# PRACTICAL RECTIFIERS

4-1 The vacuum diode

The vacuum diode comprises a hot *cathode* surrounded by a metal *anode* inside an evacuated enclosure, usually glass, as sketched in Fig. 4-2. At sufficiently high temperatures electrons are emitted from the cathode and are attracted to the positive anode. Electrons moving from the cathode to the anode constitute a current; they do so when the anode is positive with respect

FIGURE 4-1 *Current-voltage characteristic of an ideal rectifier.*
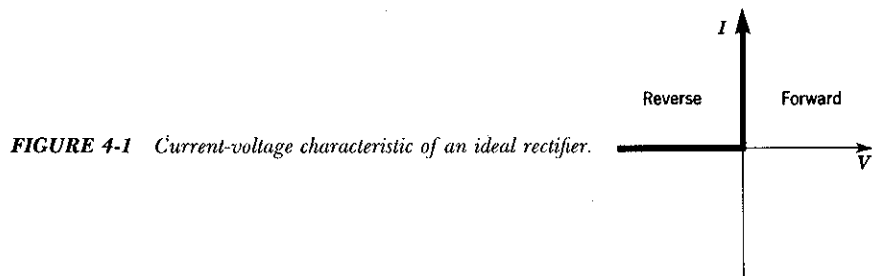
to the cathode. When the anode is negative with respect to the cathode, electrons are repelled by the anode and the reverse current is zero. The space between the anode and cathode is evacuated, so that electrons may move between the electrodes unimpeded by collisions with gas molecules.

Cathodes in vacuum diodes take several different forms. The free electrons in any conductor, given sufficient energy by heating, will escape from the solid. Some materials are much more satisfactory in this respect than others, however, either because it is relatively easy for electrons to escape, or because the material can safely withstand high temperatures. Tungsten, for example, is a useful cathode material, because it retains its mechanical strength at extreme temperatures. A thin layer of thorium on the surface of a tungsten cathode filament increases electron emission, and appreciable current is attained at temperatures of about 1900°K. This form of cathode is directly heated by an electric current the way the filament in an incandescent lamp is. It is used in vacuum diodes suitable for high-voltage applications. The conventional symbol for a vacuum diode of this type is shown in Fig. 4-3a.

The modern oxide-coated cathode, which consists of a metal sleeve coated with a mixture of barium and strontium oxides, is the most efficient electron emitter developed to date. Copious electron emission is obtained at temperatures near 1000°K, which means that the power required is much less than for tungsten.

Usually the oxide cathode is indirectly heated by a separate heater located inside the metal sleeve, as in Fig. 4-2. This isolates the heater current electrically from the cathode connection, which is a considerable advantage in electronic circuits. Also, an ac heater
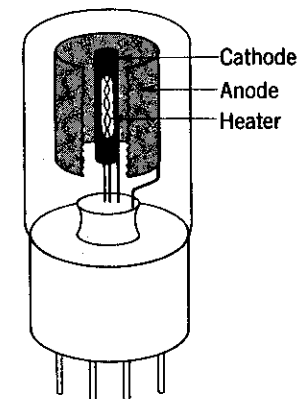

FIGURE 4-2 *Sketch of vacuum diode.*

current may be used without introducing undesirable temperature variations in the cathode at the frequency of the heater current. Oxide cathodes are employed in the great majority of vacuum tubes. The heater is often omitted in the conventional circuit symbol, Fig. 4-3b, since it is not directly an active part of the rectifier.

When the anode is negative with respect to the cathode, the electron current is zero and the reverse characteristic is essentially that of an ideal diode. The forward characteristic is determined by the motion of electrons in the space between the cathode and the anode when the anode has a positive potential. For simplicity, consider a plane cathode and a plane parallel anode separated by a distance d, as in Fig. 4-4. Assume that the potential difference between anode and cathode is V_b and that electrons are emitted from the cathode with zero velocity, which is very nearly true in practice.

If one electron is present in the cathode-anode space, it experiences a force caused by the electric field V_b/d and is uniformly accelerated to the anode. The total current is not just the sum of the currents due to many individual electrons, however. The

FIGURE 4-3 *Circuit symbols for vacuum diodes of (a) filamentary-cathode and (b) heated-cathode types.*
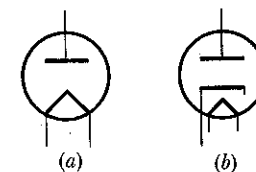
number of electrons emitted from the cathode is so great that the electric fields due to the electron charges drastically alter the uniform field V_b/d set up by the applied voltage. Therefore, the effect
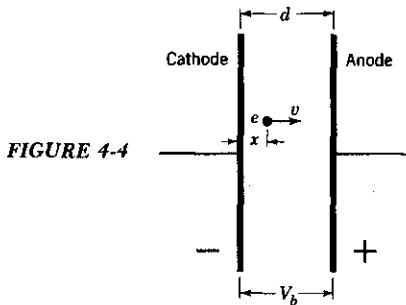

FIGURE 4-4

fect of all the electrons must be considered simultaneously. This is done by applying *Poisson's equation*, which is basically a reformulation of Coulomb's law relating electric field to charge distribution. The solution of Poisson's equation gives the potential V at any point in a region containing a volume density of charge ne. Here, n is the number of electrons per unit volume, and e is the charge on each electron. In one dimension, suitable for Fig. 4-4, Poisson's equation is written

$$\frac{d^2V}{dx^2} = -\frac{ne}{\epsilon_0} \tag{4-1}$$

where ϵ_0 is the permittivity of free space.

The current density between cathode and anode is, from Eq. (1-13),

$$J = -nev \tag{4-2}$$

where v is the velocity of the electrons and the minus sign accounts for the negative charge of the electrons. According to the definition of potential (work per unit charge), the kinetic energy of an electron at any point is related to the potential at that point by

$$\tfrac{1}{2}mv^2 = eV \tag{4-3}$$

where m is the electronic mass. Introducing the current density from Eq. (4-2) and v from Eq. (4-3), Poisson's equation becomes

$$\frac{d^2V}{dx^2} = \frac{J}{\epsilon_0 v} = \frac{J}{\epsilon_0}\left(\frac{m}{2eV}\right)^{1/2} \tag{4-4}$$

The solution to this differential equation for V is

$$V^{3/4} = \frac{3}{2}\left(\frac{J}{\epsilon_0}\sqrt{\frac{m}{2e}}\right)^{1/2} x \tag{4-5}$$

which can be verified by direct substitution. In arriving at Eq. (4-5) it has been assumed that the cathode is at zero potential and

that the electric field at the cathode is zero. These approximations are quite accurate in practical diodes.

Since we are interested in the current between cathode and anode, put $x = d$ and $V = V_b$ in Eq. (4-5) and solve for the current density. The result is

$$J = \frac{4\epsilon_0}{9d^2}\left(\frac{2e}{m}\right)^{1/2}V_b^{3/2} \tag{4-6}$$

This equation, known as *Child's law*, shows that the current varies as the $3/2$ power of the voltage, rather than as the first power, as in Ohm's law. The current-voltage characteristic of a vacuum
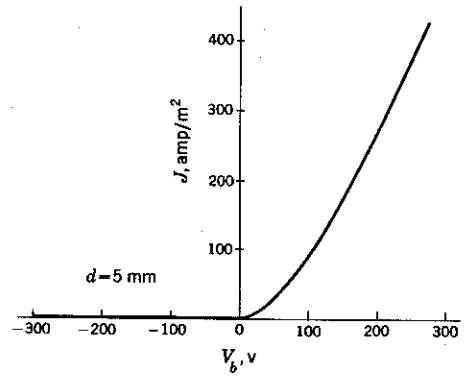

FIGURE 4-5 *Current-voltage characteristic of vacuum diode according to Child's law. Anode-cathode distance is chosen to be 5 mm.*

diode is therefore a horizontal line in the reverse direction and Eq. (4-6) in the forward direction, as shown in Fig. 4-5.

Actually the forward characteristics of practical vacuum diodes depart somewhat from Eq. (4-6) because the electrodes are normally cylindrical rather than planar and because of the simplifications introduced in deriving Child's law. For this reason the forward characteristics of practical diodes are determined experimentally. These characteristic current-voltage curves are presented graphically in manufacturer's compilations known as *tube manuals*.

According to Child's law, the current density in a vacuum diode depends upon the separation d between anode and cathode as well as upon the anode voltage. In addition, the total current of any diode varies directly with the area of the cathode. Thus it is possible to obtain different current-voltage characteristics by altering the geometric shape of the anode and cathode. Two examples of practical diode characteristics are given in Fig. 4-6. The type 5U4 diode is a medium-power rectifier; the type 1V2 diode is designed for high-voltage low-current power supplies. The anode-cathode separation is considerable in the 1V2 in order to minimize the possibility of a discharge passing between anode and cathode on the reverse portion of the voltage cycle. Consequently,

the current in the forward direction is much smaller than in the case for the 5U4, in conformity with Child's law. The 5U4 is designed for use at lower voltages, so the separation between anode and cathode is smaller. The forward current is correspondingly greater at the same forward voltage.
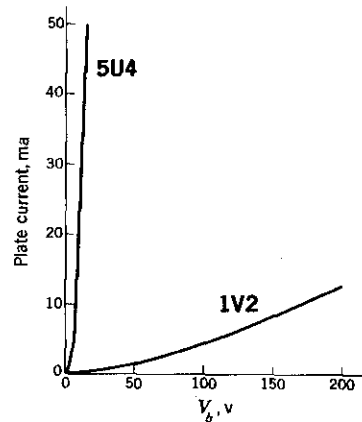

FIGURE 4-6 *Experimental forward characteristics of two practical vacuum diodes are much different because of different geometric construction.*

In any practical vacuum diode the reverse current is not truly zero, because of leakage currents on the surfaces of the glass insulators and similar secondary effects. Typically, the reverse resistance is of the order of 10 MΩ. Since the forward resistance may be of the order of 100 Ω at a suitable operating potential, the ratio of the reverse resistance to the forward resistance, or the *rectification ratio*, is appreciable. The interelectrode capacitance between the cathode and anode limits the maximum frequency at which a vacuum diode is useful. The cathode-anode capacitive reactance is effectively in parallel with the electron current and tends to short out the reverse resistance at high frequencies (see Exercise 4-2).

4-2 The junction diode

As will be shown in Chap. 6, the addition of certain foreign atoms to otherwise pure semiconductor materials such as germanium or silicon produces free electrons which carry electric current. A semiconductor crystal containing such foreign atoms is called an *n-type* crystal because of the negative charge of the current carrying electrons. Similarly, by incorporating certain other kinds of foreign atoms, the semiconductor appears to conduct current by positive current carriers. Such a crystal is called a *p-type* conductor because the current carriers have a positive charge. It is possible to change from n-type to p-type conductivity in the same crystal by introducing an abrupt change from one impurity type

to the other across a given region of the semiconductor. The junction between an n-type region and a p-type region in a semiconductor crystal, called a *pn junction*, is a very good rectifier. The rectifying characteristics of a *junction diode* can be described in the following way.

Electrons in the n-type region tend to diffuse into the p-type region at the junction. In equilibrium, this is compensated by an equal flow of electrons in the reverse direction, for there is a minor, though important, concentration of electrons in the p region. Since the concentration of electrons is much larger in the n-type material than in the p-type, the electron current from the n region would dominate if it were not for the presence of a potential rise at the junction which reduces the current flow in this direction. The polarity and magnitude of this internal potential difference are such as to make the two currents equal. A similar argument applies to the positive carriers crossing the junction between the n and p regions.

When the pn junction is in equilibrium the internal potential barrier V_0 exists between the n and p regions, as indicated schematically in Fig. 4-7a. The current I_1 resulting from electrons diffusing from the n side is equal to the current I_2 which arises from electrons leaving the p side. Suppose now an external potential is applied to the junction so as to increase the internal barrier, as in Fig. 4-7b. The number of electrons diffusing across the junction from the n region is much reduced since very few electrons have sufficient energy to surmount the larger potential barrier. On the other hand, the number moving from the p to the n side is not affected because these electrons encounter no barrier. Thus a net current exists, but it is limited by the small number of electrons in the p region. If the polarity of the external potential is reversed, Fig. 4-7c, the internal barrier is reduced and I_1 is large because the number of electrons in the n region is so great. Again, the electron current I_2 from p-type to n-type remains unaffected. The net current in this case is large and corresponds to the forward direction. The reverse polarity increases the potential barrier and results in only a small current.

In determining the current-voltage characteristic of the junction diode, the same considerations apply to the current carried by the positive carriers as to that carried by electrons, and the total current is the sum of the two. Focusing attention on the electrons first, the current from the p region to the n region is proportional to the electron concentration in the p region n_p, so that

$$I_2 = -C_1 n_p \qquad (4-7)$$

where C_1 is a constant involving the junction area and properties of the semiconductor crystal which are not of direct interest here. The minus sign in Eq. (4-7) accounts for the negative charge on the electron. According to the preceding discussion, I_2 is inde-

pendent of the applied potential V. The current I_1, from the n side to the p side, is proportional to the number of electrons in the n region with sufficient energy to surmount the barrier. This

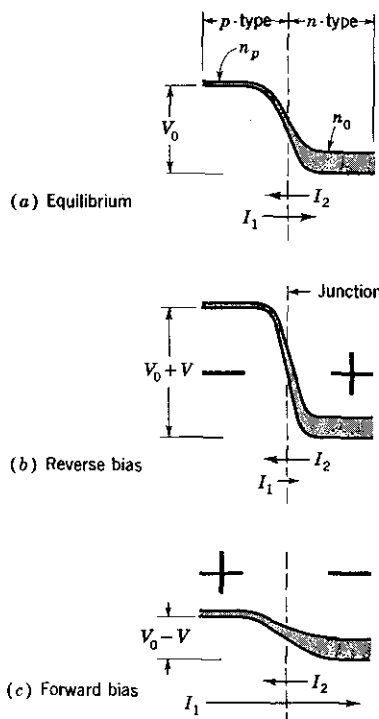(a) Equilibrium

(b) Reverse bias

(c) Forward bias

FIGURE 4-7 *Current across a pn junction depends upon height of potential rise at junction, which is (b) increased under reverse bias and (c) decreased by forward bias.*

number may be determined from the Boltzmann distribution, which relates the concentrations in two regions having different potential energies. That is,

$$I_1 = -C_1 n_0 e^{-e(V_0 - V)/kT} \qquad (4-8)$$

where n_0 is the concentration of electrons in the n region, and the exponential represents the Boltzmann relation. When the applied potential is zero, $I_1 = I_2$. From Eqs. (4-7) and (4-8)

$$n_p = n_0 e^{-eV_0/kT} \qquad (4-9)$$

The net electron current is therefore

$$I_n = I_2 - I_1$$

$$= C_1 n_0 (e^{eV/kT} - 1) e^{-eV_0/kT}$$

$$= C_1 n_p (e^{eV/kT} - 1) \qquad (4-10)$$

An identical expression can be derived for the positive carrier current. The result is

$$I_p = C_2 p_n (e^{eV/kT} - 1) \qquad (4-11)$$

where p_n is the concentration of positive carriers in the n region and C_2 is a constant analogous to C_1. The total current is the sum of Eqs. (4-10) and (4-11),

$$I = I_n + I_p$$

$$= (C_1 n_p + C_2 p_n)(e^{eV/kT} - 1) \qquad (4-12)$$

$$I = I_0 (e^{eV/kT} - 1) \qquad (4-13)$$

where I_0 is called the *saturation current*. Equation (4-13) is known as the *rectifier equation*.

The polarity of the applied potential is such that the p region is positive for forward bias. According to Eq. (4-13) the current increases exponentially in the forward direction. In contrast, the reverse current is essentially equal to I_0, independent of reverse potentials greater than a few volts. A plot of the rectifier equation for small values of applied voltage is shown in Fig. 4-8. It

FIGURE 4-8 *Current-voltage characteristic of pn junction according to rectifier equation.*
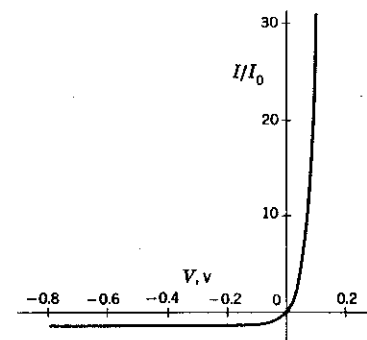
turns out that experimental current-voltage characteristics of practical junction diodes are in good agreement with the rectifier equation.

The junction diode is very nearly an ideal rectifier for the voltages commonly encountered in practical applications. This can be illustrated by comparing the current-voltage characteristics of a typical silicon junction diode with the type 5U4 vacuum diode, Fig. 4-9. Note that the voltage drop in the forward direction is much less for the junction diode. The size of this junction is only about 2 mm², so the junction diode is physically very much smaller than the vacuum diode. A third major advantage of the semiconductor device is that a hot cathode is not required. This means that heater power is not wasted and also that no warmup time is necessary after applying power to the circuit.

The reverse current of a *pn* junction is somewhat greater than that of the *vacuum diode*, although it is still small enough to be negligible. Of greater concern is the fact that the reverse satura-

FIGURE 4-9 *Forward characteristics of silicon pn-junction rectifier, type 1N1615, and vacuum diode, type 5U4.*
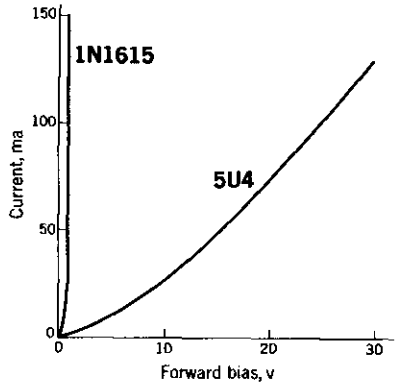
tion current increases rapidly with temperature. According to Eq. (4-13)

$$I_0 = C_1 n_p + C_2 p_n \qquad (4\text{-}14)$$

Substituting from Eq. (4-9) and its equivalent for the positive carrier current

$$I_0 = (C_1 n_0 + C_2 p_0)e^{-eV_0/kT} \qquad (4\text{-}15)$$

According to Eq. (4-15), the reverse current of a junction diode increases exponentially as the temperature increases. Even though I_0 is of the order of only 10 μa at room temperature for a typical silicon diode, the rapid increase means that only a modest temperature rise can be tolerated. Silicon junction diodes, for example, are inoperative above about 200°C. Junction diodes used in circuits at power levels above a few watts or so are cooled in order to dissipate joule heat caused by the current in the device. It is common practice to attach junction diodes firmly to metal heat sinks having radiator fins to conduct the heat away from the diode.

In spite of their temperature sensitivity, a characteristic common to most semiconductor devices, junction diodes are extremely good rectifiers and enjoy wide application. Commercial junction diodes are made of either silicon or germanium by processes similar to those described in Chap. 6. Practical devices are completely encapsulated to protect the semiconductor surface from contamination. This is accomplished by placing the diode in a small metal can filled with an inert atmosphere or by encasing the diode in plastic. Because of their superior electrical characteristics, silicon and germanium junction diodes have largely supplanted older forms of semiconductor rectifiers made of selenium or copper

oxide. It is interesting to note that the latter rectifiers were the first commercially useful semiconductor devices. The circuit symbol for all semiconductor diodes, given in Fig. 4-10, has an arrow to indicate the direction of conventional forward current in the device.

FIGURE 4-10 *Circuit symbol for semiconductor diode.*

The excellent forward conductivity of a *pn* junction means that practical diodes can be quite small. Therefore the stray capacitive reactances are correspondingly small and junction diodes are useful at high frequencies. This feature is enhanced in *point-contact* diodes, in which a metal probe is placed in contact with a semiconductor crystal. During processing a minute *pn* junction is formed immediately under the point. Such tiny devices can operate at frequencies corresponding to millimeter wavelengths and are used, for example, in radar and high-speed computer circuits. A typical point-contact diode is shown in Fig. 4-11.

FIGURE 4-11 *Point-contact gold-bonded diode. (Courtesy Ohmite Manufacturing Company.)*

RECTIFIER CIRCUITS

4-3 Half-wave rectifier

An elementary rectifier circuit, Fig. 4-12, has a vacuum diode in series with an ac source and a resistive load. When the polarity of the source makes the anode positive with respect to the cathode, the diode conducts and produces a current in the load. On the

reverse half-cycle the diode does not conduct and the current is zero. The output current is therefore a succession of half-sine waves, as indicated in Fig. 4-12, and the circuit is called a *half-wave* rectifier. The average value of the half-sine waves is clearly not zero, so that the output current has a dc component. That is, the input sine wave has been rectified.
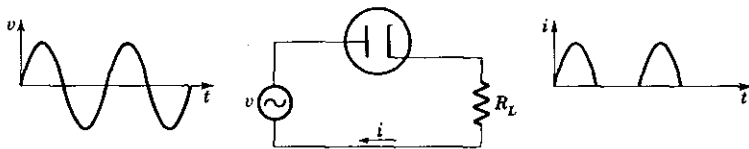

FIGURE 4-12 *Elementary half-wave rectifier circuit.*

The current in the circuit is determined from the voltage equation

$$v = iR_L + v_b \qquad (4\text{-}16)$$

where v_b is the voltage across the diode. Solving for i,

$$i = \frac{v - v_b}{R_L} \qquad (4\text{-}17)$$

This equation and Child's law, Eq. (4-6), must be solved to determine the current which satisfies both equations. Actually, the solution is carried out graphically, because the current-voltage characteristic of a practical vacuum diode is determined experimentally. Starting with a given instantaneous value of the input voltage v_1
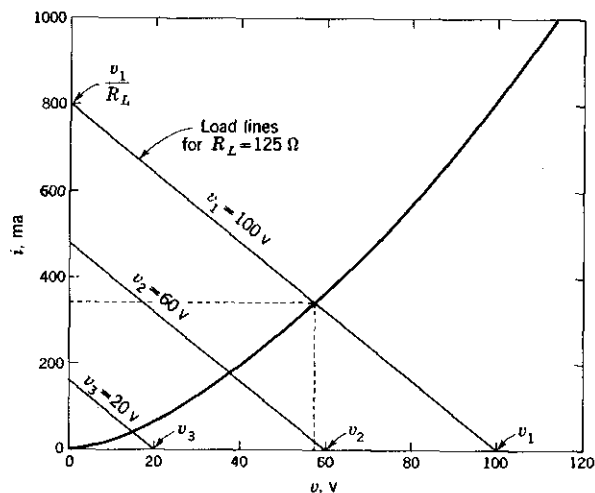

FIGURE 4-13 *Intersection of load line with diode characteristic gives current in circuit.*

Eq. (4-17) is plotted together with the characteristic curve of the diode. The plot of Eq. (4-17) is a straight line, as shown in Fig. 4-13, with a slope $-1/R_L$ and intercepts at $v = v_1$ and at $i = v/R_L$. The intersection of this so-called *load line* with the diode characteristic gives the current at the time the instantaneous input voltage is v_1. As the input voltage swings through the positive half of the cycle the current at every instant can be determined from a similar load line having a voltage intercept corresponding to the instantaneous voltage of the source.

A plot of the current as a function of applied voltage, Fig. 4-14, is called the *dynamic characteristic* of the circuit. The *static character-*
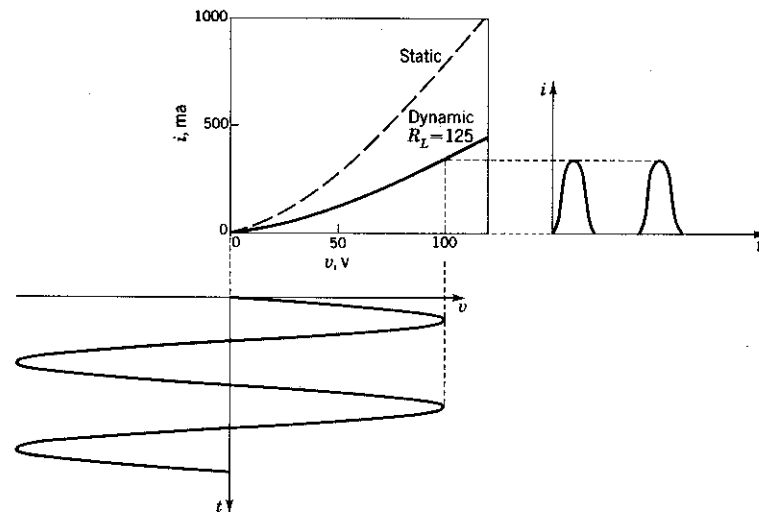

FIGURE 4-14 *Current waveform is nonsinusoidal because of curvature in rectifier dynamic characteristic.*

istic, which is simply the current-voltage curve of the diode, and the dynamic characteristic differ because of the voltage drop across the load resistance. The voltage drop reduces the anode-cathode potential for a given input voltage.

The dynamic characteristic can be used to plot the current waveform resulting from any input voltage waveform, as illustrated in Fig. 4-14. In particular, if the input voltage is sinusoidal, the output current is not truly a half-sine wave because of curvature of the diode characteristic. For many purposes, however, this curvature may be ignored and the forward characteristic replaced by a straight line approximating the true curve. In this case the diode is represented by a fixed resistance R_p in the forward direction. The approximate current in the circuit may then be found immediately from Eq. (4-16) after replacing v_b by iR_p.

4-4 Full-wave rectifier

The half-wave rectifier is inactive during one-half of the input cycle and is therefore less efficient than is possible. By arranging two diodes as in Fig. 4-15 so that each diode conducts on alternate
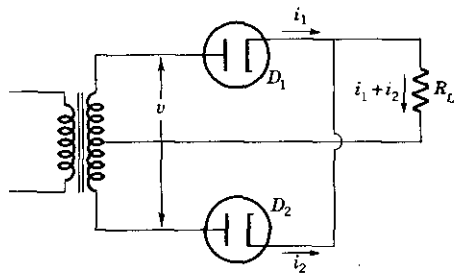

FIGURE 4-15 Full-wave rectifier.

half-cycles, *full-wave* rectification results. This is accomplished by using a center-tapped transformer winding. Then, the anode of diode D_1 is positive with respect to the center tap (and hence its cathode) when the anode of diode D_2 is negative with respect to the center tap. On the alternate half-cycle, the conditions are reversed so that the output-current waveform, Fig. 4-16, has only momentary zero values, in contrast to the half-wave circuit.
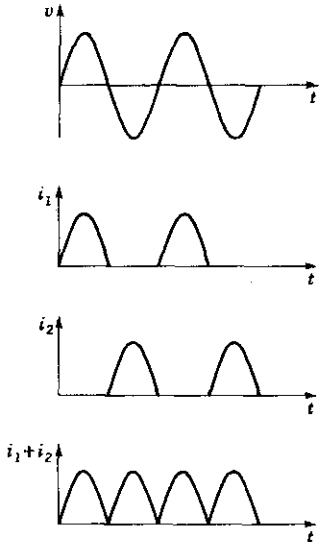

FIGURE 4-16 Waveforms in full-wave rectifier.

The full-wave rectifier, a widely used circuit, can be analyzed exactly as in the case of the half-wave rectifier. Note that each diode in the diagram of Fig. 4-15 must withstand the full end-to-end voltage of the transformer winding. Therefore the *peak inverse voltage* rating of the diodes must be at least twice the peak out-

put voltage. This is usually not a serious drawback except for circuits designed to operate at highest voltages. Comparing the output waveforms of half-wave and full-wave rectifiers, Figs. 4-12 and 4-16, reveals that the fundamental frequency equals the supply voltage in the half-wave rectifier but is twice the supply frequency in the case of the full-wave circuit. This is an important consideration in power-supply circuits, as will be demonstrated in a later section. The center-tapped transformer in the full-wave circuit supplies current on both half-cycles of the input voltage, which permits a more efficient transformer design than is possible in the case of the half-wave circuit. Note, however, that the output voltage is only one-half the total voltage of the transformer secondary.

4-5 Bridge rectifier

Full-wave rectification without a center-tapped transformer is possible with the *bridge rectifier,* Fig. 4-17. The operation of this circuit may be described by tracing the current on alternate half-
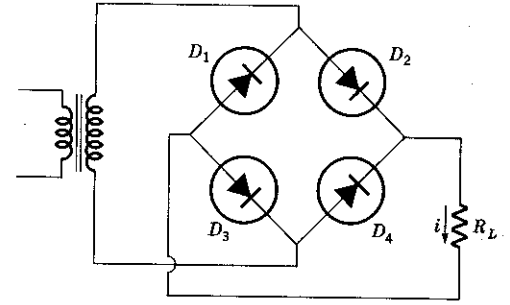

FIGURE 4-17 Bridge rectifier is full-wave circuit without center-tapped transformer.

cycles of the input voltage. Suppose, for example, the upper terminal of the transformer is positive. This means that diode D_2 conducts, as does diode D_3, and current is present in the load resistor. On the alternate half-cycle diodes D_4 and D_1 conduct and the current direction in the load resistance is the same as before. The voltage across R_L corresponds to full-wave rectification and the peak voltage is equal to the transformer voltage less the potential drops across the diodes.

Note that if vacuum diodes are used in a bridge rectifier circuit, the heater currents must be supplied from separate sources, since the diode cathodes are not at the same potential. For this reason it is common to employ junction diodes in this circuit. Since two diodes are in series with the load, the output voltage is reduced by twice the diode drop. On the other hand, the peak inverse voltage rating of the diodes need only be equal to the transformer voltage, in contrast to the previous full-wave circuit.

4-6 Voltage doubler

Consider the circuit of Fig. 4-18, in which two diodes are connected to the same voltage source but in the opposite sense. On the half-cycle during which the upper terminal of the source is
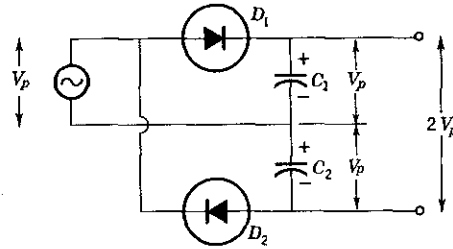

FIGURE 4-18 *Voltage-doubler rectifier yields dc output voltage equal to twice peak input voltage.*

positive, diode D_1 conducts and capacitor C_1 charges to the peak value of the input voltage. On the reverse half-cycle D_2 conducts and C_2 also charges to the full input voltage. Meanwhile, the charge on C_1 is retained, since the potential across D_1 is in the reverse direction. Thus both C_1 and C_2 charge to the peak supply voltage and the dc output voltage is equal to twice the peak input voltage. Accordingly, this circuit is called a *voltage doubler*.

The above analysis applies to the case when no current is delivered to the load. When a load resistance is connected, current is supplied by the discharge of the capacitors. On alternate half-cycles the capacitors are subsequently recharged. This implies, however, that the output voltage under load is no longer dc, but has an ac component. It is necessary to make the capacitance of C_1 and C_2 large enough to minimize this variation in output voltage, taking into account the current drain and the supply frequency. The filtering action of the capacitors in this circuit is treated in greater detail in the next section.

FILTERS

It is usually desirable to reduce the alternating component of the rectified waveform so that the output is primarily a dc voltage. This is accomplished by means of *filters* which are composed of suitably connected capacitors and inductances. A power-supply filter is a low-pass filter which reduces the amplitudes of all alternating components in the rectified waveform and passes the dc component. A measure of the effectiveness of a filter is given by the *ripple factor* r, which is defined as the ratio of the rms value of the ac component to the dc or average value. That is,

$$r = \frac{V_{\text{rms}}}{V_{\text{dc}}} \qquad (4\text{-}18)$$

According to this definition it is desirable to make the ripple factor as small as possible. Although rectified waveforms contain many harmonics of the input voltage frequency, it is generally satisfactory to determine the ripple factor at the fundamental frequency only. This is so because the fundamental frequency is predominant and also because higher harmonics are attenuated more than the fundamental by the low-pass filter characteristic.

4-7 Capacitor filter

The simplest filter circuit consists of a capacitor connected in parallel with the load resistance, as in the typical low-voltage bridge-rectifier power supply shown in Fig. 4-19. If the reactance
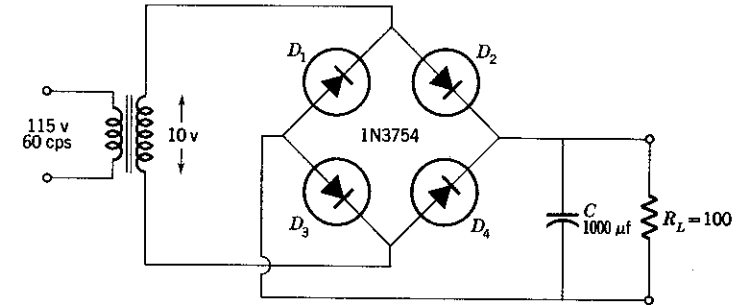

FIGURE 4-19 *Practical low-voltage power supply using capacitor filter.*

of the capacitor at the power-line frequency is small compared with the load resistance R_L, the ac component is shorted out and only dc current remains in the load resistor.

Characteristics of the *capacitor filter* are determined by examining waveforms in the circuit, Fig. 4-20. The capacitor is charged to the peak value of the rectified voltage V_p and begins to discharge
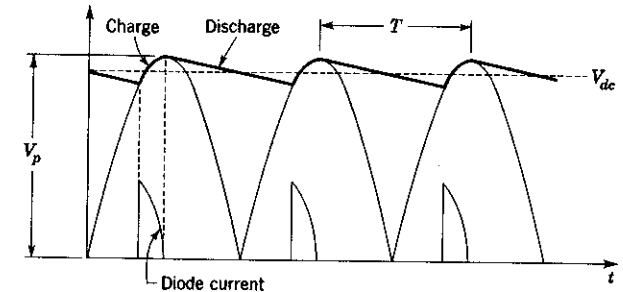

FIGURE 4-20 *Output voltage of capacitor filter is dc voltage and small triangular ripple voltage.*

through R_L after the rectified voltage decreases from the peak value. The decrease in capacitor voltage between charging pulses depends upon the relative values of the RC time constant and the period of the input voltage. A small time constant means the decrease is large and the ripple voltage is also large. On the other hand, a large time constant results in a small ripple component. The diodes conduct only during the portion of the cycle that the capacitor is charging, because only during this interval is the sum of the supply voltage and the capacitor voltage such that the potential across the diodes is in the forward direction.

The ripple voltage is approximately a triangular wave if the RC time constant is long compared with the period, $R_LC \gg T$. In this case the exponential decrease of capacitor voltage during one period is given approximately by $V_p \times T/R_LC$. The dc output voltage is the peak capacitor voltage minus the average ripple component, or

$$V_{dc} = V_p - \frac{V_pT}{2R_LC} = V_p\left(1 - \frac{1}{2fR_LC}\right) \tag{4-19}$$

where f is the main frequency of the rectified waveform. The effective voltage of a triangular wave is calculated in Appendix 2. In present notation

$$V_{rms} = \frac{1}{2\sqrt{3}}\frac{V_pT}{R_LC} \tag{4-20}$$

Therefore, using Eq. (4-18), the ripple factor becomes

$$r = \frac{1}{2\sqrt{3}}\frac{1}{fR_LC} \tag{4-21}$$

where the approximation $R_LC \gg T$ has been used again.

This result shows that ripple is reduced by increasing the value of the filter capacitor. When the load current is equal to zero ($R_L \to \infty$), the ripple factor becomes zero, which means that the output voltage is pure dc. As the load current is increased (smaller values of R_L), the ripple factor increases. Inserting component values given on the circuit diagram of Fig. 4-19, the ripple factor is found to be 0.05. Thus the ripple voltage is about 5 percent of the dc output. Note that the ripple voltage of a full-wave rectifier is approximately one-half that of the half-wave circuit, because the frequency of the rectified component is twice as great.

The dc output voltage may be written, using Eq. (4-19), as

$$V_{dc} = V_p - \frac{1}{2fC}I_{dc} \tag{4-22}$$

where the approximation $I_{dc} \approx V_p/R_L$ has been used. According to Eq. (4-22), the dc output voltage decreases linearly as the dc current drawn by the load increases. The constancy in output

voltage with current is called the *regulation* of the power supply; Eq. (4-22) shows that a large value of filter capacitance improves the regulation. It should be noted that the decrease in output voltage given by this equation refers only to the change accompanying the increase in the ac ripple component. The IR drops associated with diode resistances and the resistance of the transformer winding further reduce the output voltage as the load current increases.

The simple capacitor filter provides very good filtering action at low currents and is often used in high-voltage low-current power supplies. Because of its simplicity, the circuit also is found in those higher current supplies where ripple is relatively less important. The dc output voltage is high, equal to the peak value of the supply voltage. The disadvantages of the capacitor filter are poor regulation and increased ripple at large loads.

4-8 L-section filter

It is useful to add a series inductor to the capacitance filter, as in the circuit of Fig. 4-21. The series inductance in this *L-section* or *choke-input* filter opposes rapid variations in the current and so

FIGURE 4-21 *L-section filter.*
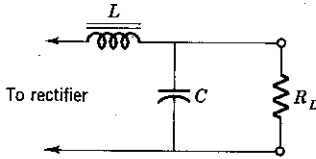
contributes to the filtering action. The ripple factor may be determined by noting that the ac voltage components of the rectified waveform divide between the inductance and the impedance Z of the resistor-capacitor combination. Therefore

$$r = \left(\frac{V_{rms}}{V_{dc}}\right)_f = \frac{(V_{rms})_r[Z/(Z + X_L)]}{(V_{dc})_r} \tag{4-23}$$

where the subscript f refers to the voltage ratio at the output of the filter and the subscript r refers to the rectified waveform. Equation (4-23) may be put in a more instructive form by calculating the magnitude of the impedance and rearranging,

$$r = \left(\frac{V_{rms}}{V_{dc}}\right)_r\frac{1}{1 + X_L/Z} = \left(\frac{V_{rms}}{V_{dc}}\right)_r\frac{1}{1 + \omega L\sqrt{(1/R_L)^2 + (\omega C)^2}} \tag{4-24}$$

$$r = \left(\frac{V_{rms}}{V_{dc}}\right)_r\frac{1}{1 + \omega^2 LC\sqrt{1 + (1/\omega R_LC)^2}} \tag{4-25}$$

Note that the dependence upon R_L, that is, upon the load current, is much smaller than is the case for the capacitor filter. In fact, if $\omega R_L C > 1$, as is done in practice, the ripple factor is independent of the load. Inserting the rms/dc ratio appropriate for a full-wave rectifier (Appendix 2), Eq. (4-25) becomes

$$r = \frac{\sqrt{2}}{3} \frac{1}{\omega^2 LC} \qquad (4\text{-}26)$$

According to Eq. (4-26) large values of L and large values of C improve the filtering action.

The inductance chokes off alternating components of the rectified waveform and the dc output voltage is simply the average, or dc, value of the rectified wave. For a series of half-sinusoids this means that the dc output voltage is (Appendix 2)

$$V_{dc} = \frac{2}{\pi} V_p \cong 0.9V \qquad (4\text{-}27)$$

where V_p is the peak and V is the rms value of the transformer voltage. Therefore, the output voltage of the choke-input filter is considerably less than that of the capacitor filter.

According to Eq. (4-26) the ripple is independent of the load current. This means that there is no decrease in output voltage due to a decrease in filtering action at high currents, such as in Eq. (4-22) for the capacitor filter. Therefore, the output voltage is independent of the load, except for IR drops in the diodes and transformer windings. For this reason the L-section filter is used in applications where wide variations in the load current are expected. The advantage of good regulation must be balanced against the comparatively low output voltage in considering any given application.
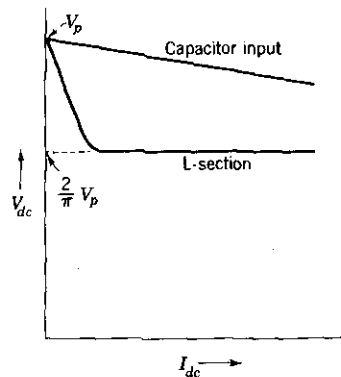
The effectiveness of the inductance is reduced at low currents, so at very light loads the L-section filter acts like a simple capacitance filter. Therefore the output voltage rises to the peak value of the input voltage. In order to prevent this, it is common practice to include a resistance across the capacitor. This resistor draws sufficient current to make the inductance effective even when the load current drops to zero. Such a *bleeder* resistor is also useful in draining the charge from the filter capacitor after the power supply is turned off, which reduces the injury hazard. A comparison of the voltage-regulation curves of a capacitor filter and a choke-input filter, Fig. 4-22, shows the poor regulation but high output of the former and the good regulation but lower voltage of the latter. The minimum current necessary to ensure good filtering action in the case of the L-section filter is clearly evident.

4-9 π-section filter

The combination of a capacitor-input filter with an L-section filter, as shown in the power-supply diagram of Fig. 4-23, is a very popular circuit. The output voltage of this π-section filter is nearly that of the capacitor-input filter, and the regulation characteristics are about the same. The ripple is very much reduced by the double filtering action, however. In fact, the overall ripple factor
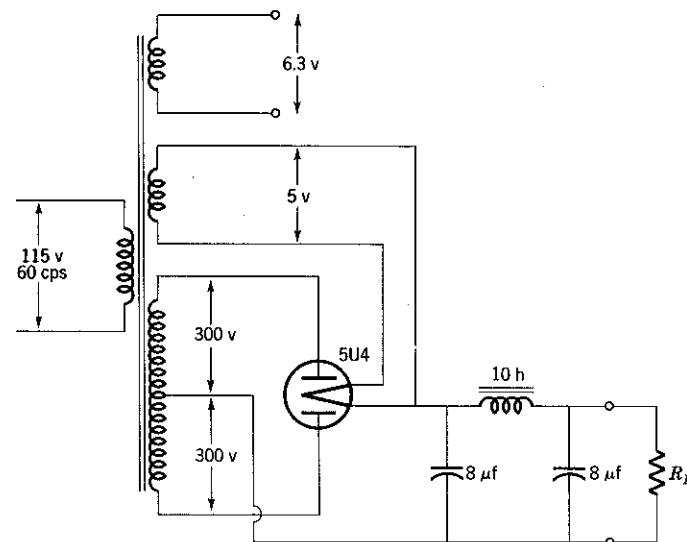

FIGURE 4-22 *Voltage-regulation characteristics of capacitor-input filter and L-section filter.*

FIGURE 4-23 *A practical 300-volt power supply.*

is essentially the product of the ripple factor of the capacitor filter times the impedance ratio of the L-section filter. Therefore, using Eq. (4-21) and the impedance ratio developed for (4-26),

$$r = \frac{\pi}{\sqrt{3}} \frac{1}{\omega R_L C_1} \frac{1}{\omega^2 L C_2}$$

$$= \frac{\pi}{\sqrt{3}} \frac{X_{C_1} X_{C_2}}{X_L R_L} \qquad (4\text{-}28)$$

According to Eq. (4-28), if the reactances of both capacitors are small at the ripple frequency, and that of the inductance is large, the ripple factor is small. Note that the ripple increases with load as R_L is made smaller. In spite of the poor regulation properties of the π-section filter, it is widely used, because of its excellent filtering action.

The practical power-supply diagram of Fig. 4-23 has several other features of note. Both rectifiers are enclosed in the same glass envelope of the type 5U4 diode to simplify full-wave rectification. The 5-volt heater-cathode (the oxide emitter is coated directly on the heater) is heated by a separate secondary winding on the power transformer. Finally, a 6.3-volt winding is also provided to supply the heaters of other vacuum tubes in the associated circuitry. A circuit similar to Fig. 4-23 is very common in electronic devices.

Other combinations of filter types are also used. Two L-section filters, Fig. 4-24a for example, provide the good regulation of the choke-input filter together with very low ripple. The ripple factor of any filter design is calculated by considering each simple filter separately, as in the case of the π-section filter described above.

A useful variant of the π-section filter particularly suitable for low-current circuits replaces the inductance with a resistor, Fig. 4-24b; the ripple factor is simply Eq. (4-28) with X_L replaced by the value of the series resistor R. This circuit is useful only if the current is low enough so that the IR drop across the resistor is not excessive and if regulation is of secondary concern. Within these restrictions, the circuit provides better filtering action than a simple capacitance filter.
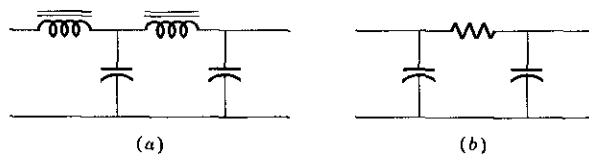

FIGURE 4-24 (a) Double L-section filter, and (b) π-section RC filter.

VOLTAGE REGULATORS

It is often desirable that the voltage of a power supply remain fixed independent of the load current. Although the regulation of the choke-input filter is very good, the output voltage decreases with increasing current even in this circuit, because of the IR drops across the transformer winding resistance and the rectifier diodes. Other variables, such as changes in the line voltage and component aging, may also contribute to variations in the power-supply voltage. In order to maintain the output voltage constant, *voltage regulator* devices are made part of the power supply. These are fairly elaborate electronic circuits if very precise regulation is necessary, as discussed in a later chapter. In other cases, special components that maintain modest voltage stability are used.

4-10 Zener diodes

At some particular reverse-bias voltage the reverse current in a *pn* junction increases very rapidly. This happens when electrons are accelerated to high velocities by the field at the junction and produce other free electrons by ionization collisions with atoms. These electrons are similarly accelerated by the field and in turn cause other ionizations. This avalanche process leads to very large current and the junction is said to have suffered breakdown. The breakdown is not destructive, however, unless the power dissipation is allowed to increase the temperature to the point where local melting destroys the semiconductor. The voltage across the junction remains quite constant over a wide current range in the breakdown region. This effect can be used to maintain the output of a power supply at the breakdown voltage.
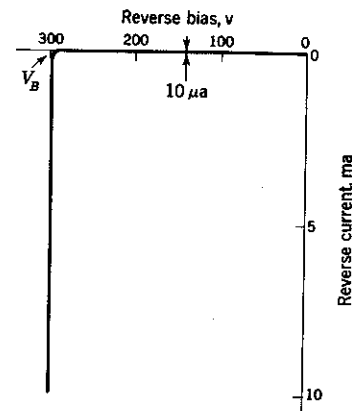

FIGURE 4-25 Reverse breakdown characteristic of pn-junction Zener diode.

These *pn* junctions are called *Zener diodes*, because Clarence Zener first suggested an explanation for the rapid current increase at breakdown. The current-voltage curve of a Zener diode, Fig. 4-25, has a sharp transition potential and a flat current plateau above breakdown. Zener diodes may be obtained with breakdown voltages ranging from about two volts to several hundred volts and with current ratings of a few milliamperes to many amperes.

The way in which a Zener diode is used to regulate the output voltage of a power supply is illustrated in Fig. 4-26. The unregulated output voltage of the supply V must be greater than the
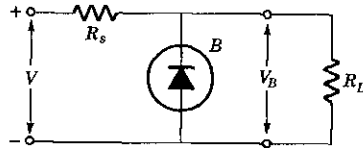
FIGURE 4-26 *Zener diode used as voltage regulator.*

Zener breakdown voltage of the diode. Then the voltage drop across R_S caused by the diode current plus the diode breakdown potential adds up to the power-supply potential. As the load current increases, the diode current decreases, so the drop across R_S always is the difference between the breakdown potential and the power-supply voltage. Even if the power-supply voltage varies under load, the regulated output voltage remains constant.

The minimum value of the series resistor R_S is determined by the maximum permissible diode current I_M.

$$R_S = \frac{V}{I_M} \tag{4-29}$$

If it is known that the load current never falls below a certain minimum value, the series resistance can profitably be increased above that given by Eq. (4-29). This minimizes the power lost in the series resistor.

4-11 VR tubes

An older form of voltage regulator, the *VR tube*, is based on the properties of an electric glow discharge in a low-pressure inert gas between two unheated electrodes. If the potential between the electrodes is high enough, electrons are accelerated sufficiently to ionize gas atoms by collision and a current avalanche results. As in the case of the Zener diode, the electrode potential is nearly independent of the current in the discharge, so the device can be used as a voltage regulator in a power supply. It turns out that the electrode potential required to initiate the gas discharge is about 10 percent greater than the sustaining potential, so that the un-

regulated power-supply voltage must be correspondingly greater.

VR tubes are available with 75-, 90-, 105-, and 150-volt ratings and with a nominal current range of 5 to 40 ma. It is therefore possible to exert control over load-current variations covering this same range. Different regulated voltages can be obtained by using two or more VR tubes in series (Fig. 4-27). This can also be done
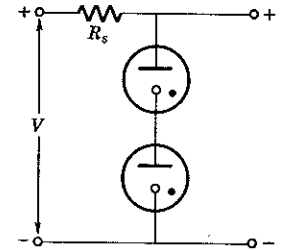
FIGURE 4-27 *Two voltage-regulator tubes may be placed in series to regulate dc voltage equal to sum of individual breakdown voltages.*

with Zener diodes, if necessary, although the wider selection of voltage ratings available usually makes this unnecessary. Parallel connection of regulators to increase the current range is not feasible, for one of the parallel-connected units unavoidably has a slightly lower operating potential and therefore carries all of the current. The circuit symbol for the VR tube, shown in Fig. 4-27, has an open circle symbolizing the cold cathode. The solid dot indicates a gas-filled tube.

Both VR tubes and Zener diodes also provide filtering action since they tend to maintain the output voltage constant against changes in the power-supply voltage, including ripple. For this reason it is often possible to employ only rudimentary, inductance-capacitance filtering in conjunction with voltage regulators.

4-12 Controlled rectifiers

It is often necessary to control the power delivered to some load, such as an electric motor or the heating element of a furnace. Series resistances or potentiometers waste power, a serious drawback in high-power circuits. *Controlled rectifiers* have been developed which are capable of adjusting the transmitted power with little waste.

The most satisfactory unit of this kind is the *silicon controlled rectifier*, or *SCR*. This semiconductor device contains four parallel *pn* junctions; it is described in detail in Chap. 6. For present purposes it is sufficient to note that it is similar to a junction rectifier in which forward conduction is controlled by the current in a control electrode, called the *gate*. The inclusion of the gate electrode is shown on the SCR symbol (Fig. 4-28).

The current-voltage characteristic of a typical SCR, Fig. 4-29, is identical to a junction rectifier in the reverse direction. The forward direction has both an ON state, which is equivalent to normal

FIGURE 4-28   *Circuit symbol for silicon controlled rectifier.*
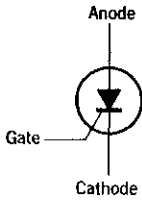
forward conduction in a junction rectifier, and a low-current OFF state. So long as the forward anode-cathode potential remains below a certain critical value (actually it is the corresponding current which is critical), the forward current is small. Above this
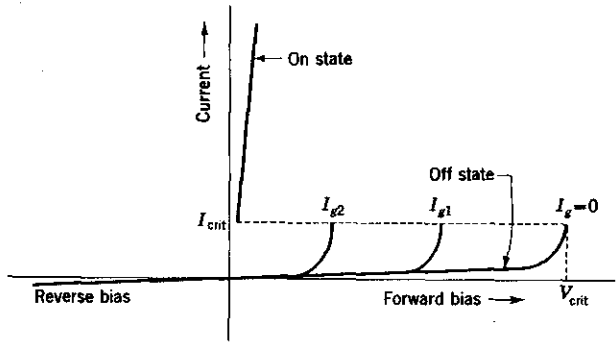

FIGURE 4-29   *Current-voltage characteristic of SCR. Unit can be triggered into ON state by gate current. Here $I_{g2} > I_{g1} > 0$.*

critical value the SCR is in the high-current low-voltage ON condition. The critical current may be supplied by the gate electrode, which means that the SCR may be triggered into the ON position by a small current (as low as 100 μa) supplied to the gate terminal. It is not necessary to maintain the gate current, for once the SCR is triggered into the high-conduction state it remains in this condition until the anode potential is reduced to zero.

Consider the simple SCR circuit of Fig. 4-30, in which the motor speed is controlled by the power delivered to it. The peak value of the transformer voltage V_p is less than the critical value, so that, unless the SCR is triggered by a gate current, the load current is zero. Suppose the gate is supplied with a current pulse each cycle that lags the transformer voltage by a phase angle α, as illustrated in Fig. 4-31. Forward conduction is delayed until this point in each cycle and the rectified output waveform is similar to that of a
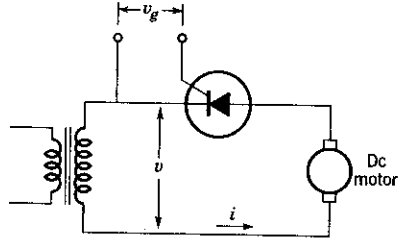
half-wave rectifier, except that the first part of each cycle is missing. The average, or dc, value of the current in the motor is found by integrating the output current over a complete cycle, or

$$I_{dc} = I_p \int_{\alpha}^{\pi} \sin \omega t \, d\omega t \qquad (4\text{-}30)$$

where I_p is the peak value of the current. Integrating,

$$I_{dc} = \frac{I_p}{2\pi} (1 + \cos \alpha) \qquad (4\text{-}31)$$

FIGURE 4-30   *Simple SCR circuit to control speed of dc motor.*

According to Eq. (4-31) the motor current can be adjusted from a maximum ($\alpha = 0$) to zero ($\alpha = \pi$) simply by changing the phase angle of the gate pulses. Simple circuits are available to generate such pulses, as described in a later chapter.
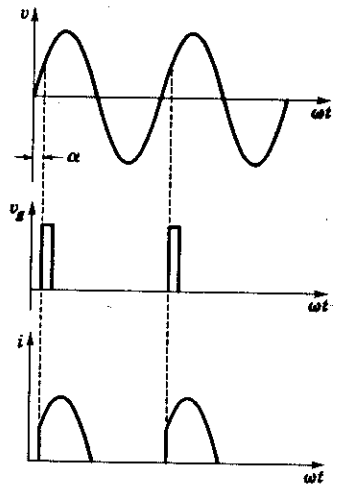

FIGURE 4-31   *Output current waveform depends upon phase angle between transformer voltage and gate current pulse.*

It is not necessary to use pulses in the gate circuit, so long as care is taken to limit the power dissipated by the control electrode below that which might damage the SCR. Consider the *phase-shift control* circuit of Fig. 4-32, in which the gate current is shifted

in phase with respect to the anode voltage. Since the gate voltage is the sum of the secondary voltage v_1 and the drop across R, the gate voltage is

$$v_g \angle \phi_g = v_1 - Ri \angle \phi \qquad (4\text{-}32)$$

This is easily solved for the phase angle between the gate voltage and the anode voltage, which has the same phase as v_1. The result

$$\phi_g = \arctan \frac{2R\omega C}{(R\omega C)^2 - 1} \qquad (4\text{-}33)$$

shows that the gate voltage may be put in phase with the anode potential ($R = 0$) or nearly $180°$ out of phase ($R = \infty$).
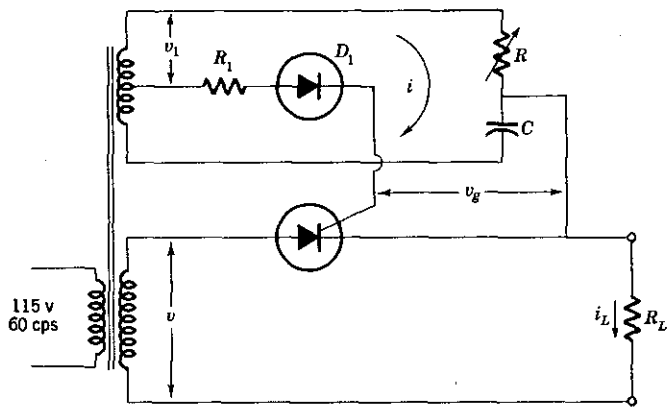
FIGURE 4-32 *Phase-shift control of SCR.*

The waveforms of Fig. 4-33 show how variation of ϕ_g alters the time at which the diode conducts on each forward cycle as the gate voltage becomes larger than the critical value. Note that the gate signal remains *on* during part of the reverse half-cycle as well, but this has no effect upon the reverse characteristic. The purpose of resistor R_1 and diode D_1 in the circuit (Fig. 4-32) is to limit the gate current to a safe value and to prevent reverse gate current, both of which are deleterious to the SCR.

Characteristics similar to those of the SCR are also obtained in hot-cathode gas tubes provided with a control electrode, called the *grid*. The circuit symbol of such a so-called *thyratron* is shown in Fig. 4-34. Proper operation of these tubes is based on a gas discharge similar to that in the VR tube, except that a heated cathode is used and the grid determines the extent to which the current avalanche can proceed at a given anode potential. Electron emission from the heated cathode greatly increases the current-carrying ability of the discharge. The anode-to-cathode potential necessary to maintain the discharge is quite low. In fact, hot-cathode gas-filled diodes are used as rectifiers in many applications.

The control electrode initiates the discharge, but the anode potential must maintain it. Thus, the action is quite similar to that of the gate electrode in an SCR. Many different kinds of thyra-

FIGURE 4-33 *Waveforms in phase-shift control SCR circuit.*

trons have been developed, some of which require a positive grid potential with respect to the cathode whereas others employ a negative grid potential. The former do not conduct unless the
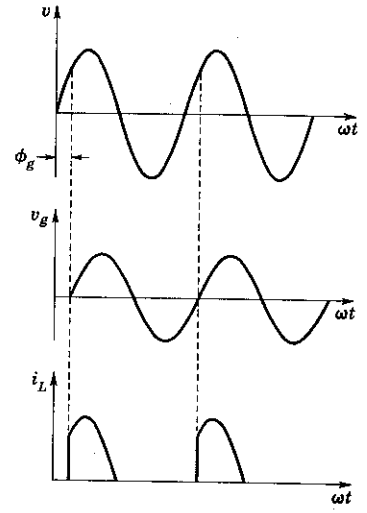
FIGURE 4-34 *Circuit symbol for thyratron.*

control grid is made momentarily positive to initiate the discharge. The negative control type starts conduction whenever the anode voltage is high enough for a given negative grid potential. In either type the discharge is maintained independent of the grid potential until the anode voltage is removed.
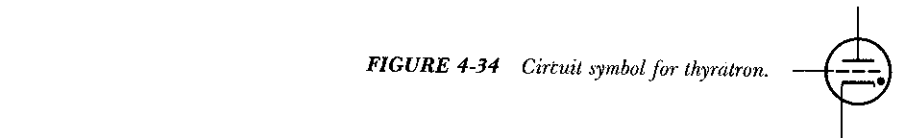
Thyratrons are used in circuits similar to those appropriate for semiconductor controlled rectifiers. Actually SCRs are gradually replacing thyratrons in many applications, because of their small size, savings in heater power, smaller internal voltage drop, and faster switching speed. In applications involving high ambient temperatures, or extreme operating conditions, thyratrons and other gas-filled devices are unsurpassed, however.

DIODE CIRCUITS

Diodes prove to be useful in circuits other than power supplies. A major application is in circuits designed to operate with square-

wave pulses, as will be discussed in a later chapter. The rectification characteristics of diodes are also put to work in circuits dealing with sine-wave signals. In these applications the nonlinear diode modifies the sine-wave signals in a specified manner.

4-13 Clippers

Consider the diode *clipper* circuit, Fig. 4-35, in which two diodes are connected in parallel with an ac voltage source. Batteries V_1 and V_2 bias each diode in the reverse direction. Whenever the
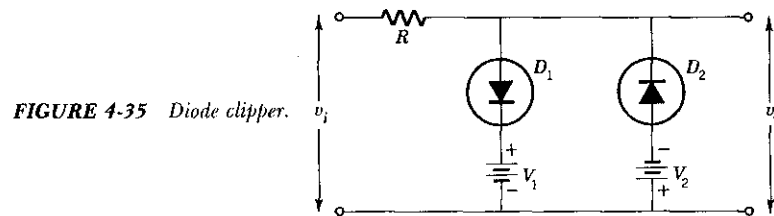
FIGURE 4-35 *Diode clipper.*

input voltage signal is greater than V_1, diode D_1 conducts and causes a voltage drop across the series resistor R. Similarly, every time the input voltage becomes more negative than V_2, diode D_2 conducts. Thus, the output waveform is *clipped* or *limited* to the voltages set by the reverse biases V_1 and V_2. The clipping action is most efficient when the series resistance is much greater than the load impedance.

Clipper circuits produce square waves from a sine-wave generator, as illustrated by the waveforms in Fig. 4-36. If $V_1 = V_2$ and
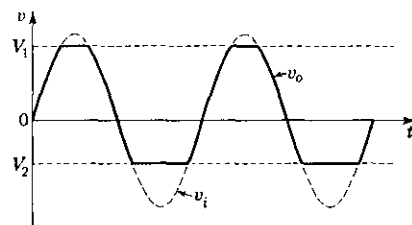
FIGURE 4-36 *Maximum amplitudes in output waveform of diode clipper are limited to values of bias voltages.*

the amplitude of the input signal is considerably greater than the bias voltages, the output waveform is very nearly a square wave. Note that if, say, $V_2 = 0$, the output can never swing negative, so the waveform is a train of positive-going square pulses. The clipping action operates on any input waveform; it can be adjusted by altering the reverse-bias voltages V_1 and V_2.

The clipper circuit is also a useful protective device which limits input voltages to a safe value set by the bias voltage. In radio-receiver circuits limiters are often used to reduce the effect of strong noise pulses by limiting their amplitude to that of the

desired signal. The signal waveform is transmitted undistorted so long as the instantaneous amplitude remains smaller than the bias voltages.

4-14 Clamps

The diode *clamp* circuit is shown in Fig. 4-37. Consider first the situation in which the bias voltage V is set equal to zero. The diode conducts on each negative cycle of input voltage and charges the
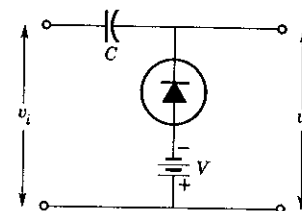
FIGURE 4-37 *Diode clamp.*

capacitor to a voltage equal to the negative peak value of the input signal. If the load current is zero, the capacitor retains its charge on the positive half-cycle, since the diode voltage is in the reverse direction. Thus, the output voltage is

$$v_o = v_i + V_p \tag{4-34}$$

where V_p is the negative peak value of the input voltage. According to Eq. (4-34) the output voltage waveform replicates the input signal except that it is shifted by an amount equal to the dc voltage across the capacitor.

The output waveform corresponding to a sinusoidal input, Fig. 4-38, has the negative peaks of the sine wave *clamped* at zero volt-

FIGURE 4-38 *Negative peak of output waveform is clamped at zero when V = 0 in diode clamp circuit of Fig. 4-37.*

age. This is always the case, independent of the amplitude of the input voltage. Furthermore, the negative peaks are always clamped at zero volts no matter what form the input waveshape takes. When the terminals of the diode are interchanged, the same circuit analysis applies; the positive peaks of the output wave are clamped at zero.

If the bias voltage V in Fig. 4-37 is set at a potential other than zero, the capacitor charges to a voltage equal to $V_p + V$. Therefore, the negative peaks are clamped at the voltage V (see Exercise 4-16). Similarly, when the bias is $-V$, the negative peaks are

clamped at this potential. Reversing the diode polarity makes it possible to clamp the positive peaks of the input wave at the voltage equal to the bias potential.

The diode clamp is used in circuits which require that the voltages at certain points have fixed peak values. This is important, for example, if the waveform is subsequently clipped at a given voltage level. In most diode clamp circuits it is useful to connect a large resistance across the output terminals so that the charge on the capacitor can eventually drain away. This makes it possible for the circuit to adjust to changes in amplitude of the input voltage.

4-15 Ac voltmeters

The rectifying action of diodes makes it possible to measure ac voltages with a dc voltmeter, such as the d'Arsonval meter discussed in the first chapter. Two ways in which this is accomplished in VOM meters are illustrated in Fig. 4-39. In Fig. 4-39a diode
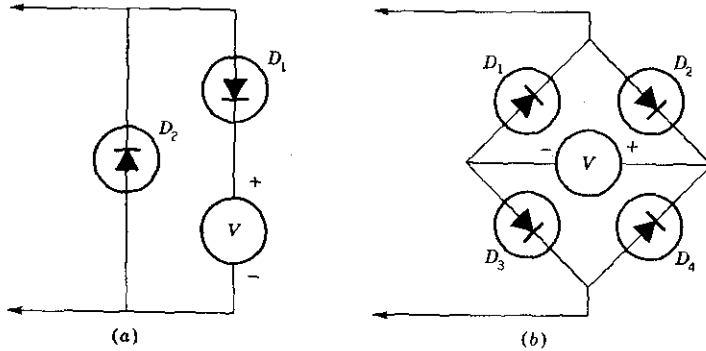

FIGURE 4-39   (a) Half-wave and (b) full-wave ac voltmeters.

D_1 is a half-wave rectifier and the dc component of the rectified waveform registers on the dc voltmeter. Diode D_2 is included to rectify the negative cycle of the input waveform. Although no meter current results from the action of diode D_2, it is included so that both halves of the ac voltage are rectified. This ensures that the voltmeter loads the circuit equally on both half-cycles and avoids possible waveform distortion. The bridge circuit, Fig. 4-39b, has greater sensitivity than the half-wave circuit because it is a full-wave rectifier.

The deflection of a d'Arsonval meter is proportional to the average current, so that the voltmeters of Fig. 4-39 measure the average value of the ac voltage. The meter scale is commonly

calibrated in terms of rms readings, however, in order to facilitate comparison with dc readings. This calibration assumes that the ac waveform is sinusoidal, and, if this is not the case, the meter readings must be interpreted in terms of the actual waveform measured. Series resistor multipliers are used with ac voltmeters to increase the range, just as in the case of dc meters.

The half-wave rectifier, Fig. 4-40, makes a useful *peak-reading* voltmeter if the current drain through the meter circuit is small. In this case the capacitor charges to the positive peak value of the
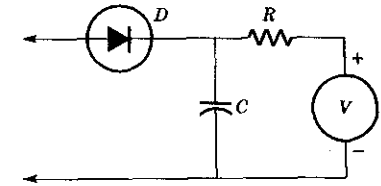

FIGURE 4-40   Peak-reading ac voltmeter.

input waveform on each half-cycle, and the dc meter deflection corresponds to this value. Here again, it is common practice to calibrate the meter scale in terms of rms readings, assuming that the unknown voltage is sinusoidal. This is accomplished by designing the scale to indicate the peak value divided by $\sqrt{2}$. In case the unknown voltage is not sinusoidal, the peak value is obtained by multiplying the scale reading by $\sqrt{2}$. A difficulty with this circuit is that the system being measured must contain a dc path. If this is not so and a series capacitor is present, the voltage across C depends upon the relative values of the two capacitors, since they are effectively in series during the forward cycle. This results in erroneous voltage readings.

The diode clamp is a peak-reading circuit which does not have this difficulty. It is customary to include an RC filter, Fig. 4-41, to
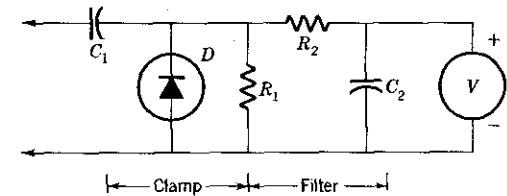

FIGURE 4-41   Diode clamp used as peak-reading voltmeter.

remove the ac component of the clamped wave. Referring to the waveform in Fig. 4-38, the average value is equal to the negative peak of the input voltage, so the dc meter reading corresponds to the negative peak voltage. Note that a series capacitor in the circuit to be measured in effect becomes part of the clamping action and does not result in a false meter indication. If the filter in Fig. 4-41 is replaced with a diode peak rectifier, Fig. 4-40, the combination reads the peak-to-peak value of the input wave (Exercise

4-17). Although usually calibrated in terms of rms readings for a sine wave, a peak-to-peak voltmeter is very useful in measuring complex waveforms.

4-16 Detectors

Nonlinear diode properties are useful in other ways than those corresponding to actual rectification. Suppose that the amplitude of the ac voltages applied is very small, so the diode current-voltage characteristic, Eq. (4-6) or (4-13), can be represented by the expression

$$i = a_1 v + a_2 v^2 \qquad (4-35)$$

where a_1 and a_2 are constants. An expression of this form is a satisfactory approximation to the diode characteristic for sufficiently small values of applied voltage. According to Eq. (4-35), one part of the current is proportional to the square of the input voltage.

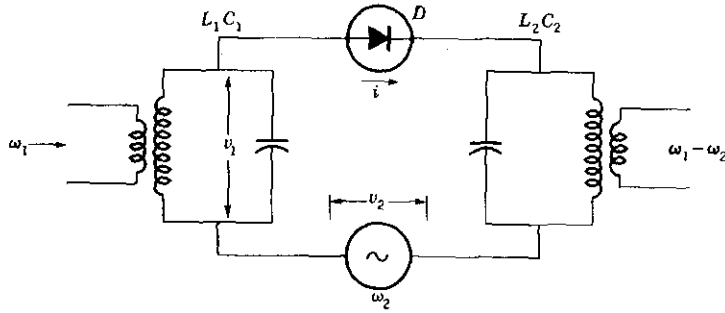Consider the circuit shown in Fig. 4-42, in which two sinusoidal

FIGURE 4-42 *Diode mixer uses nonlinear properties of diode.*

signals of somewhat different frequencies, ω_1 and ω_2, are supplied to a diode. The total diode voltage is the sum of the individual waves,

$$v = V_1 \sin \omega_1 t + V_2 \sin \omega_2 t \qquad (4-36)$$

where V_1 and V_2 are the peak amplitudes. The current is found by substituting Eq. (4-36) into Eq. (4-35), assuming for the moment that the diode impedance is greater than other impedances in the circuit,

$$i = a_1 V_1 \sin \omega_1 t + a_1 V_2 \sin \omega_2 t + a_2 V_1^2 \sin^2 \omega_1 t$$
$$+ a_2 V_2^2 \sin^2 \omega_2 t + 2 a_2 V_1 V_2 \sin \omega_1 t \sin \omega_2 t \qquad (4-37)$$

Rearranging terms and introducing trigonometric substitutions for $\sin^2$ and $\sin \omega_1 t \sin \omega_2 t$,

$$i = \frac{a_2}{2}(V_1^2 + V_2^2) + a_1(V_1 \sin \omega_1 t + V_2 \sin \omega_2 t)$$

$$- \frac{a_2}{2}(V_1^2 \cos 2\omega_1 t + V_2^2 \cos 2\omega_2 t)$$

$$+ a_2 V_1 V_2 \cos (\omega_1 - \omega_2)t - a_2 V_1 V_2 \cos (\omega_1 + \omega_2)t \qquad (4-38)$$

The first term in Eq. (4-38) is a direct current, while the second corresponds to the input voltages. The last two terms are at frequencies not present in the original inputs. Suppose ω_1 and ω_2 are not too different; then the term involving $\omega_1 - \omega_2$ is a comparatively low frequency.

In the *diode mixer* circuit of Fig. 4-42 the output circuit $L_2 C_2$ is tuned to the frequency $\omega_1 - \omega_2$, so only this component has an appreciable amplitude at the output terminals. In effect, an input signal of frequency ω_1 is mixed with a constant-amplitude sine wave at ω_2 and is converted to an output frequency $\omega_1 - \omega_2$. This is called *heterodyning* and is of considerable importance in, for example, radio receivers. The incoming high-frequency signal is converted into a lower-frequency signal which is easier to amplify. Furthermore, changing the frequency ω_2 means that the receiver is tuned to a different ω_1, such that $\omega_1 - \omega_2$ is a constant. This is important because the amplification of the $\omega_1 - \omega_2$ signal is accomplished by fixed tuned circuits that are inexpensive and easy to keep in adjustment, yet the receiver may be tuned to different input frequencies. In this application Fig. 4-42 is called a *first detector* because the signal frequency is modified in the first stages of the receiver.

This circuit has another important use, which may be illustrated by rearranging Eq. (4-38). Using the trigonometric substitution,

$$2 \sin a \sin b = \cos (a - b) - \cos (a + b) \qquad (4-39)$$

Equation (4-38) may be put in the form

$$i = \frac{a_2}{2}(V_1^2 + V_2^2) + a_1 V_2 \sin \omega_2 t - \tfrac{1}{2}(V_1^2 \cos 2\omega_1 t + V_2^2 \cos 2\omega_2 t)$$

$$+ a_1 V_1 \sin \omega_1 t + 2 a_2 V_1 (V_2 \sin \omega_2 t) \sin \omega_1 t \qquad (4-40)$$

Suppose ω_2 is much smaller than ω_1 and that the output circuit $L_2 C_2$ is tuned to the frequency ω_1. Then only the last two terms in Eq. (4-40) have an appreciable output amplitude. The first of these is simply the current resulting from the input voltage at the frequency ω_1. Careful inspection of the last term in Eq. (4-40) shows that it is a sinusoidal waveform at a frequency ω_1 but with an amplitude that varies sinusoidally at a frequency corresponding to ω_2.

The waveform of the last term in Eq. (4-40), illustrated in Fig. 4-43, shows how the amplitude varies. The frequency ω_1 is said to be modulated at the frequency of ω_2. This is the basis for radio
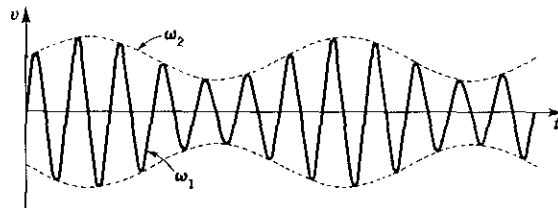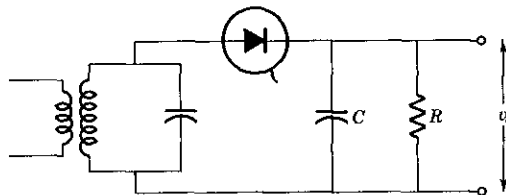

FIGURE 4-43 *Modulated waveform.*

and television communication wherein a high-frequency *carrier* ω_1 is modulated in accordance with a low-frequency signal ω_2 corresponding to the sound or picture to be transmitted. The high-frequency voltage is radiated from the sending station to a receiver where the waveform of the signal is recovered.

The signal is recovered by *demodulating* the modulated waveform with a diode peak rectifier, Fig. 4-44. Since the output of the peak

FIGURE 4-44 *Diode peak rectifier used to demodulate modulated waveform.*

rectifier is equal to the peak value of the input voltage, the waveform across the output capacitor corresponds to that of the modulating signal, as in Fig. 4-45. In this application the peak
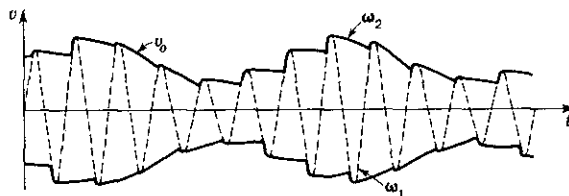

FIGURE 4-45 *Modulating waveform recovered at output of second detector.*

rectifier is called a second detector, because the waveform is changed for the second time in the receiver circuit. The second detector circuit, Fig. 4-44, is widely used in radio and television receivers.

EXERCISES

4-1 Calculate the forward resistance of a 5U4 vacuum diode at a voltage of 50 volts. Assuming the reverse resistance is 10 MΩ, what is the reverse to forward resistance ratio? *Ans.:* 115 Ω; 8.6×10^4

4-2 Assuming the interelectrode capacitance of a 5U4 diode is 4 pf and the reverse resistance is 10 MΩ, what is the approximate upper limit to the frequency at which the diode is useful?
Ans.: 2.5×10^4 cps

4-3 By differentiating the rectifier equation for a junction diode, Eq. (4-13), determine an expression for the junction resistance $R = (dI/dV)^{-1}$. Given that $e/kT = 38$ volts^{-1} at room temperature, calculate the rectification ratio at a potential of 1 volt. *Ans.:* 10^{33}

4-4 Approximate the forward characteristic of a 5U4 diode with a straight line, using Fig. 4-6, and determine the peak rectified current in a half-wave circuit with a 5000-Ω load and an input voltage of 100 volts rms. Determine the direct current by averaging the rectified half-sine wave over a full cycle and compare with the peak current. *Ans.:* 27.5 ma; 8.7 ma

4-5 Repeat Exercise 4-4 for a full-wave circuit in which the transformer secondary voltage is 200 volts rms center-tapped. Note that the direct current is found by averaging a rectified half-sine wave over one-half a cycle. *Ans.:* 27.5 ma; 17.5 ma

4-6 In the voltage-doubler circuit, Fig. 4-18, the input voltage is 115 volts rms at a frequency of 60 cps. If the load resistance is 10,000 Ω, what is the minimum value of the capacitors necessary to be sure the voltage drop between charging pulses is less than 10 percent of the output voltage? *Ans.:* 1.85×10^{-6} farad

4-7 Calculate the output voltage and ripple factor of the circuit of Fig. 4-19 after diodes D_1 and D_4 are removed. *Ans.:* 11.8 volts; 0.096

4-8 Plot the dc output and the ripple voltage of the rectifier circuit of Fig. 4-19 as a function of load current up to a current of 1 amp. Repeat for the circuit of Exercise 4-7.

4-9 Plot the dc output and the ripple voltage of the full-wave rectifier circuit given in Fig. 4-23 up to a current of 200 ma.

4-10 Determine the ripple factor of an L-section filter comprising a 10-henry choke and a 8-μf capacitor used with a full-wave rectifier. Compare with a simple 8-μf capacitor filter at a load current of 50 ma and also 150 ma, assuming an output of 50 volts.
Ans.: 0.042; 0.604; 1.82

4-11 What are the ripple factor and ripple voltage of the full-wave circuit of Fig. 4-23 if the load resistance is 2000 Ω? Repeat, assuming the output capacitor is defective and is open-circuited.

<div align="right"><i>Ans.:</i> 0.026; 11 volts</div>

4-12 In the Zener-diode voltage regulator of Fig. 4-26 determine the range of load resistances over which the circuit is useful if $R_S = 1500\ \Omega$ and the supply voltage $V = 150$ volts; the diode breakdown voltage is 100 volts and maximum rated current is 100 ma. For a fixed $R_L = 10,000\ \Omega$, over what range of input voltages does the circuit regulate? *Ans.:* Greater than 3 kΩ; 101.5 to 251.5 volts

4-13 In Exercise 4-12, what is the proper value of R_S if the maximum diode current is 10 ma? *Ans.:* 15 kΩ

4-14 The power-supply voltage in Fig. 4-27 is 350 volts, the series resistor is 2500 Ω, and the VR tubes are rated at 105 volts. If the load current is 20 ma at 210 volts, what is the maximum current possible in a second load resistor connected to the point between the tubes so that this output voltage is 105 volts? *Ans.:* 36 ma

4-15 Using the phase-shift controlled rectifier circuit, Fig. 4-32, plot the dc load current as a function of control resistance R if the transformer secondary voltage is 100 volts rms, the load resistance is 10 Ω, and $C = 0.1\ \mu f$. Assume the SCR turns on whenever the gate voltage is positive with respect to the cathode.

4-16 Sketch the waveforms of the sinusoidal input voltage, the diode voltage, and the output voltage for the diode clamp, Fig. 4-37, with a positive bias voltage V. Repeat for $-V$.

4-17 Analyze the peak-to-peak voltmeter circuit, Fig. 4-46, by sketching

FIGURE 4-46
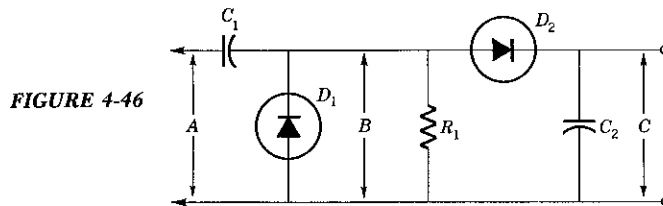
the voltage waveforms at points A, B, and C, assuming the input voltage A is sinusoidal. Note that the circuit is a diode clamp followed by a peak rectifier.

4-18 Can you suggest a power-supply use for the circuit of Exercise 4-17? What advantage does it have over the one discussed in the text? Are there any disadvantages? (*Hint:* Consider the voltages across the diodes and capacitors in both circuits.)

4-19 Determine the expression corresponding to Eq. (4-35) for the rectifier equation (4-13). *Ans.:* $(e/kT)v + \frac{1}{2}(e/kT)^2 v^2$

4-20 Select suitable values of R and C for the diode second-detector circuit, Fig. 4-44, if f_1 is equal to 1 Mc and f_2 is 1000 cps. Note that the value of R should be reasonably large to minimize loading the resonant circuit. *Ans.:* $10^5\ \Omega$, 100 pf