for basic converters

Converter	F_g	F_v
Buck	$\frac{D^2 T_s}{2L}$	$\frac{\left(1-2D\right) T_s}{2L}$
Boost	$\frac{\left(2D-1\right) T_s}{2L}$	$\frac{D'^2 T_s}{2L}$
Buck-boost	$\frac{D^2 T_s}{2L}$	$-\frac{D'^2 T_s}{2L}$

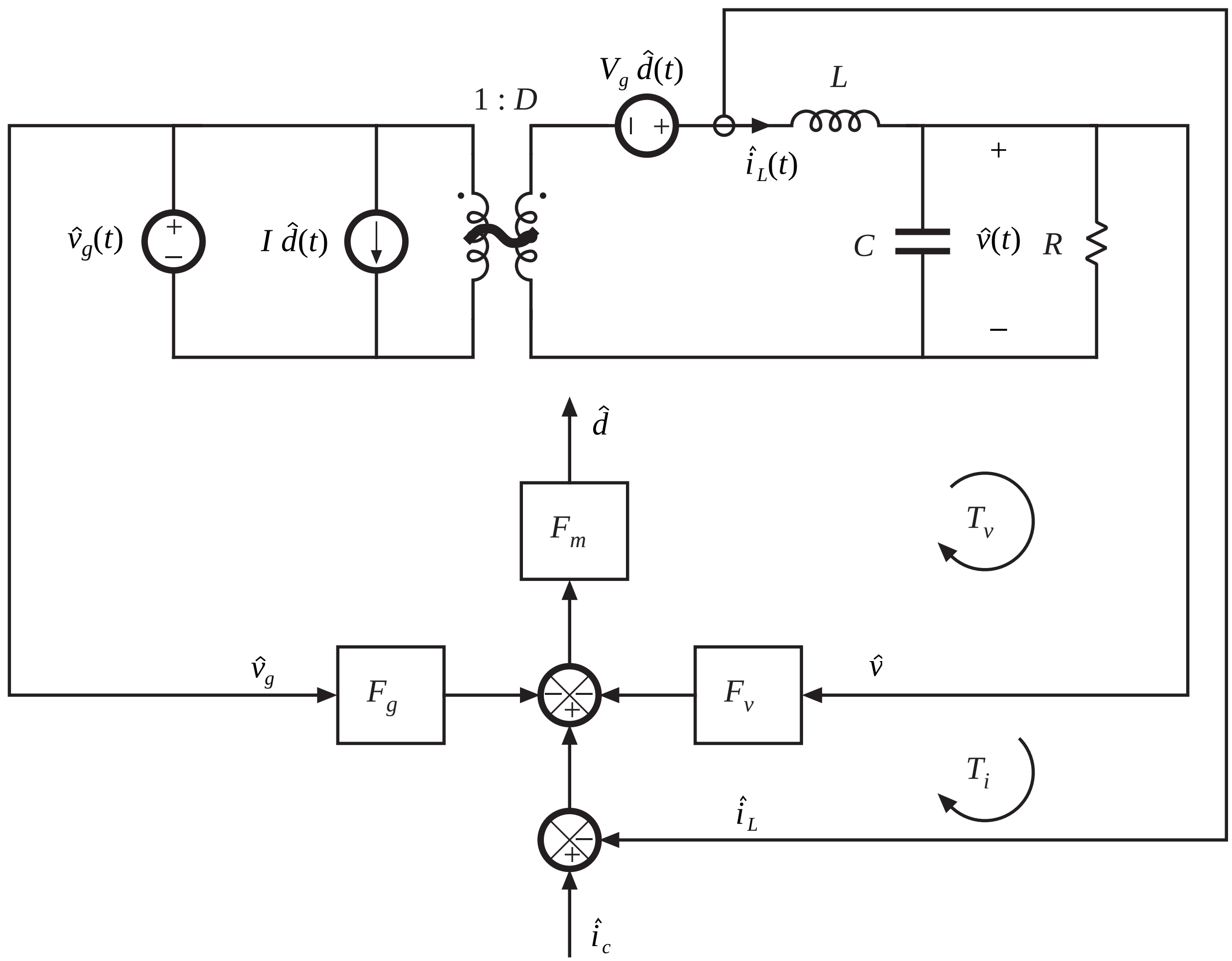
Block diagram of the current programmed controller

$$\hat{d}(t) = F_m \left[\hat{i}_c(t) - \hat{i}_L(t) - F_g \hat{v}_g(t) - F_v \hat{v}(t) \right]$$

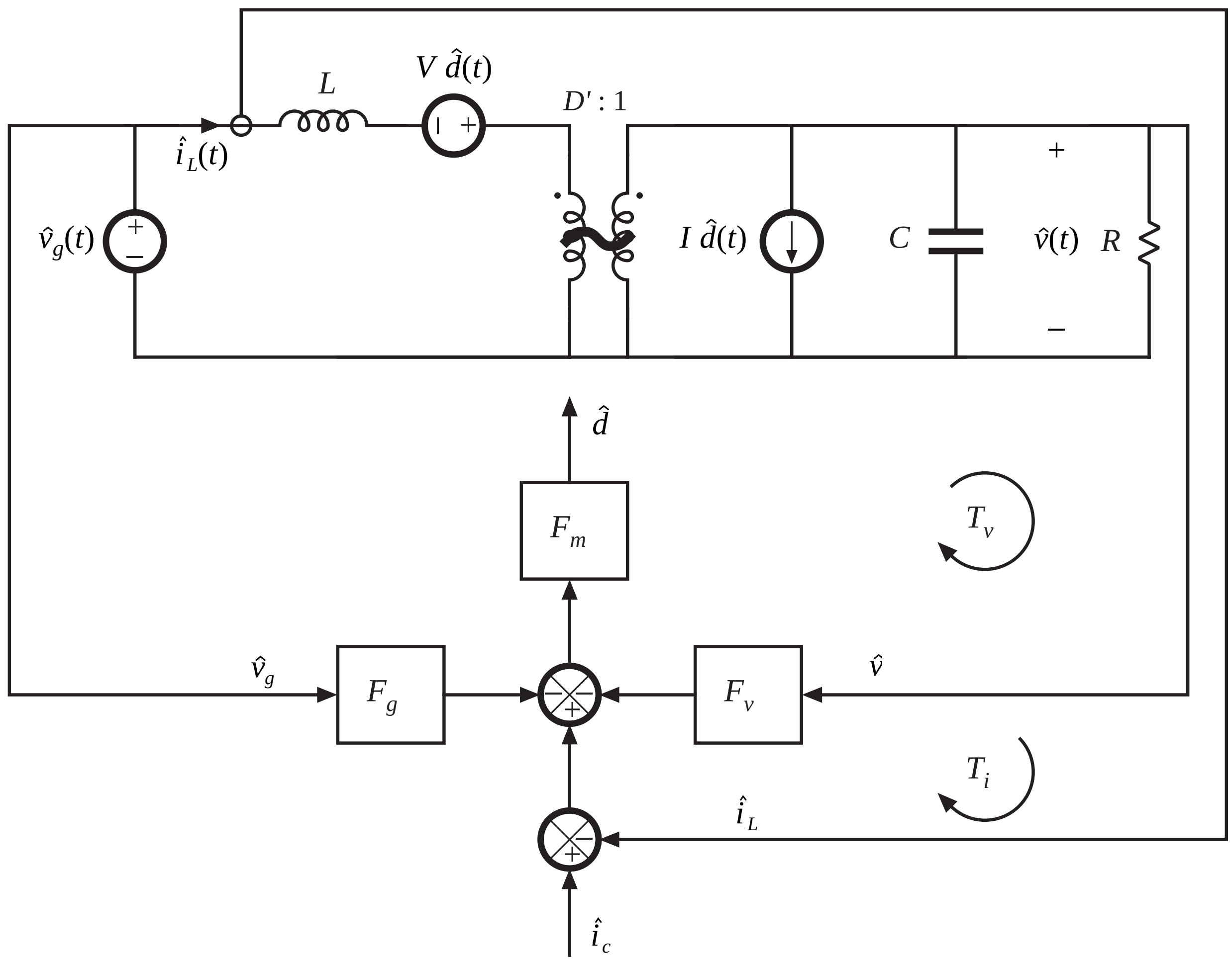


- Describes the duty cycle chosen by the CPM controller
- Append this block diagram to the duty cycle controlled converter model, to obtain a complete CPM system model

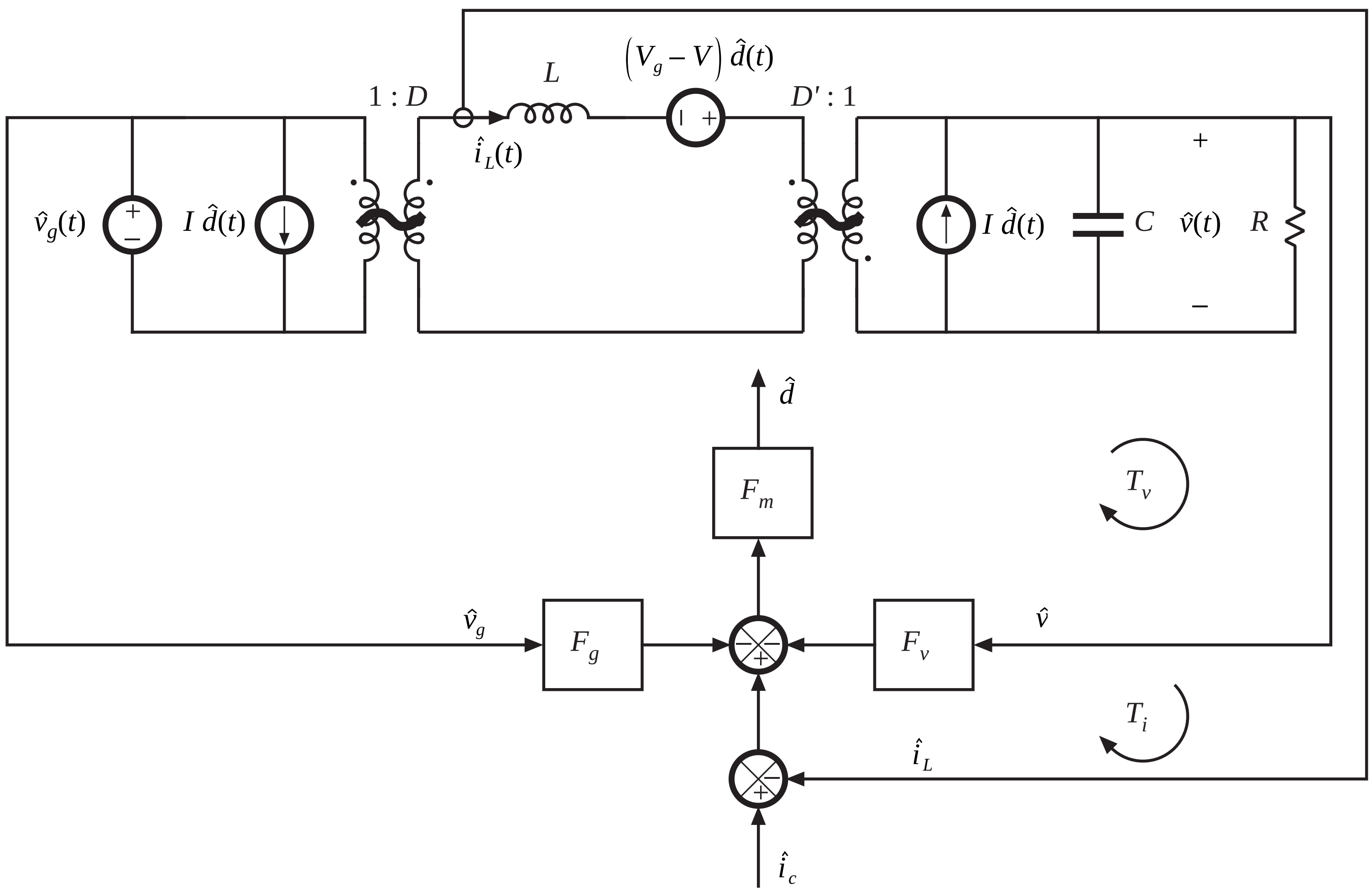
CPM buck converter model



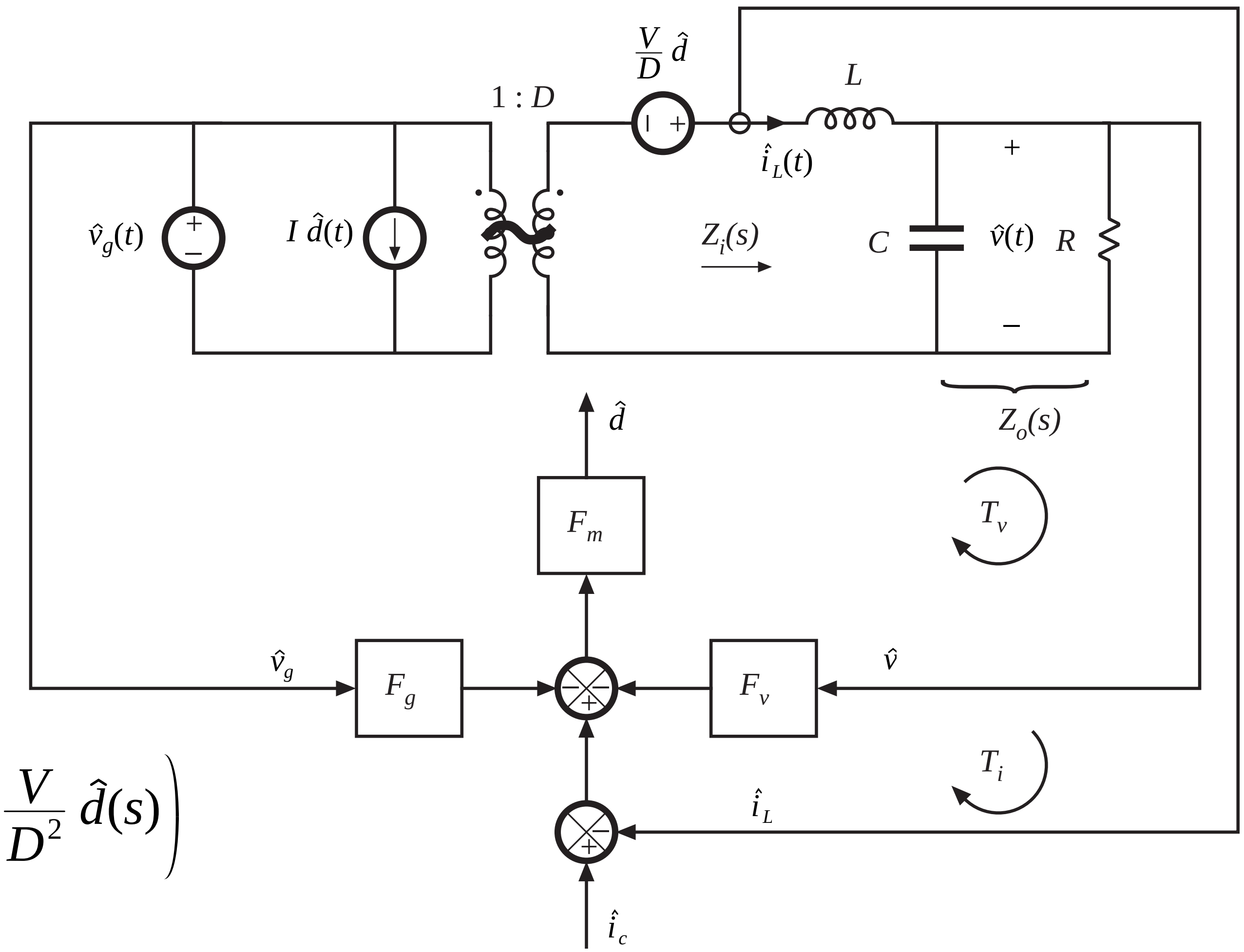
CPM boost converter model



CPM buck-boost converter model



11.3.2 Example: analysis of CPM buck converter



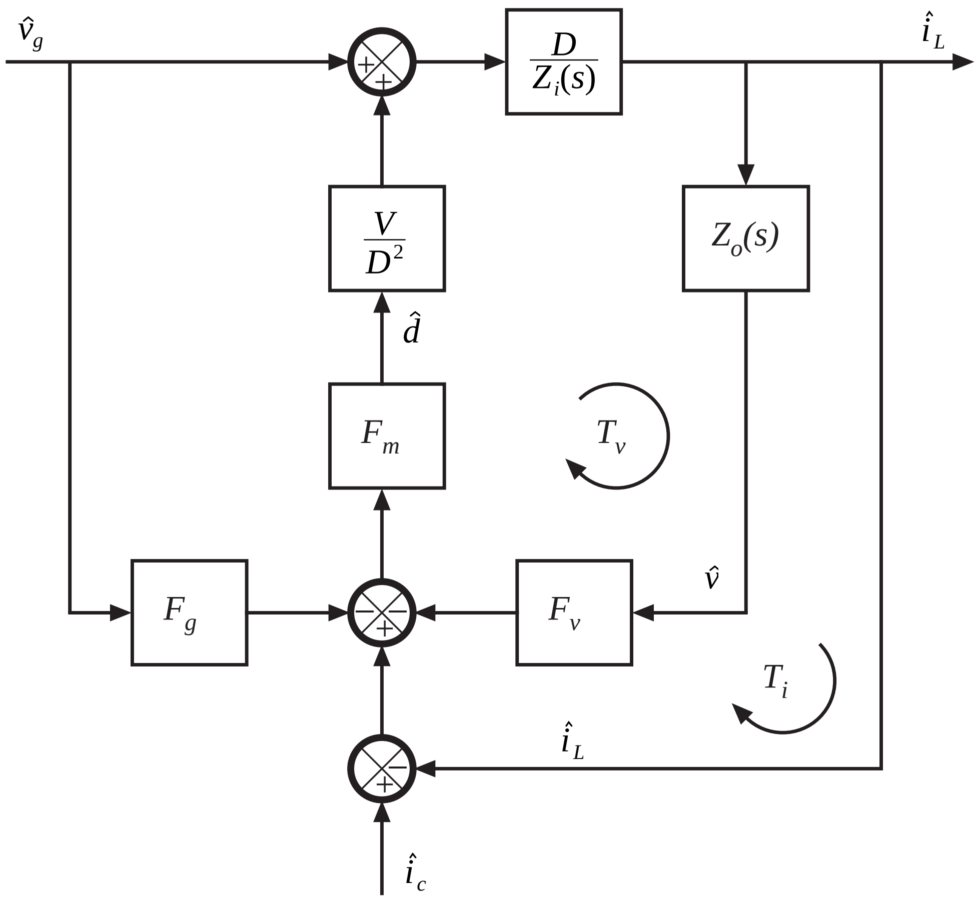
$$Z_o = R \parallel \frac{1}{sC}$$

$$Z_i = sL + \left(R \parallel \frac{1}{sC} \right)$$

$$\hat{i}_L(s) = \frac{D}{Z_i(s)} \left(\hat{v}_g(s) + \frac{V}{D^2}\,\hat{d}(s) \right)$$

$$\hat{v}(s) = \hat{i}_L(s)\, Z_o(s)$$

Block diagram of entire CPM buck converter



Voltage loop gain:

$$T_v(s) = F_m \frac{V}{D^2} \frac{D}{Z_i(s)} Z_o(s) F_v$$

Current loop gain:

$$T_i(s) = \frac{1}{Z_o(s) F_v} \frac{T_v(s)}{1 + T_v(s)}$$

$$T_i(s) = \frac{F_m \frac{V}{D^2} \frac{D}{Z_i(s)}}{1 + F_m \frac{V}{D^2} \frac{D}{Z_i(s)} Z_o(s) F_v}$$

Transfer function from $\hat{i}_c$ to $\hat{i}_L$:

$$\frac{\hat{i}_L(s)}{\hat{i}_c(s)} = \frac{T_i(s)}{1 + T_i(s)}$$

Discussion: Transfer function from $\hat{i}_c$ to $\hat{i}_L$

- When the loop gain Ti(s) is large in magnitude, then $\hat{i}_L$ is approximately equal to $\hat{i}_c$. The results then coincide with the simple approximation.

$$\frac{\hat{i}_L(s)}{\hat{i}_c(s)} = \frac{T_i(s)}{1 + T_i(s)}$$

- The control-to-output transfer function can be written as

$$G_{vc}(s) = \frac{\hat{v}(s)}{\hat{i}_c(s)} = Z_o(s)\,\frac{T_i(s)}{1 + T_i(s)}$$

which is the simple approximation result, multiplied by the factor $T_i/(1 + T_i)$.

Loop gain $T_i(s)$

Result for the buck converter:

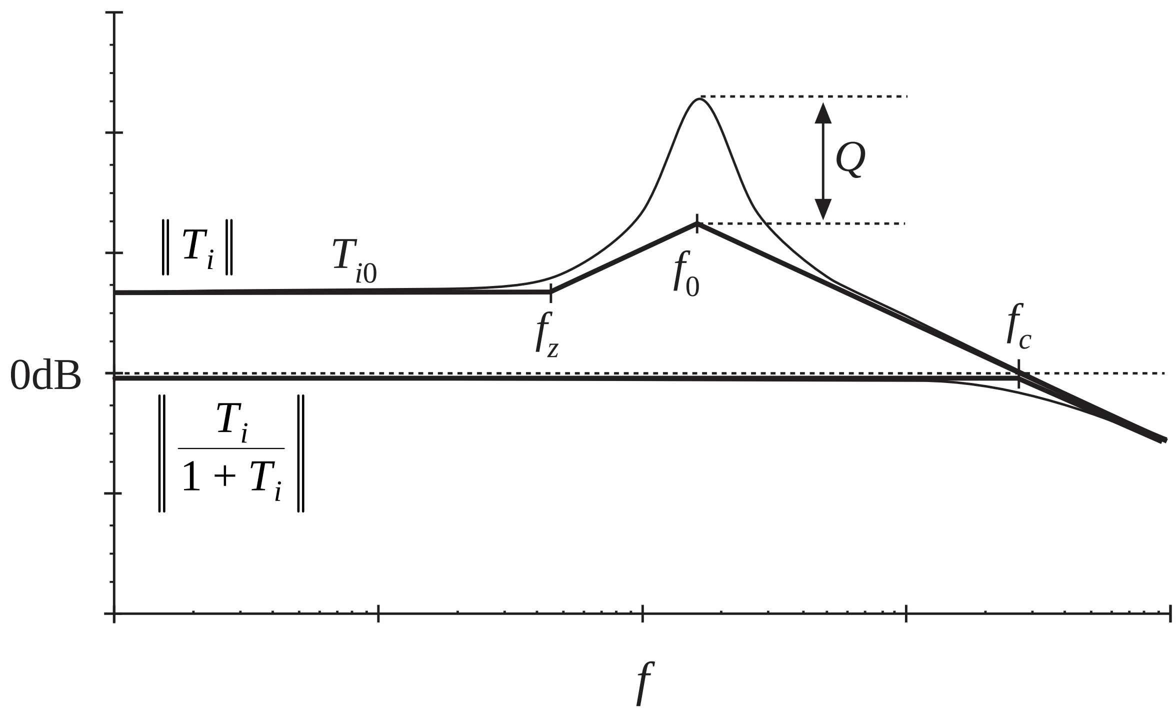
$$T_i(s) = \left(K \frac{1-\alpha}{1+\alpha}\right) \frac{\left(1 + sRC\right)}{1 + s\left(\frac{L}{R}\,\frac{2DM_a}{M_2}\,\frac{1-\alpha}{1+\alpha}\right) + s^2\left(LC\,\frac{2DM_a}{M_2}\,\frac{1-\alpha}{1+\alpha}\right)}$$

Characteristic value $\alpha = -\dfrac{1 - \dfrac{m_a}{m_2}}{\dfrac{D'}{D} + \dfrac{m_a}{m_2}}$ Controller stable for $|\,\alpha\,| < 1$

$K = 2L/RT_s$

CCM for $K > D'$

Loop gain



$$T_i(s) = T_{i0} \frac{\left(1 + \frac{s}{\omega_z}\right)}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$T_{i0} = K \frac{1-\alpha}{1+\alpha}$$

$$\omega_z = \frac{1}{RC}$$

$$\omega_0 = \frac{1}{\sqrt{LC \frac{2DM_a}{M_2} \frac{1-\alpha}{1+\alpha}}}$$

$$Q = R \sqrt{\frac{C}{L}} \sqrt{\frac{M_2}{2DM_a} \frac{1+\alpha}{1-\alpha}}$$

Crossover frequency f_c

High frequency asymptote of T_i is

$$T_i(s) \approx T_{i0} \frac{\left(\frac{s}{\omega_z}\right)}{\left(\frac{s}{\omega_0}\right)^2} = T_{i0} \frac{\omega_0^2}{s\omega_z}$$

Equate magnitude to one at crossover frequency:

$$\| T_i(j2\pi f_c) \| \approx T_{i0} \frac{f_0^2}{2\pi f_c f_z} = 1$$

Solve for crossover frequency:

$$f_c = \frac{M_2}{M_a} \frac{f_s}{2\pi D}$$

Construction of transfer functions

$$\frac{\hat{i}_L(s)}{\hat{i}_c(s)} = \frac{T_i(s)}{1 + T_i(s)} \approx \frac{1}{\left(1 + \frac{s}{\omega_c}\right)}$$

$$G_{vc}(s) = Z_o(s)\,\frac{T_i(s)}{1 + T_i(s)} \approx R\,\frac{1}{\left(1 + sRC\right)\left(1 + \frac{s}{\omega_c}\right)}$$

—the result of the simple first-order model, with an added pole at the loop crossover frequency

Exact analysis

$$G_{vc}(s) = R\left(\frac{T_{i0}}{1+T_{i0}}\right)\frac{1}{1+s\left(\frac{T_{i0}}{1+T_{i0}}\frac{1}{\omega_z}+\frac{1}{\left(1+T_{i0}\right)Q\omega_0}\right)+\frac{1}{1+T_{i0}}\left(\frac{s}{\omega_0}\right)^2}$$

for $|\, T_{i0}\,| >> 1$,

$$G_{vc}(s) \approx R\,\frac{1}{1+\frac{s}{\omega_z}+\frac{s^2}{T_{i0}\,\omega_0^2}}$$

Low Q approximation:

$$G_{vc}(s) \approx R\,\frac{1}{\left(1+\frac{s}{\omega_z}\right)\left(1+\frac{s\omega_z}{T_{i0}\omega_0^2}\right)}$$

which agrees with the previous slide

Line-to-output transfer function

Solution of complete block diagram leads to

$$\hat{v}(s) = \hat{i}_c(s)\, Z_o(s)\, \frac{T_i(s)}{1 + T_i(s)} + \hat{v}_g(s)\, \frac{G_{g0}(s)}{1 + T_i(s)}$$

with

$$G_{g0}(s) = Z_o(s)\, \frac{D}{Z_i(s)}\; \frac{\left(1 - \frac{V}{D^2} F_m F_g\right)}{\left(1 + \frac{V}{D^2} F_m F_v Z_o(s)\, \frac{D}{Z_i(s)}\right)}$$

Line-to-output transfer function

$$G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\hat{i}_c(s) = 0} = \frac{G_{g0}(s)}{1 + T_i(s)}$$

$$G_{vg}(s) = \frac{\left(\frac{1}{1 + T_{i0}}\right) \frac{(1-\alpha)}{(1+\alpha)}\, 2D^2 \left(\frac{M_a}{M_2} - \frac{1}{2}\right)}{1 + s\left(\frac{T_{i0}}{1 + T_{i0}}\, \frac{1}{\omega_z} + \frac{1}{\left(1 + T_{i0}\right) Q\omega_0}\right) + \frac{1}{1 + T_{i0}} \left(\frac{s}{\omega_0}\right)^2}$$

Poles are identical to control-to-output transfer function, and can be factored the same way:

$$G_{vg}(s) \approx \frac{G_{vg}(0)}{\left(1 + \frac{s}{\omega_z}\right)\left(1 + \frac{s}{\omega_c}\right)}$$

Line-to-output transfer function: dc gain

$$G_{vg}(0) \approx \frac{2D^2}{K}\left(\frac{M_a}{M_2} - \frac{1}{2}\right)$$ for large T_{i0}

For any T_{i0} , the dc gain goes to zero when M_a/M_2 = 0.5

Effective feedforward of input voltage variations in CPM controller then effectively cancels out the v_g variations in the direct forward path of the converter.

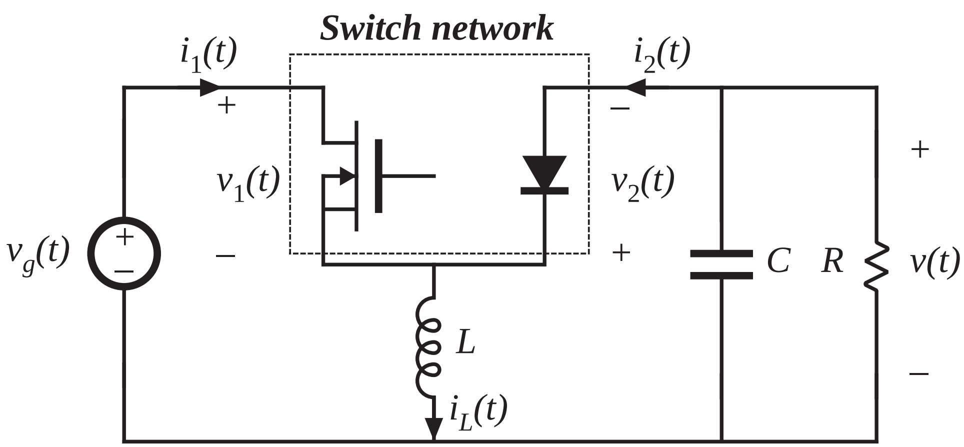
11.4 Discontinuous conduction mode

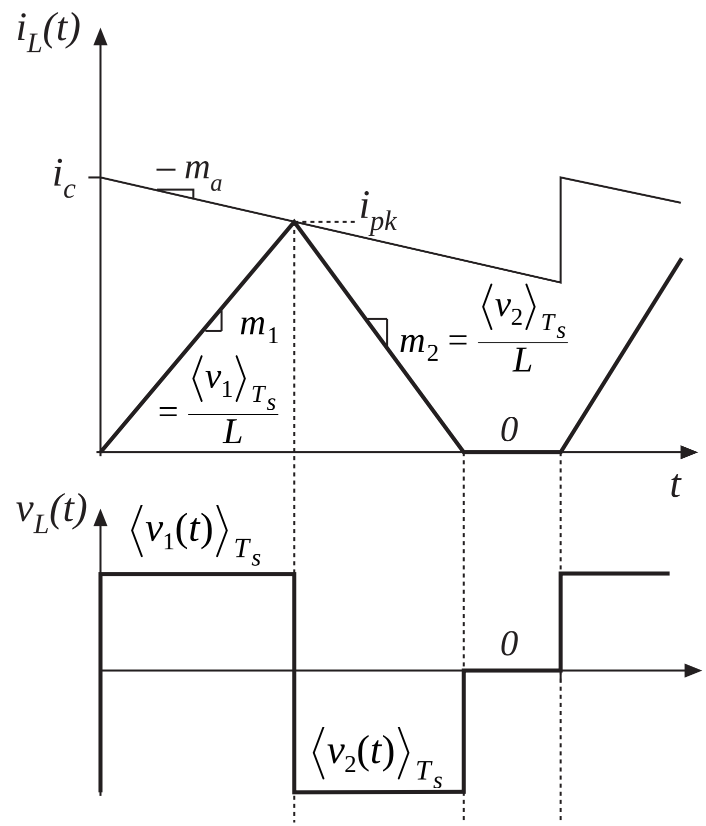
- Again, use averaged switch modeling approach
- Result: simply replace

  Transistor by power sink

  Diode by power source
- Inductor dynamics appear at high frequency, near to or greater than the switching frequency
- Small-signal transfer functions contain a single low frequency pole
- DCM CPM boost and buck-boost are stable without artificial ramp
- DCM CPM buck without artificial ramp is stable for $D < 2/3$. A small artificial ramp $m_a \geq 0.086 m_2$ leads to stability for all D.

DCM CPM buck-boost example





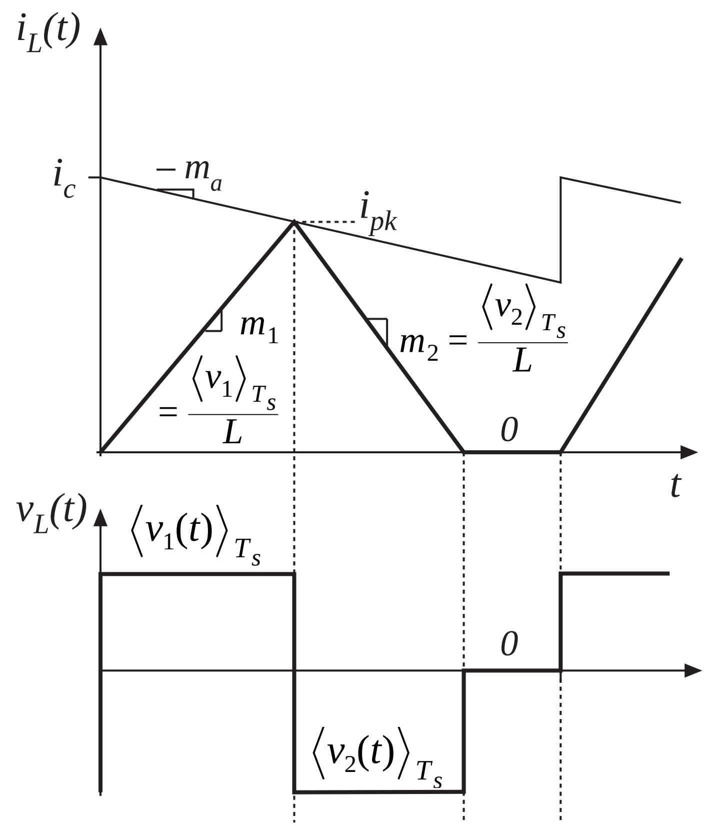
Analysis

$$i_{pk} = m_1 d_1 T_s$$

$$m_1 = \frac{\left\langle v_1(t) \right\rangle_{T_s}}{L}$$

$$i_c = i_{pk} + m_a d_1 T_s$$
$$= \left(m_1 + m_a \right) d_1 T_s$$

$$d_1(t) = \frac{i_c(t)}{\left(m_1 + m_a \right) T_s}$$



Averaged switch input port equation

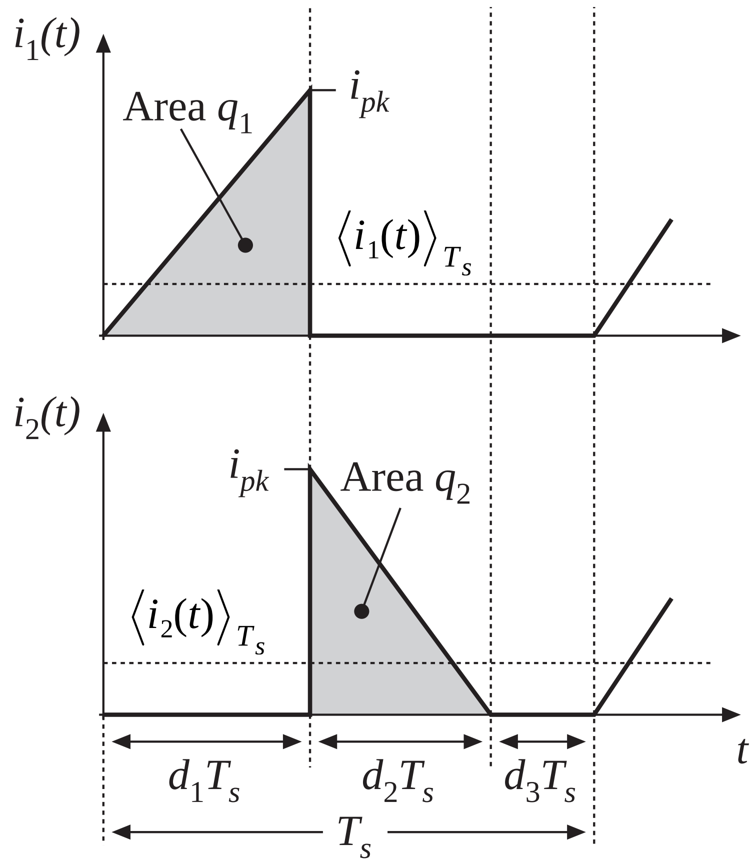
$$\left\langle i_1(t) \right\rangle_{T_s} = \frac{1}{T_s} \int_t^{t+T_s} i_1(\tau) d\tau = \frac{q_1}{T_s}$$

$$\left\langle i_1(t) \right\rangle_{T_s} = \frac{1}{2} i_{pk}(t) d_1(t)$$

$$\left\langle i_1(t) \right\rangle_{T_s} = \frac{1}{2} m_1 d_1^2(t) T_s$$

$$\left\langle i_1(t) \right\rangle_{T_s} = \frac{\frac{1}{2} L i_c^2 f_s}{\left\langle v_1(t) \right\rangle_{T_s} \left(1 + \frac{m_a}{m_1} \right)^2}$$

$$\left\langle i_1(t) \right\rangle_{T_s} \left\langle v_1(t) \right\rangle_{T_s} = \frac{\frac{1}{2} L i_c^2 f_s}{\left(1 + \frac{m_a}{m_1} \right)^2} = \left\langle p(t) \right\rangle_{T_s}$$



Discussion: switch network input port

- Averaged transistor waveforms obey a power sink characteristic
- During first subinterval, energy is transferred from input voltage source, through transistor, to inductor, equal to

$$W = \frac{1}{2} L i_{pk}^2$$

This energy transfer process accounts for power flow equal to

$$\left\langle p(t) \right\rangle_{T_s} = W f_s = \frac{1}{2} L i_{pk}^2 f_s$$

which is equal to the power sink expression of the previous slide.

Averaged switch output port equation

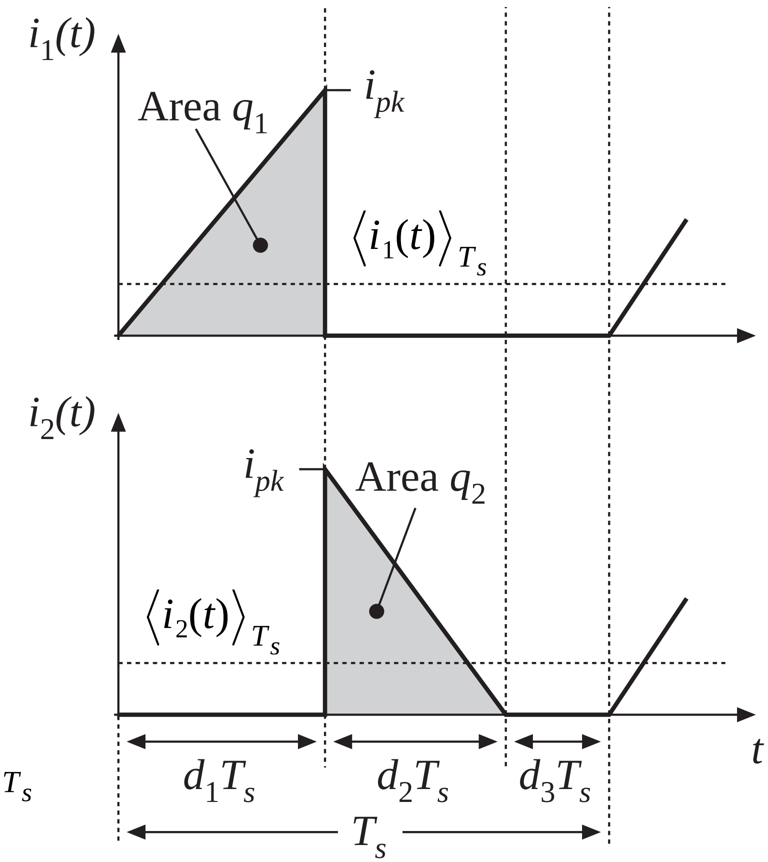
$$\left\langle i_2(t)\right\rangle_{T_s} = \frac{1}{T_s}\int_t^{t+T_s} i_2(\tau)d\tau = \frac{q_2}{T_s}$$

$$q_2 = \frac{1}{2}\, i_{pk} d_2 T_s$$

$$d_2(t) = d_1(t)\,\frac{\left\langle v_1(t)\right\rangle_{T_s}}{\left\langle v_2(t)\right\rangle_{T_s}}$$

$$\left\langle i_2(t)\right\rangle_{T_s} = \frac{\left\langle p(t)\right\rangle_{T_s}}{\left\langle v_2(t)\right\rangle_{T_s}}$$

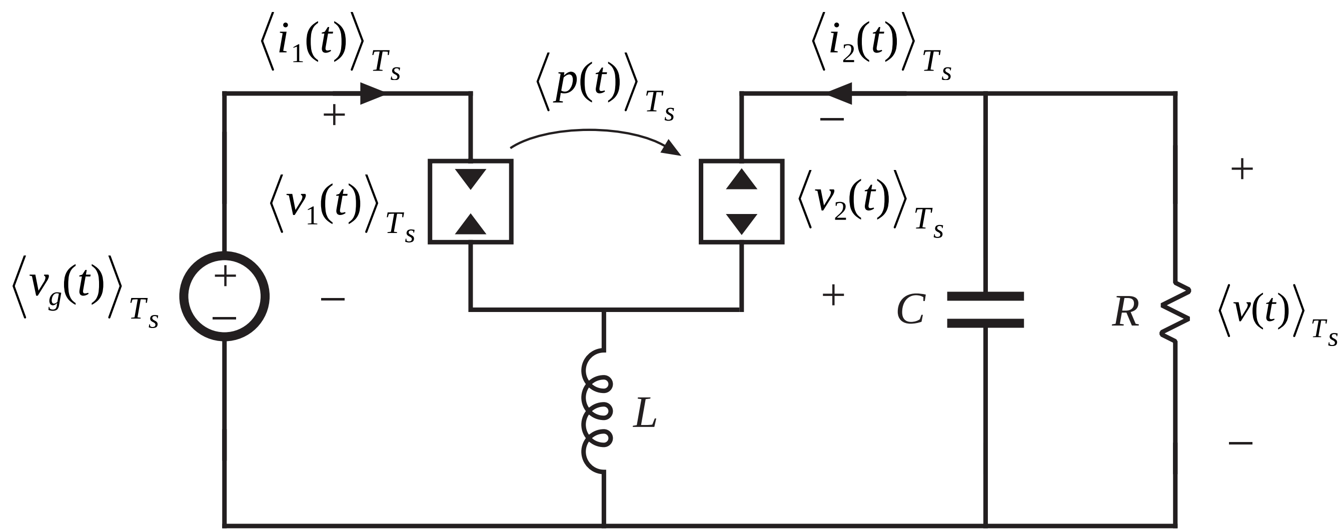
$$\left\langle i_2(t)\right\rangle_{T_s}\left\langle v_2(t)\right\rangle_{T_s} = \frac{\frac{1}{2} L i_c^2(t) f_s}{\left(1 + \frac{m_a}{m_1}\right)^2} = \left\langle p(t)\right\rangle_{T_s}$$



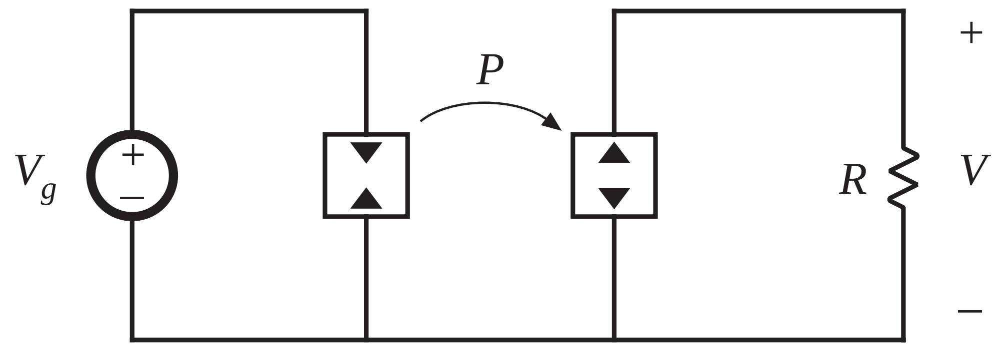
Discussion: switch network output port

- Averaged diode waveforms obey a power sink characteristic
- During second subinterval, all stored energy in inductor is transferred, through diode, to load
- Hence, in averaged model, diode becomes a power source, having value equal to the power consumed by the transistor power sink element

Averaged equivalent circuit



Steady state model: DCM CPM buck-boost



Solution

$$\frac{V^2}{R} = P$$

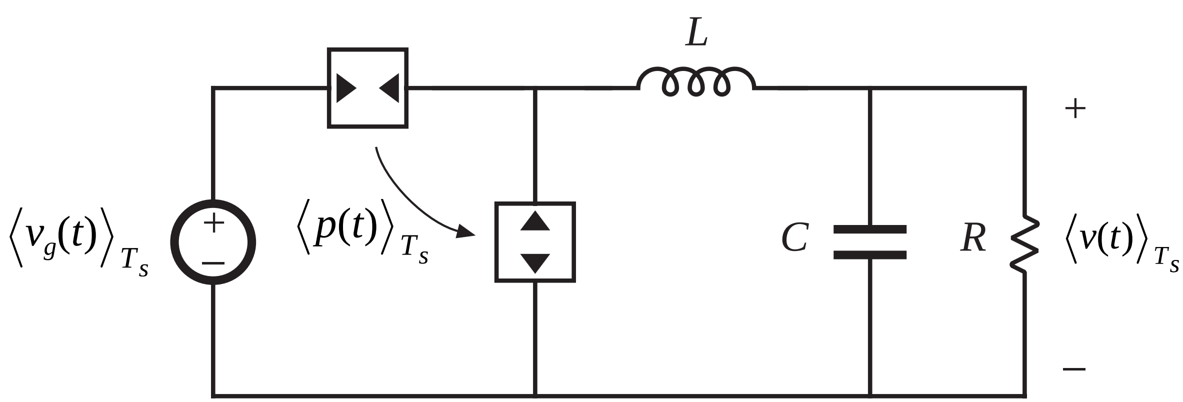
$$P = \frac{\frac{1}{2} L I_c^2(t) f_s}{\left(1 + \frac{M_a}{M_1}\right)^2}$$

$$V = \sqrt{PR} = I_c \sqrt{\frac{R L f_s}{2\left(1 + \frac{M_a}{M_1}\right)^2}}$$

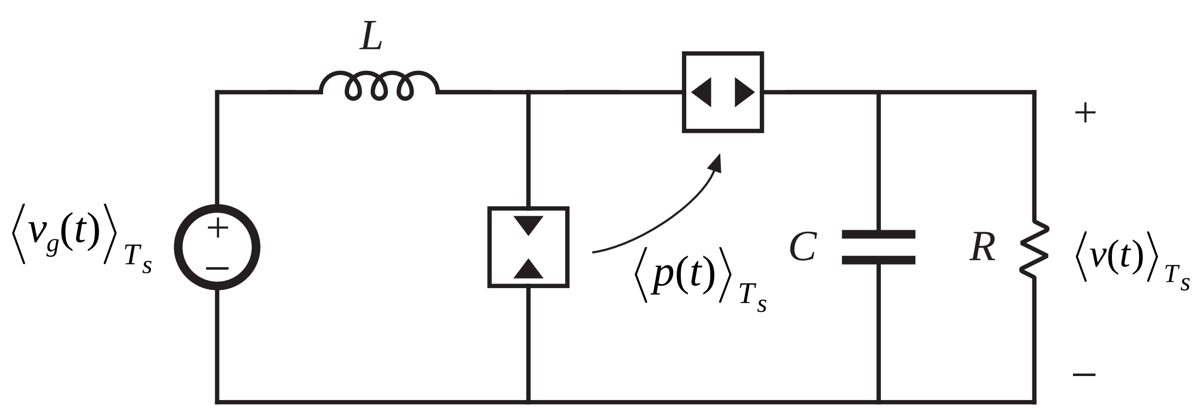
for a resistive load

Models of buck and boost

Buck



Boost



Summary of steady-state DCM CPM characteristics

Table 11.3. Steady-state DCM CPM characteristics of basic converters

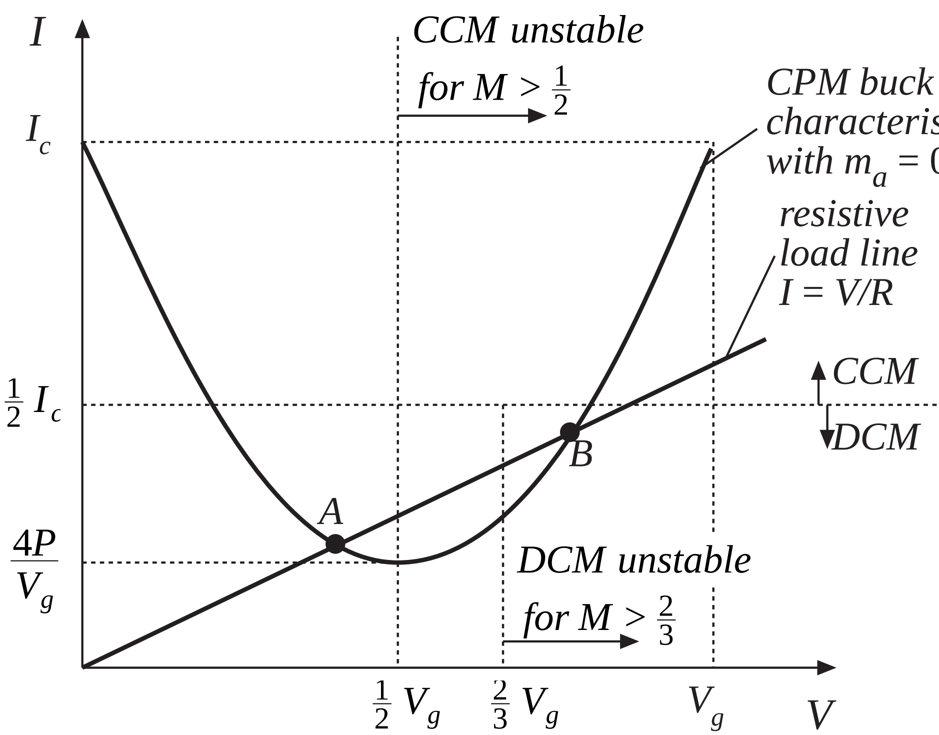
Converter	M	I_{crit}	Stability range when $m_a = 0$
Buck	$\frac{P_{load} - P}{P_{load}}$	$\frac{1}{2}\left(I_c - M\, m_a\, T\right.$	$0 \le M < \frac{2}{3}$
Boost	$\frac{P_{load}}{P_{load} - P}$	$\frac{\left(I_c - \frac{M-1}{M}\, m_a\, T_s\right)}{2\, M}$	$0 \le D \le 1$
Buck-boost	Depends on load characteristic: $P_{load} = P$	$\frac{\left(I_c - \frac{M}{M-1}\, m_a\, T_s\right)}{2\left(M-1\right)}$	$0 \le D \le 1$

$$\left|I\right| > \left|I_{crit}\right| \quad \text{for CCM}$$

$$\left|I\right| < \left|I_{crit}\right| \quad \text{for DCM}$$

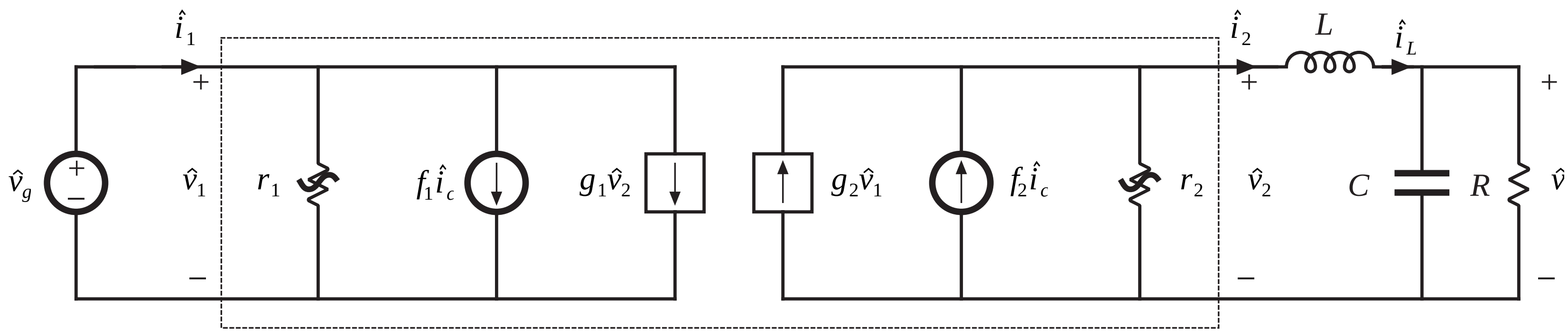
Buck converter: output characteristic with $m_a = 0$

$$I = \frac{P}{V_g - V} + \frac{P}{V} = \frac{P}{V\left(1 - \frac{V_g}{V}\right)}$$

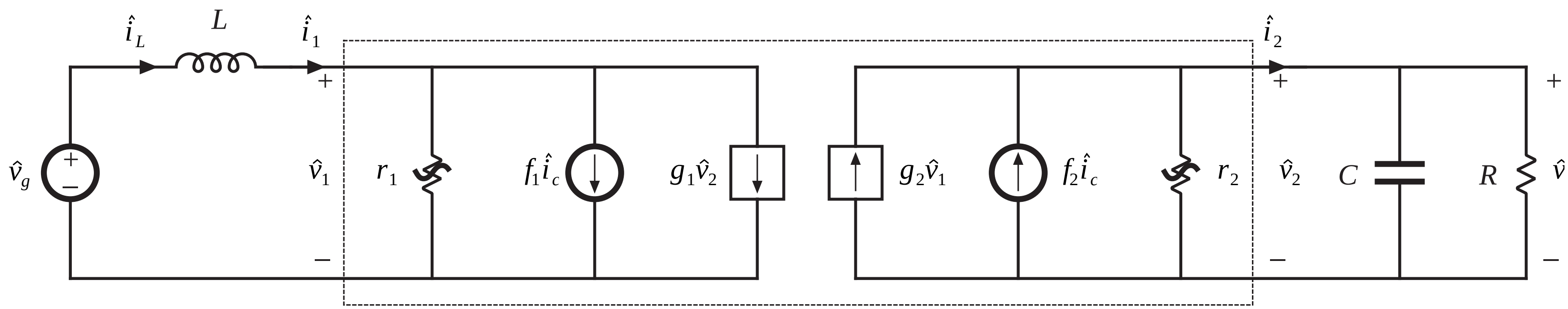


- with a resistive load, there can be two operating points
- the operating point having $V > 0.67V_g$ can be shown to be unstable

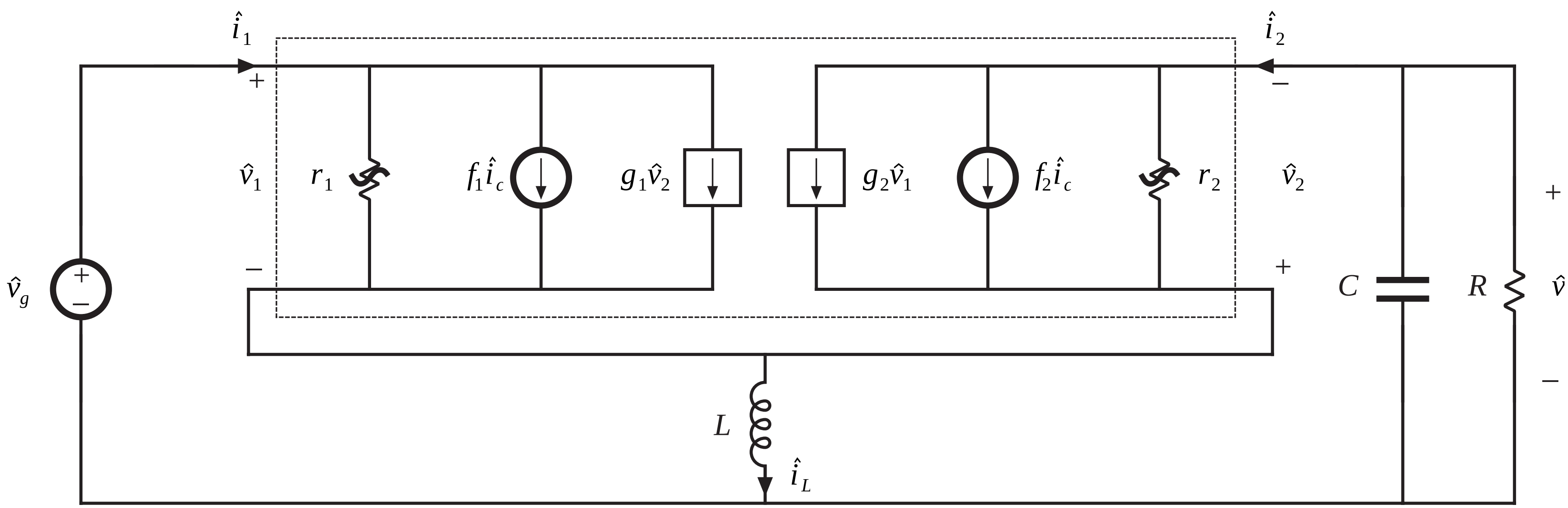
Linearized small-signal models: Buck



Linearized small-signal models: Boost



Linearized small-signal models: Buck-boost



DCM CPM small-signal parameters: input port

Table 11.4. Current programmed DCM small-signal equivalent circuit parameters: input port

Converter	g_1	f_1	r_1
Buck	$\frac{1}{R}\left(\frac{M^2}{1-M}\right)\frac{\left(1-\frac{m_a}{m_1}\right)}{\left(1+\frac{m_a}{m_1}\right)}$	$2\,\frac{I_1}{I_c}$	$-R\left(\frac{1-M}{M^2}\right)\frac{\left(1+\frac{m_a}{m_1}\right)}{\left(1-\frac{m_a}{m_1}\right)}$
Boost	$-\frac{1}{R}\left(\frac{M}{M-1}\right)$	$2\,\frac{I}{I_c}$	$\frac{R}{M^2\left(\frac{2-M}{M-1}+\frac{2\,\frac{m_a}{m_1}}{1+\frac{m_a}{m_1}}\right)}$
Buck-boost	0	$2\,\frac{I_1}{I_c}$	$\frac{-R}{M^2}\,\frac{\left(1+\frac{m_a}{m_1}\right)}{\left(1-\frac{m_a}{m_1}\right)}$

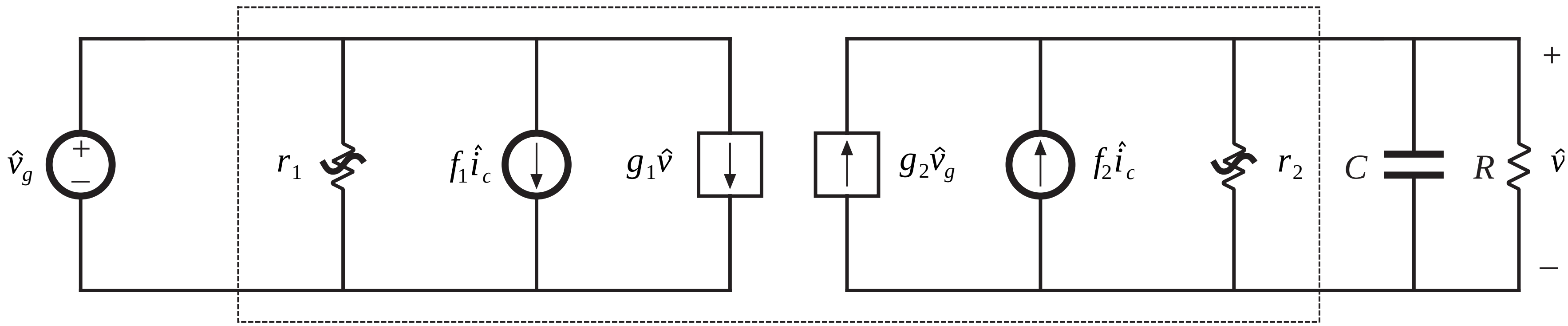
DCM CPM small-signal parameters: output port

Table 11.5. Current programmed DCM small-signal equivalent circuit parameters: output port

Converter	g_2	f_2	r_2
Buck	$\frac{1}{R}\left(\frac{M}{1-M}\right)\frac{\left(\frac{m_a}{m_1}\right)\left(2-M\right)-M}{\left(1+\frac{m_a}{m_1}\right)}$	$2\,\frac{I}{I_c}$	$R\,\frac{\left(1-M\right)\left(1+\frac{m_a}{m_1}\right)}{\left(1-2M+\frac{m_a}{m_1}\right)}$
Boost	$\frac{1}{R}\left(\frac{M}{M-1}\right)$	$2\,\frac{I_2}{I_c}$	$R\left(\frac{M-1}{M}\right)$
Buck-boost	$\frac{2M}{R}\,\frac{\left(\frac{m_a}{m_1}\right)}{\left(1+\frac{m_a}{m_1}\right)}$	$2\,\frac{I_2}{I_c}$	R

Simplified DCM CPM model, with $L = 0$

Buck, boost, buck-boost all become



$$G_{vc}(s) = \left.\frac{\hat{v}}{\hat{i}_c}\right|_{\hat{v}_g = 0} = \frac{G_{c0}}{1 + \frac{s}{\omega_p}}$$

$$G_{vg}(s) = \left.\frac{\hat{v}}{\hat{v}_g}\right|_{\hat{i}_c = 0} = \frac{G_{g0}}{1 + \frac{s}{\omega_p}}$$

$$G_{c0} = f_2 \left(R \parallel r_2\right)$$

$$\omega_p = \frac{1}{\left(R \parallel r_2\right) C}$$

$$G_{g0} = g_2 \left(R \parallel r_2\right)$$

Buck ω_p

Plug in parameters:

$$\omega_p = \frac{1}{RC} \frac{\left(2 - 3M\right)\left(1 - M\right) + \frac{m_a}{m_2} M\left(2 - M\right)}{\left(1 - M\right)\left(1 - M + M \frac{m_a}{m_2}\right)}$$

- For $m_a = 0$, numerator is negative when $M > 2/3$.
- ω_p then constitutes a RHP pole. Converter is unstable.
- Addition of small artificial ramp stabilizes system.
- $m_a > 0.086$ leads to stability for all $M \leq 1$.
- Output voltage feedback can also stabilize system, without an artificial ramp

11.5 Summary of key points

1. In current-programmed control, the peak switch current $i_s(t)$ follows the control input $i_c(t)$. This widely used control scheme has the advantage of a simpler control-to-output transfer function. The line-to-output transfer functions of current-programmed buck converters are also reduced.

2. The basic current-programmed controller is unstable when $D > 0.5$, regardless of the converter topology. The controller can be stabilized by addition of an artificial ramp having slope ma. When $m_a \geq 0.5\, m_2$, then the controller is stable for all duty cycle.

3. The behavior of current-programmed converters can be modeled in a simple and intuitive manner by the first-order approximation $\langle\, i_L(t)\, \rangle_{T_s} \approx i_c(t)$. The averaged terminal waveforms of the switch network can then be modeled simply by a current source of value i_c , in conjunction with a power sink or power source element. Perturbation and linearization of these elements leads to the small-signal model. Alternatively, the small-signal converter equations derived in Chapter 7 can be adapted to cover the current programmed mode, using the simple approximation $i_L(t) \approx i_c(t)$.

Summary of key points

4. The simple model predicts that one pole is eliminated from the converter line-to-output and control-to-output transfer functions. Current programming does not alter the transfer function zeroes. The dc gains become load-dependent.

5. The more accurate model of Section 11.3 correctly accounts for the difference between the average inductor current $\langle i_L(t) \rangle_{T_s}$ and the control input $i_c(t)$. This model predicts the nonzero line-to-output transfer function $G_{vg}(s)$ of the buck converter. The current-programmed controller behavior is modeled by a block diagram, which is appended to the small-signal converter models derived in Chapter 7. Analysis of the resulting multiloop feedback system then leads to the relevant transfer functions.

6. The more accurate model predicts that the inductor pole occurs at the crossover frequency fc of the effective current feedback loop gain $T_i(s)$. The frequency f_c typically occurs in the vicinity of the converter switching frequency f_s . The more accurate model also predicts that the line-to-output transfer function $G_{vg}(s)$ of the buck converter is nulled when $m_a = 0.5\ m_2$.

Summary of key points

7. Current programmed converters operating in the discontinuous conduction mode are modeled in Section 11.4. The averaged transistor waveforms can be modeled by a power sink, while the averaged diode waveforms are modeled by a power source. The power is controlled by $i_c(t)$. Perturbation and linearization of these averaged models, as usual, leads to small-signal equivalent circuits.