8 FEEDBACK AMPLIFIERS

The performance of transistor and vacuum-tube amplifiers is enhanced in many respects by returning a fraction of the output signal to the input terminals. This process is called feedback. The feedback signal may either augment the input signal or tend to cancel it. The latter, called negative feedback, is the primary concern of this chapter. Improved frequency-response characteristics and reduced waveform distortion are attained with negative feedback. In addition, amplifier performance is much less dependent upon changes in tube or transistor parameters caused by aging or temperature effects.

NEGATIVE FEEDBACK

The circuit alterations of a standard amplifier which return a portion of the output signal to the input may be analyzed by the techniques developed in previous chapters. It is more illustrative, however, to isolate the feedback portion of the circuit and treat it separately. Consider the feedback amplifier, Fig. 8-1, comprising

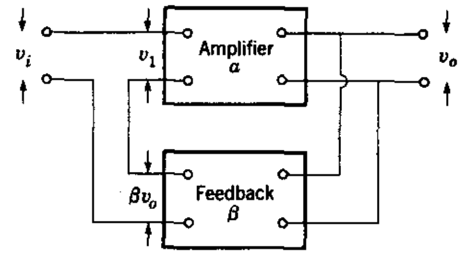


FIGURE 8-1 Block diagram of feedback amplifier.

a standard amplifier with a gain a and a feedback network indicated by the box marked β. According to this circuit, a voltage βv_o is added to the input signal v_i so that the total input signal to the amplifier is

$$v_1 = v_i + \beta v_o \tag{8-1}$$

Introducing the fact that $v_o = av_1$,

$$v_o = av_i + a\beta v_o \tag{8-2}$$

so that

$$v_o = \frac{a}{1-\beta a} v_i \tag{8-3}$$

According to Eq. (8-3) the overall gain of the amplifier with feedback,

$$a' = \frac{a}{1-\beta a} \tag{8-4}$$

may be greater or smaller than that of the amplifier alone, depending upon the algebraic sign of βa.

The condition of greatest interest in this chapter is *negative feedback*, when βa is a negative quantity. In this case, Eq. (8-4) shows that the overall gain is reduced because, in effect, the feedback voltage cancels a portion of the input signal. If the amplifier gain is very large, $a \gg 1$, the overall gain reduces to

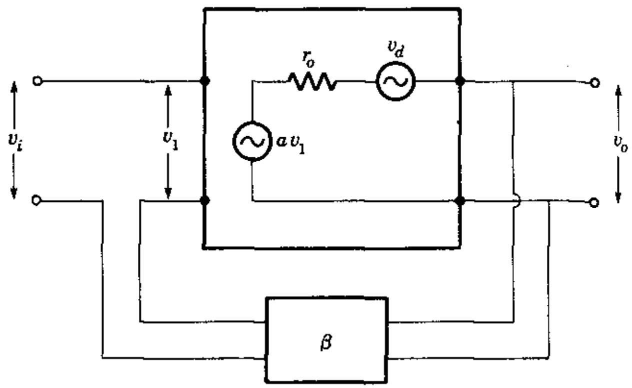
$$a' = \frac{1}{\beta} \tag{8-5}$$

This shows that the gain depends only upon the properties of the feedback circuit. Most often, the feedback network is a simple combination of resistors and/or capacitors. Therefore, the gain is independent of variations in tube or transistor parameters in the amplifier. In addition to this desirable improvement in stability, the gain may be calculated from circuit values of the feedback network alone. Thus it is not necessary to know, for example, the h parameters of all transistors in the circuit.

Negative feedback is also effective in reducing waveform distortion in amplifiers. Waveform distortion results from a nonlinear transfer characteristic, which may be interpreted as a smaller gain where the slope of the transfer characteristic is less, and as a larger gain where the slope of the transfer characteristic is greater. According to Eq. (8-5), however, the gain of an amplifier with feedback is essentially independent of variations caused by nonlinearities in tube or transistor characteristics. Therefore, the transfer characteristic is more linear and distortion is reduced.

A quantitative measure of the reduction in distortion achieved with feedback is obtained by assuming that distortion signals can be represented by a voltage generator in the amplifier, Fig. 8-2.

FIGURE 8-2



With this approximation the amplified signal av_1 is distortionless. As usual, r_o represents the internal impedance of the circuit as viewed from the output terminals of the amplifier. Under this condition, the output voltage

$$v_o = av_1 + v_d \tag{8-6}$$

includes the distortion voltages v_d. Both the amplified signal and distortion voltages are fed back, so that the input to the amplifier is

$$v_1 = v_i + \beta(v_o + v_d) \tag{8-7}$$

Introducing Eq. (8-7) into (8-6) and solving for output signal,

$$v_o = \frac{a}{1 - a\beta} v_i + \frac{1 + a\beta}{1 - a\beta} v_d \tag{8-8}$$

The ratio of undistorted output signal to distortion voltages in the case of no feedback (where $v_1 = v_i$) is, from Eq. (8-6),

$$\left(\frac{S}{D}\right)_a = \frac{av_i}{v_d} \tag{8-9}$$

Including feedback, this ratio is, from Eq. (8-8),

$$\left(\frac{S}{D}\right)_f = \frac{a}{1 + a\beta} \frac{v_i}{v_d} \tag{8-10}$$

Comparing Eqs. (8-9) and (8-10),

$$\left(\frac{S}{D}\right)_f = \frac{1}{1 + a\beta} \left(\frac{S}{D}\right)_a \tag{8-11}$$

According to Eq. (8-11), feedback reduces the relative importance of distortion signals in the output by the factor $1 + a\beta$. Since $a\beta$ is a large number, this improvement is significant. In effect, feedback results in an amplified distortion signal that cancels the original distortion voltages to a large extent. This result is particularly useful in power amplifiers where transistors or tubes are used over the full range of their characteristics.

The benefits of negative feedback are obtained at the expense of reduced gain, according to Eq. (8-4). This is not a serious loss, however, because large amplifications are easily obtained in transistor and vacuum-tube circuits. In practice, the maximum usable gain is limited by random noise effects anyway and it is not difficult to achieve the maximum amplification that can be effectively used, even with feedback included.

The gain a and the feedback factor β are inherently complex numbers. That is, phase shifts associated with coupling capacitors and stray capacitance effects are present, particularly at frequencies outside of the passband of the amplifier. These phase shifts cause a departure from the 180° phase shift necessary for the feedback voltage to interfere destructively with the input signal. It can happen that the overall phase shift becomes zero (i.e., 360°) so that $a\beta$ is positive and the feedback voltage augments the input signal. This is called *positive feedback* and leads to serious instability effects in feedback amplifiers.

Note, particularly, that if $a\beta = +1$ the output voltage, Eq. (8-3), becomes very large, even in the absence of an input signal. This means that positive feedback may cause an amplifier to oscillate, as previously discussed in connection with the feedback effects of the grid-plate capacitance in triodes and the collector capacitance in transistors. Oscillation is deleterious in amplifiers since the output voltage is not a replica of the input signal. Positive feedback is, however, a useful condition in oscillator circuits, as discussed in the next chapter.

8-1 Voltage feedback

In the foregoing, a portion of the output voltage is returned to the input terminals, a condition referred to as *voltage feedback.* The two-stage transistor amplifier, Fig. 8-3, uses voltage feedback intro-

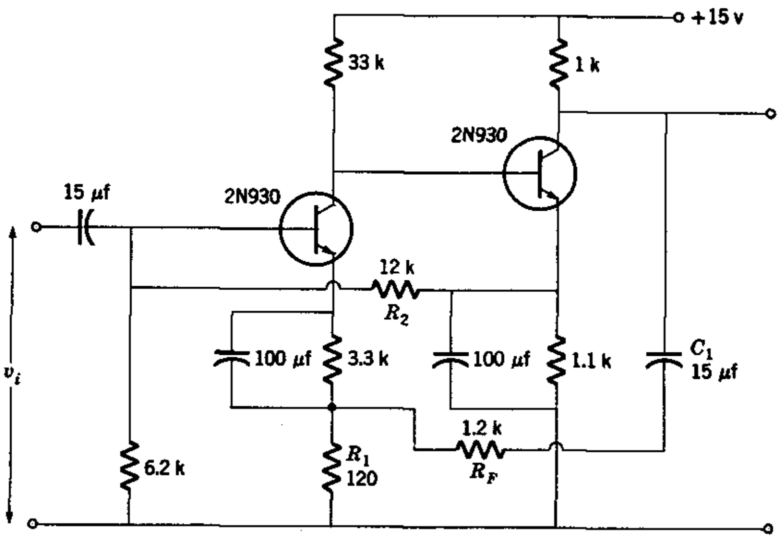


FIGURE 8-3 Two-stage feedback amplifier. Feedback is determined by ratio R_1/R_F.

duced by the resistor R_F connecting the output terminal with the input circuit of the first stage. The feedback voltage is introduced into the emitter circuit of the first stage because the output voltage of a two-stage amplifier is in phase with the input signal. The feedback factor is the result of the resistor divider made up of R_F and R_1, so that

$$\beta = \frac{R_1}{R_F + R_1} \cong \frac{R_1}{R_F} \tag{8-12}$$

Capacitor C_1 isolates the dc components of the two stages. Also, note that dc interstage coupling is used in this amplifier, which is quite independent of negative-feedback considerations. Resistor R_2 sets the bias on the first stage and also provides a dc feedback effect which helps stabilize the dc-coupled amplifier against slow drifts. It is not part of the ac feedback circuit.

The effect of the feedback ratio on the frequency-response characteristic is illustrated in Fig. 8-4. Without feedback (resistor R_F removed) the midband gain of the amplifier is 1000 and the high-frequency cutoff is 100 kc. As feedback is increased by using smaller values of R_F the midband gain is reduced and the frequency response is extended to lower and higher frequencies. With a 1200-Ω feedback resistor the gain is 10 and the upper frequency cutoff is extended to 15 Mc. Under this condition $\beta = 120/1200 = 0.1$, so $a\beta = 100$. According to Eq. (8-5) the midband gain is $1/\beta$, in agreement with the experimental response

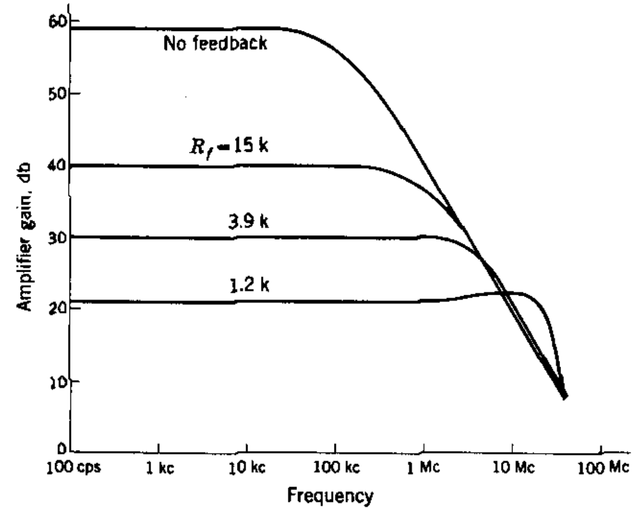


FIGURE 8-4 Effect of feedback on response characteristic of amplifier in Fig. 8-3.

curves, Fig. 8-4. Thus, a very stable wide-band amplifier is possible through the use of feedback.

Note that in Fig. 8-4 the gain increases at the extreme of the response curve when a large feedback ratio is used. This is a result of phase shift in the amplifier at these frequencies. In effect, the negative feedback is reduced since the feedback voltage is no longer exactly 180° out of phase with the input signal. The phase characteristic of feedback amplifiers at frequencies outside of the passband is very important. Such irregularities in the response curve should be small, particularly when large feedback ratios are employed. It is possible for the phase shifts to become great enough to cause positive feedback at the frequency extremes, a condition which must be avoided if the amplifier is to remain stable. This matter is considered further in a later section.

If the feedback circuit is frequency-selective, it is possible to develop a specific frequency-response characteristic for the amplifier. Consider, for example, an amplifier having a bridged-T feedback network in one stage, Fig. 8-5. In this circuit cathode followers are used to provide a high input impedance and a low output impedance so that the frequency-selective stage is isolated from disturbing influences of input and output loads. A difference amplifier is used in the first two stages to simplify coupling between stages in the presence of the feedback network.

Since the bridged-T filter is connected from plate to grid, the feedback voltage is 180° out of phase with the input signal, as required for negative feedback. According to the response curve of the bridged-T filter, Exercise 3-16, the feedback voltage is a minimum at the characteristic frequency of the filter. Accordingly, the

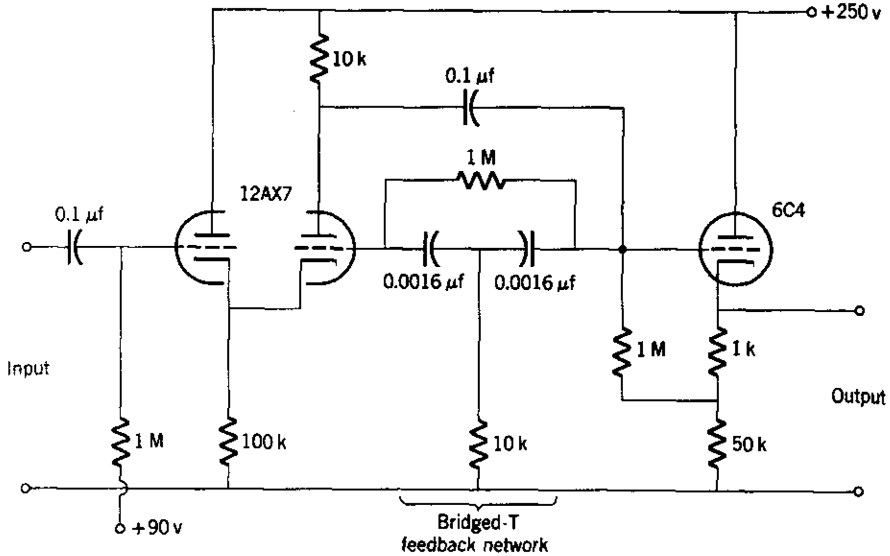


FIGURE 8-5 *Tuned amplifier using a frequency-selective feedback network consisting of bridged-T filter.*

amplifier gain is a maximum at this frequency and the result is a tuned amplifier. The feedback ratio β is a function of frequency because of the characteristics of the bridged-T filter, and the feedback effect considerably enhances the selectivity of the filter, as illustrated in Fig. 8-6.

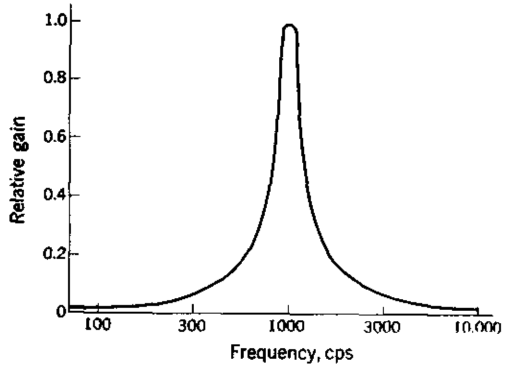


FIGURE 8-6 *Frequency-response characteristic of the bridged-T amplifier in Fig. 8-5.*

Feedback tuned amplifiers are commonly used at audio frequencies where high-Q inductances are difficult to construct because large values of inductance are required. Furthermore, it is a simple matter to tune the amplifier by making the resistors variable. This approach is also useful in integrated circuits where inductances are difficult to produce. Other frequency-selective feedback networks, such as the twin-T or Wien bridge, are also used in tuned feedback amplifiers.

Negative feedback alters the input and output impedance of an amplifier. To see how this comes about, replace the amplifier by its Thévenin equivalent, Fig. 8-7. The output impedance with feed-

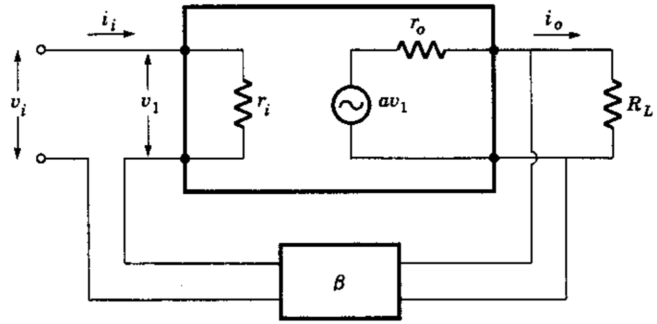


FIGURE 8-7 *Circuit to determine effective input and output impedance of amplifier with voltage feedback.*

back is determined from the ratio of the open-circuit output voltage to the short-circuit output current. The open-circuit output voltage is, from Eq. (8-3),

$$(v_o)_{oc} = \frac{av_i}{1 - a\beta} \tag{8-13}$$

The short-circuit output current is found by making $R_L = 0$ in Fig. 8-7,

$$(i_o)_{sc} = \frac{av_1}{r_o} = \frac{a(v_i + \beta v_o)}{r_o} = \frac{av_i}{r_o} \tag{8-14}$$

where the fact that $v_o = 0$ has been used. Therefore, from Eqs. (8-13) and (8-14),

$$R_o = \frac{(v_o)_{oc}}{(i_o)_{sc}} = \frac{r_o}{1 - a\beta} \tag{8-15}$$

According to this result, the effective output impedance is reduced by feedback by the same factor as the reduction in gain.

The effective input impedance is found from the ratio of the input voltage to the input current. Applying Kirchhoff's rule to the input circuit of Fig. 8-7,

$$v_i + \beta v_o - i_i r_i = 0 \tag{8-16}$$

The output voltage with no load may be written as

$$v_o = av_1 = ar_i i_i \tag{8-17}$$

Inserting Eq. (8-17) into Eq. (8-16) and solving for the ratio v_i/i_i yields

$$R_i = \frac{v_i}{i_i} = (1 - a\beta) r_i \tag{8-18}$$

which shows that the effective input impedance is increased by the same factor as the decrease in output impedance.

Both changes, increased input impedance and reduced output impedance, are desirable improvements in amplifiers, as discussed previously. Feedback is often introduced to achieve these benefits alone. Actually, the cathode follower and emitter follower, in which the input impedance is large and the output impedance is small, may be considered to be feedback amplifiers (see Exercise 8-3). In this case the full output voltage across the emitter or cathode resistor is applied to the input and the feedback ratio is -1.

8-2 Current feedback

It is possible to develop a feedback signal proportional to the output current rather than to the output voltage, and this is called *current feedback*. Consider the current-feedback circuit, Fig. 8-8,

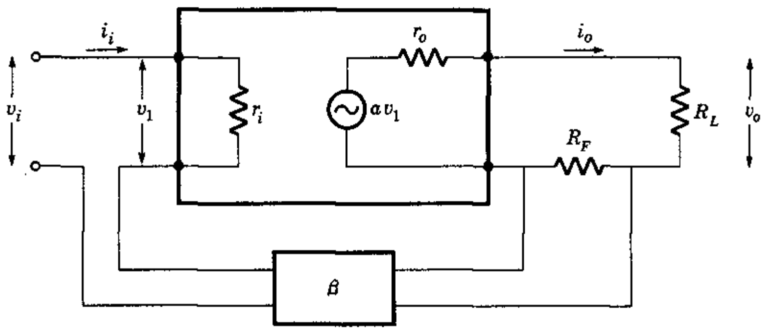


FIGURE 8-8 *Circuit to determine effective input and output impedance of amplifier with current feedback.*

in which the feedback signal results from the voltage drop across a resistor R_F in series with the load. Writing Kirchhoff's voltage equation around the output circuit,

$$-av_1 + i_o(r_o + R_F) + v_o = 0 \tag{8-19}$$

and around the input circuit,

$$-v_1 + v_i + \beta i_o R_F = 0 \tag{8-20}$$

Solving Eq. (8-20) for v_1 and inserting this into Eq. (8-19) results in

$$v_o = av_i - i_o[r_o + (1 - a\beta)R_F] \tag{8-21}$$

Interpreting Eq. (8-21) in terms of a Thévenin equivalent circuit, the open-circuit voltage is av_i, which means, in effect, that the voltage gain of the amplifier is not affected by current feedback. On the other hand, the output impedance is increased, since, from Eq. (8-21),

$$R_o = r_o + (1 - a\beta)R_F \tag{8-22}$$

Additionally, the input impedance is also increased, as can be shown by considering the input circuit as in the case of voltage feedback.

The output current is found from Eq. (8-21) by introducing $v_o = i_o R_L$ and solving for i_o,

$$i_o = \frac{av_i}{R_L + r_o + (1 - a\beta)R_F} \cong -\frac{v_i}{\beta R_F} \tag{8-23}$$

where the approximation applies when the gain is large. Note that the output current is independent of the amplifier gain and transistor parameters. This equation is analogous to Eq. (8-5) for the case of voltage feedback. It indicates that current feedback minimizes distortion in the output current, rather than the output voltage.

Omitting the emitter or cathode bypass capacitor in power amplifiers is a commonly used form of current feedback. The load current in the emitter or cathode bias resistor introduces negative current feedback into the input circuit, with a consequent reduction in distortion (see Exercise 8-5). In this connection, the fact that bypass capacitors cannot be used in push-pull class-B power amplifiers, as mentioned in the previous chapter, is actually advantageous, except for the concomitant loss of gain.

Both voltage and current feedback can be employed in the same amplifier, Fig. 8-9. Voltage relations in the output circuit of Fig. 8-9 are identical to Eq. (8-19), so that

$$v_o = av_1 - i_o(r_o + R_F) \tag{8-24}$$

Similarly, Kirchhoff's voltage equation for the input circuit yields

$$v_1 = v_i + \beta_v v_o + \beta_i i_o R_F \tag{8-25}$$

Substituting Eq. (8-25) into Eq. (8-24) and solving for the output voltage,

$$v_o = \frac{a}{1 - a\beta_v} v_i - i_o \frac{r_o + (1 - a\beta_i)R_F}{1 - a\beta_v} \tag{8-26}$$

Note that this expression incorporates both Eq. (8-3) for voltage feedback alone ($\beta_i = 0$) and also Eq. (8-21) for current feedback only ($\beta_v = 0$).

Combined current and voltage feedback permits a unique adjustment of the output impedance of the amplifier. The output impedance is given by the coefficient of i_o in Eq. (8-26) and can be made to vanish if

$$r_o + (1 - a\beta_i)R_F = 0$$

or

$$a\beta_i = 1 + \frac{r_o}{R_F} \tag{8-27}$$

If Eq. (8-27) is satisfied, the amplifier has zero internal impedance and therefore can deliver maximum power to any load impedance.

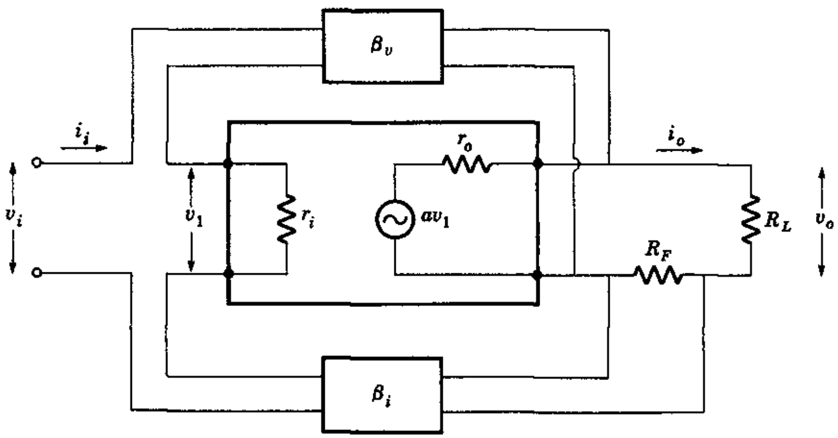


FIGURE 8-9 Combined voltage and current feedback.

Note that this requires positive current feedback, according to Eq. (8-27). Positive current feedback is permissible in this case because of the stabilizing effect of negative voltage feedback which is also present.

A practical amplifier employing both current and voltage feedback is illustrated in Fig. 8-10. Circuit-voltage feedback is taken from the secondary of the output transformer and applied to the input stage. The feedback ratio is determined by resistors R_f and R_k. Current feedback is provided by unbypassed cathode resistors in the output stage. Both current and voltage feedback effects in this circuit are negative.

Capacitor C_f shunting the feedback resistor R_f alters the feedback ratio and phase shift in order to eliminate positive-feedback effects at very high frequencies. Similarly, capacitor C_1 purposely reduces amplifier gain at frequencies beyond the normal bandpass of the amplifier (approximately 10 cps to 50 kc) to improve stability. Note that the pentode voltage amplifier is dc-coupled to the triode phase-inverter stage. This eliminates phase shift resulting from an interstage coupling network between these tubes, which also contributes to amplifier stability at low and high frequencies.

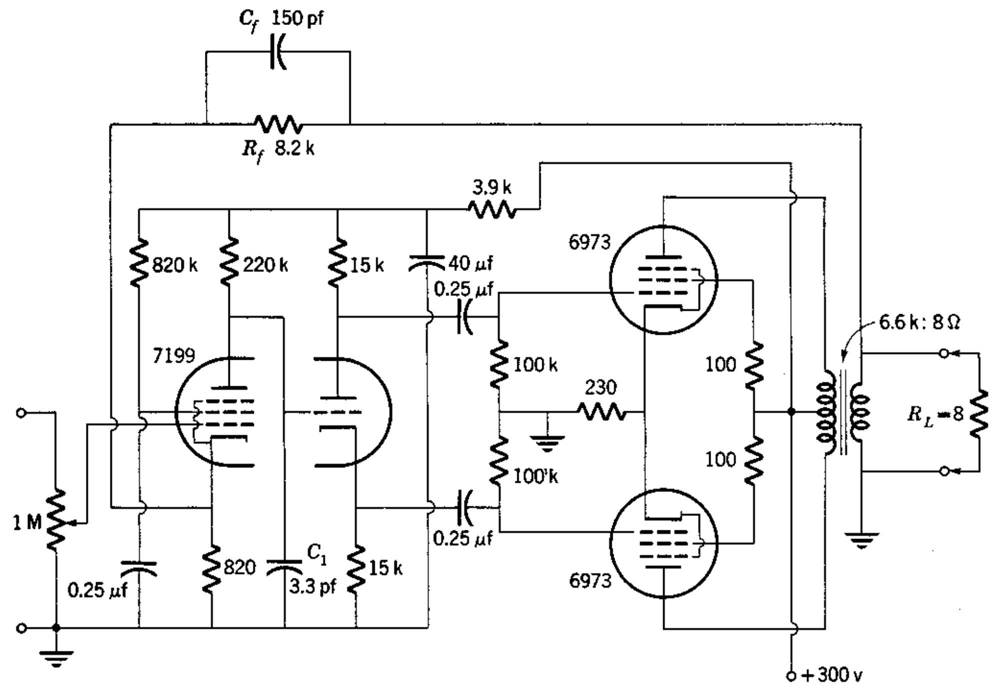


FIGURE 8-10 Practical vacuum-tube amplifier using combined current and voltage feedback.

8-3 Stability

Great care is taken in the design of feedback amplifiers to make them stable in the face of phase shifts leading to positive feedback at the frequency extremes of the amplifier bandpass. Fortunately there exists a rather straightforward criterion which can be applied to establish the stability of feedback amplifiers. According to Eq. (8-4), instability occurs if $a\beta$ is positive and equal to unity. It follows that if the absolute value of $a\beta$ drops below unity before the phase shift reaches $-360°$, the amplifier will be stable. It is convenient to plot the magnitude and phase of $a\beta$ as a function of frequency, Fig. 8-11, in order to establish the relative positions at which $a\beta = 1$ and the phase shift equals zero. By this standard, the amplifier with characteristics represented by Fig. 8-11 is stable.

In order to apply this criterion for stability, the amplifier must be stable in the absence of feedback. Also, there exist certain situations in which a feedback amplifier can be conditionally stable, that is, stable under some conditions of, say, loading, and not under others. For most practical circuit analysis, however, the stated condition is sufficient.

It is useful to consider the $a\beta$ characteristics of several amplifier types in the light of this stability requirement. Since in most cases the feedback ratio is independent of frequency, it is sufficient to examine the amplitude and phase characteristics of the amplifier gain separately. In the case of a single *RC*-coupled amplifier

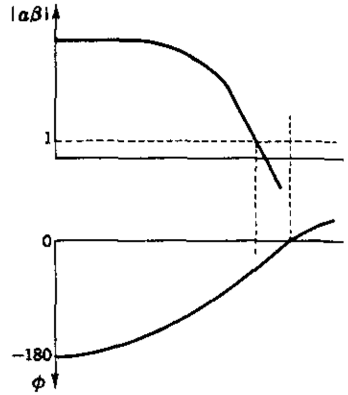


FIGURE 8-11 *Feedback amplitude and phase characteristics of stable amplifier.*

stage, Fig. 8-12*a* (compare Fig. 7-12), the phase shift never exceeds 90°. This means the phase of $a\beta$ is always less than $90 + 180 = 270°$. Since this is smaller than 360°, a single-stage feedback amplifier cannot be unstable.

Two identical *RC*-coupled stages in cascade have the ideal characteristic sketched in Fig. 8-12*b*. The phase shift reaches 180° at very low and very high frequencies, but the gain is very small at these extremes. Therefore, it is unlikely that $a\beta$ can be equal to unity, where the total phase shift is $180 + 180 = 360°$. Practical amplifiers always have stray capacitive effects that can introduce additional phase shift at high frequencies, however. It is possible that the additional phase shift caused by stray capacitances may result in unstable conditions at high frequencies if the gain is large.

The characteristics of three identical *RC*-coupled stages, Fig. 8-12*c*, are such that the extreme phase shift approaches 270°. Therefore, 180° phase shift is encountered at both low and high frequencies, and a three-stage feedback amplifier is certain to be unstable if the midband gain is large enough. It can be shown that the maximum value of $a\beta$ permitted at midband in this case is equal to 8, although even this value puts the amplifier on the verge of oscillation.

Many possibilities exist to remove this unfavorable difficulty other than minimizing the gain. For example, the bandwidth of two stages in a three-stage amplifier can be made very much greater than that of the remaining stage. This means that the phase characteristics are determined primarily by the single stage which is unconditionally stable. Alternatively, dc coupling can be used between two stages (as in Fig. 8-10) to eliminate the phase shift associated with one interstage coupling network. It is common practice to alter the feedback and gain characteristics, particularly at high frequencies, by including small capacitances to change the phase and gain characteristics in a way that improves stability. Usually such changes must be made empirically on the actual amplifier because of the unavoidable and unknown stray wiring capacitances.

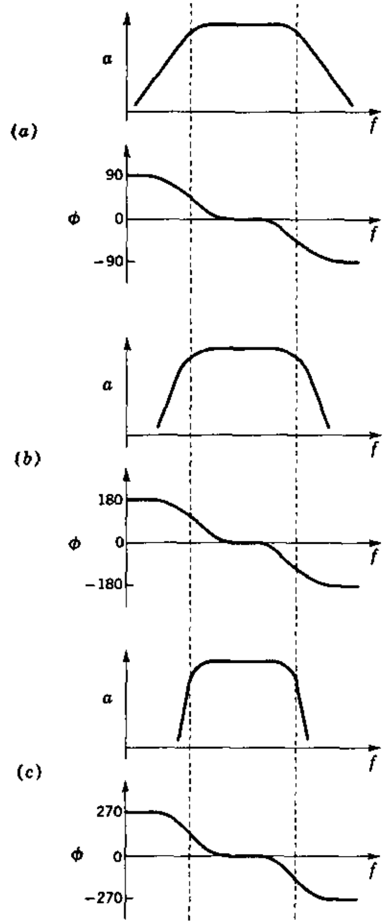


FIGURE 8-12 *Gain and phase characteristics of (a) one-stage amplifier, (b) two-stage amplifier, and (c) three-stage amplifier.*

OPERATIONAL AMPLIFIERS

8-4 Operational feedback

High-gain, dc-coupled feedback amplifiers can perform a number of operations such as multiplication, addition, and integration on an input signal. They are widely used in measurement and control applications as well as in electronic analog computers. Commercially available units have come to be known as *operational amplifiers.* In addition to dc coupling and high gain, operational amplifiers exhibit high input impedance and low output impedance and conventionally introduce a 180° phase shift between the input signal and the output voltage.

To illustrate the features of an operational amplifier, consider the feedback circuit, Fig. 8-13, in which negative voltage feedback is produced by the resistor R_f connected between the output and

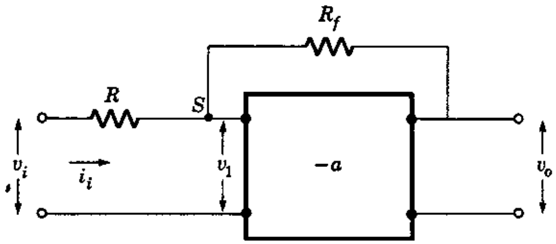


FIGURE 8-13 Block diagram of operational amplifier.

input. Note that the feedback is negative because of the phase inversion in the amplifier. The feedback ratio in *operational feedback* can vary from unity for a high-impedance source to $R/(R + R_f)$ for a low-impedance source since the feedback voltage is effectively connected in parallel with the input signal source. It is convenient to analyze the operational feedback circuit by applying Kirchhoff's current rule to the branch point S. Since the amplifier input impedance is large, the current in this branch is negligible, which means the current in R equals the current in R_f, or

$$\frac{v_i - v_1}{R} = \frac{v_1 - v_o}{R_f} \tag{8-28}$$

Introducing $v_1 = -v_o/a$ and rearranging,

$$v_o\left(1 + \frac{1}{a} + \frac{R_f}{aR}\right) = -\frac{R_f}{R}v_i \tag{8-29}$$

Since the gain is very large

$$v_o = -\frac{R_f}{R}v_i \tag{8-30}$$

which means that the output voltage is just the input signal multiplied by the constant factor $-R_f/R$. If precision resistors are used for R_f and R, the accuracy of this multiplication operation is quite good.

The branch point S has a special significance in operational amplifiers. This may be illustrated by determining the effective impedance between S and ground, which is given by the ratio of v_1 to the input current,

$$Z_s = \frac{v_1}{i_i} = \frac{v_1 R_f}{v_1 - v_o} = \frac{R_f}{1 - v_o/v_1} = \frac{R_f}{1 + a} \tag{8-31}$$

where the right side of Eq. (8-28) has been inserted for the input current. According to Eq. (8-31) the impedance of S to ground is very low if the gain is large. Typical values are $R_f = 100{,}000\ \Omega$ and $a = 10^4$, so that the impedance is 10 Ω. The low impedance results from the negative feedback voltage, which cancels the input signal at S and tends to keep the branch point at ground potential. For this reason the point S is called a *virtual ground.* Although S is kept at ground potential by feedback action, no current to ground exists at this point.

The virtual ground at S shows immediately that the impedance viewed from the input terminals is equal to R. The output impedance is very low because an emitter-follower stage is used and this impedance is further reduced by feedback. For example, the output impedance of an emitter follower is typically about 200 Ω; if the feedback ratio, $R/(R + R_f)$ (for simplicity consider the input to be shorted), is about unity and the gain is 10^4, the effective output impedance is only 0.02 Ω.

Operational amplifiers are capable of responding to signal frequencies ranging from approximately 100 kc down to dc. Conventional chopper amplifiers are not used because the upper frequency limit of a chopper amplifier is only about 1/4 of the chopping frequency. Accordingly, operational amplifiers are dc-coupled circuits with special design features to minimize slow drifts. Even minor drifts are significant in operational-amplifier applications because an output signal exists in the absence of an input signal.

The circuit of a typical transistorized operational amplifier, Fig. 8-14, includes three difference-amplifier stages together with an emitter-follower output stage. The inherently balanced circuitry of difference amplifiers minimizes drifting, as discussed in the previous chapter. The input stage is particularly significant in this connection; note that each base-bias resistor is returned to the collector of the opposite transistor in order to further stabilize the circuit. In this way any changes in one transistor are compensated by a corresponding shift of the bias on the other transistor. The input stage also employs a transistor in the common emitter lead to improve the ac balance of the stage. The reason for this is that a large emitter resistor improves balance [refer to Eq. (5-45) pertaining to a vacuum-tube difference amplifier], but a large resistance also means that the voltage drop is excessive. Introducing a transistor at this point means that the effective ac resistance is essentially equal to the collector resistance of the transistor. The dc potential drop is, however, only a few volts. In effect, transistor Q_3 is the emitter resistor for the difference amplifier comprising Q_1 and Q_2. Because of the large effective resistance of Q_3, the circuit is very well balanced.

The difference-amplifier input stage also presents two input terminals. One, called the *inverting,* or negative, input, is the normal input terminal and produces an output signal 180° out of phase with the input. The second, called the *noninverting,* or *positive,* input, results in an output signal in phase with signals applied to this terminal. Advantages of the noninverting input are discussed in a later section. The 100-Ω potentiometer in the

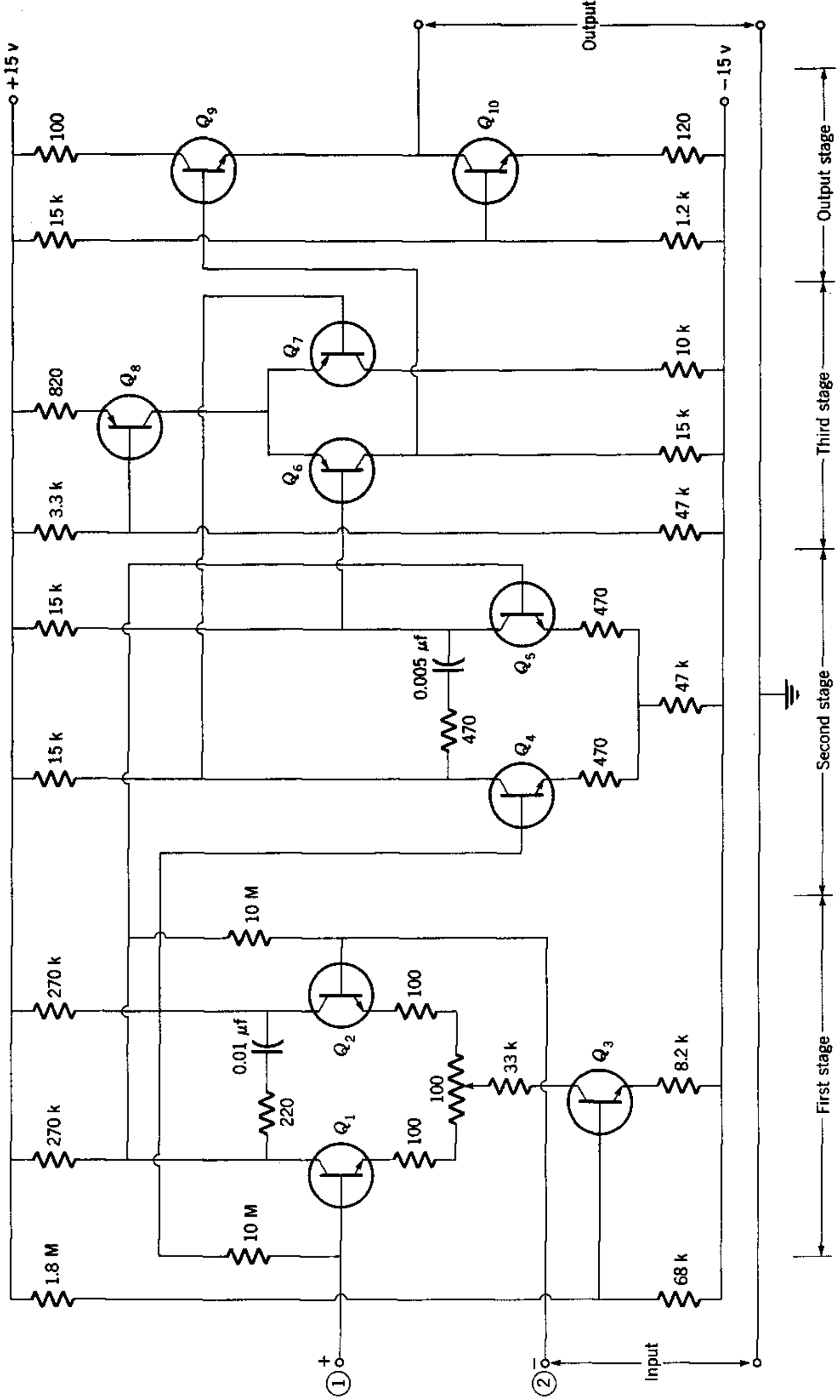


FIGURE 8-14 *Transistorized operational amplifier.* (*Courtesy Burr-Brown Research Corporation.*)

emitter circuit is a balance control which compensates for slight asymmetries in the overall amplifier and permits adjustment of the output voltage to zero in the absence of an input signal. An *RC* filter is connected between the collector of Q_1 and Q_2 in order to reduce the gain at high frequencies and thereby eliminate the possibility of high-frequency instabilities.

The second stage is a straightforward difference amplifier. Here again an *RC* filter connected between the collectors of Q_4 and Q_5 reduces the high-frequency gain. Transistors Q_6 and Q_7 form a third-stage difference amplifier, emitter-stabilized by Q_8 as in the input-stage circuit. Note that this stage uses *pnp* transistors, in contrast to *npn* devices in the first two stages. The reason for this is that the dc potential rise in a multiple-stage amplifier can be kept to reasonable limits since the *pnp* stage can be "inverted" insofar as the dc supply voltages are concerned. This is an example of the flexibility inherent in transistor circuitry resulting from the combination of *npn* and *pnp* transistors. The emitter-follower output stage, transistor Q_9, uses transistor Q_{10} as an emitter resistor. Here again the large effective ac collector resistance of Q_{10} means that the emitter-follower action of Q_9 is enhanced.

The overall gain of this amplifier is 30,000 and the bandwidth is from dc to 1 Mc. Nominally, the input impedance is 200,000 Ω, while the output impedance is only 100 Ω. The use of difference-amplifier circuitry reduces the drift rate to 50 μv per 24 hr. These characteristics result in a useful operational amplifier.

Even with very careful design and well-stabilized supply voltages, it is difficult to prevent drifts in dc amplifiers. Drift voltages at the amplifier output mean that operational feedback no longer keeps the branch point *S* at virtual ground and the error in the virtual ground is referred to as *offset*. Typically, offset voltages are of the order of 10 mv, which may be serious in critical applications. Improved performance is achieved by using a chopper amplifier to stabilize the virtual ground voltage, Fig. 8-15. Here, the chopper amplifier, which is drift-free because it is ac-coupled, measures the offset voltage at *S* and provides an amplified error signal to the positive input of the operational amplifier. This signal counteracts drifts in the operational amplifier and, in practice, offset can be reduced to about 0.1 mv.

Note that a capacitor is interposed between *S* and the operational amplifier so that offset is controlled only by the chopper amplifier. This prevents the operational amplifier itself from responding to dc signals, but the chopper amplifier does so and, in feeding its output signal to the operational amplifier, becomes part of the operational feedback circuit. In effect, the operational amplifier is now ac-coupled and handles the high-frequency signals, while the chopper amplifier handles the dc and very-low-frequency signals. The dc gain is very large since it is the product of the gains of both amplifiers. This means that the overall response curve is nonuniform in that the low-frequency

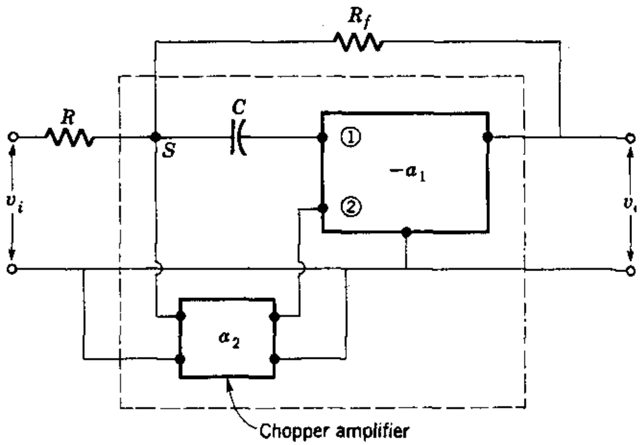


FIGURE 8-15 Chopper-stabilized operational amplifier.

gain is larger than the gain at high frequencies. According to Eq. (8-30) this is unimportant since by feedback action the gain does not appear in the expression for the output signal, so long as the gain remains large at all frequencies of interest.

The combination of an operational amplifier stabilized by a chopper amplifier is often itself referred to as an operational amplifier. Thus the dashed line in Fig. 8-15 encloses a chopper-stabilized operational amplifier. In less critical applications where chopper stabilization is unnecessary, the dc amplifier alone, such as Fig. 8-14, is the operational amplifier. Chopper amplifiers specifically designed for stabilizing operational amplifiers are also available as separate units. A typical transistor circuit and a vacuum-tube version are shown in Fig. 8-16*a* and *b*, respectively. Both circuits are rather conventional ac-coupled amplifiers and use a synchronous chopper to recover the dc signal. Note that unbypassed emitter and cathode resistors provide current feedback to improve phase characteristics so that the synchronously chopped output is in phase with the chopped input signal. In fact, dc coupling is used for convenience in the transistor version for the same reason. In this case, input and output capacitors are present, so that the amplifier is actually ac-coupled.

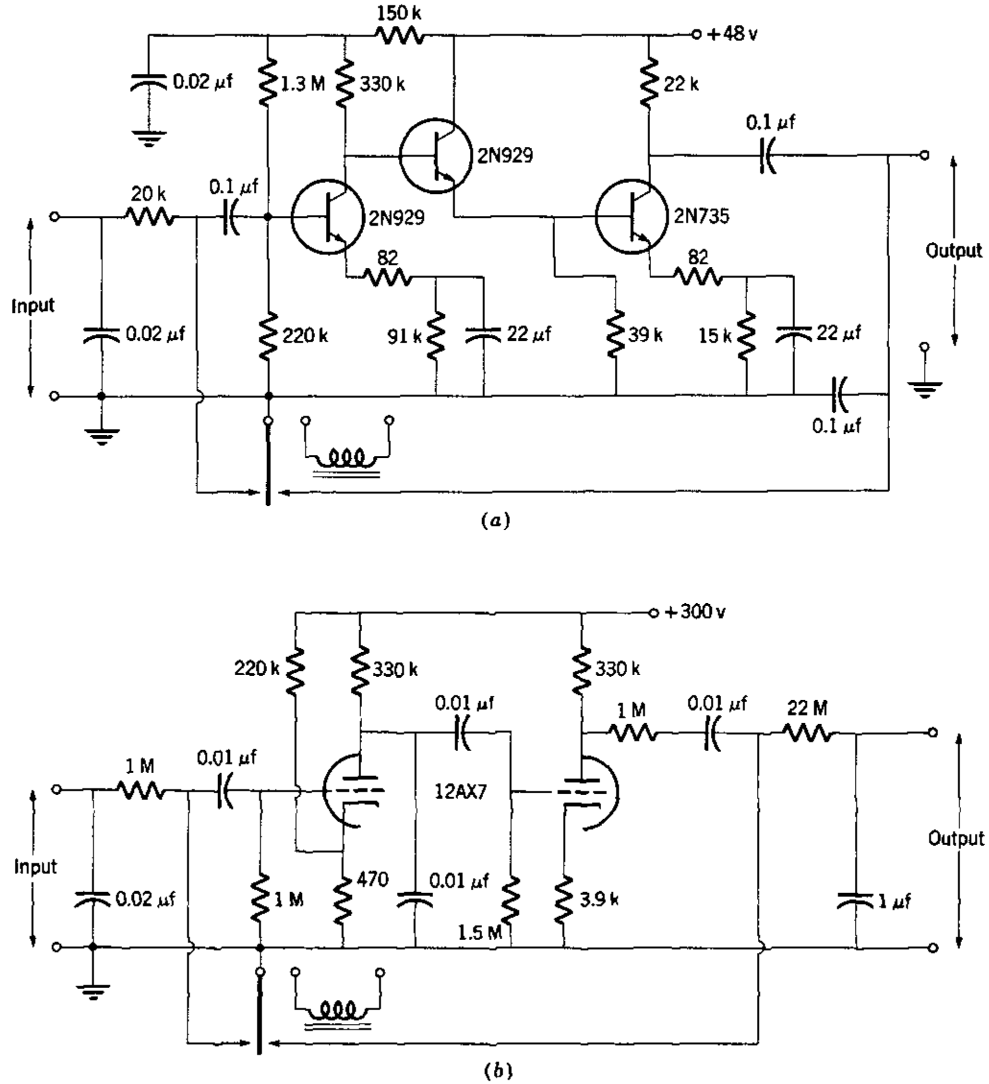


FIGURE 8-16 Typical (a) transistor and (b) vacuum-tube chopper amplifiers.

8-5 Mathematical operations

The operational feedback circuit, Fig. 8-13, multiplies the input signal by the constant $-R_f/R$. It is conventional to simplify the circuit diagram, as in Fig. 8-17, by not showing the ground terminals specifically. The operational amplifier is indicated by a triangle pointing toward the output terminal. It is understood that the simplified circuit diagram, Fig. 8-17, actually implies the corresponding complete circuit, Fig. 8-13. The operational amplifier in Fig. 8-17 may actually be the combination of a dc amplifier and a chopper amplifier, as explained previously.

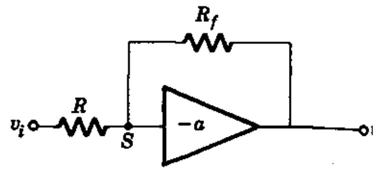


FIGURE 8-17 *Conventional circuit symbol for operational amplifier.*

If $R_f = R$ in the multiplier circuit, the signal is simply multiplied by -1. This is often useful in obtaining a signal multiplied by a positive constant wherein this operational amplifier precedes one which determines the multiplier. Multiplicative factors ranging from -0.1 to -10 are possible, and the precision is determined principally by the accuracy of the two resistors.

The operational amplifier can also be used to sum several signals by connecting individual resistors to the branch point S, Fig. 8-18.

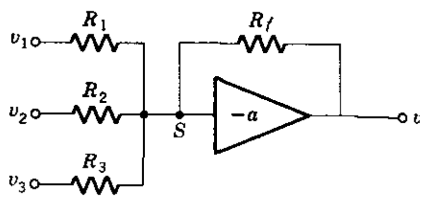


FIGURE 8-18 *Summing circuit using operational amplifier.*

In this circuit, the sum of the currents in R_1, R_2, and R_3 equals the current in R_f, since no current to ground exists at S. Furthermore, S is at ground potential, so that [compare Eq. (8-28)]

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} = -\frac{v_o}{R_f} \tag{8-32}$$

$$v_o = -R_f\left(\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}\right) \tag{8-33}$$

Many signals can be added in this way. There is no interaction between individual signal sources since S is a virtual ground. Because the addition appears to take place at this point, S is commonly called the *summing point*. Note that each signal may be multiplied by the same factor (or -1), if $R_1 = R_2 = R_3$, etc., or that individual factors may be selected, as convenient.

If the feedback resistor is replaced by a capacitor, Fig. 8-19, the circuit performs the operation of integration. Using the fact that S is at ground potential,

$$v_o = \frac{q}{C} = \frac{1}{C}\int_0^t i\,dt = -\frac{1}{RC}\int_0^t v_i\,dt \tag{8-34}$$

where Eq. (2-20) has been inserted for the voltage across a capacitor. According to Eq. (8-34) the output voltage is the integral of the input signal. Note that there is no restriction placed on the frequency components of the input signal, as in the case of the simple RC integrator discussed in Chap. 2. It is only necessary that the amplifier bandwidth be large enough to handle all signal fre-

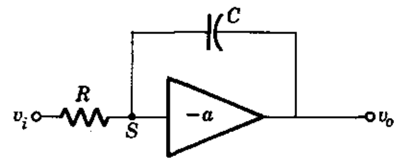


FIGURE 8-19 *Integrating circuit.*

quencies. The integral of the sum of several signals is obtained by introducing several input resistors, as in Fig. 8-18. Integrating circuits are particularly susceptible to offset voltages because, as Eq. (8-34) shows, a constant offset voltage results in an ever-increasing output signal. If this is not counteracted, the amplifier eventually is driven into saturation.

Interchanging R and C, Fig. 8-20, results in a differentiating cir-

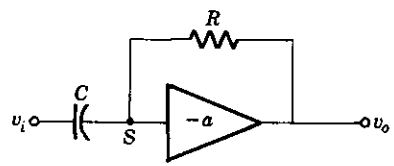


FIGURE 8-20 *Differentiating circuit.*

cuit which gives the time derivative of the input signal. Again equating currents at the summing point,

$$-\frac{v_o}{R} = \frac{dq}{dt} = \frac{d}{dt}Cv_i = C\frac{dv_i}{dt} \tag{8-35}$$

$$v_o = -RC\frac{dv_i}{dt} \tag{8-36}$$

The differentiator circuit is not affected by offset errors since, in effect, the capacitor makes the circuit ac-coupled. However, spurious noise pulses, which characteristically have a fast rate of change, may dominate in the output because of Eq. (8-36) and lead to a great deal of high-frequency noise.

8-6 Analog computers

The circuits of the preceding section can be assembled into *analog computers* to perform mathematical operations on voltages that are analogous to the numbers in a mathematical problem or equation. Although analog computers are not as accurate as their digital counterparts, they are much less expensive. The interconnection of operational amplifier circuits into a computer designed to solve the mathematical equation describing any physical system can be looked upon as simulating the actual system. In this way, the response of any system can be easily determined and the effect of changes in the system investigated by simply altering the values of electric components or potentials in the simulator.

Consider the analog-computer solution of the two simultaneous equations

$$\begin{aligned} a_1x + b_1y &= c_1 \\ a_2x + b_2y &= c_2 \end{aligned} \tag{8-37}$$

Solving for x and y,

$$\begin{aligned} x &= \frac{c_1}{a_1} - \frac{b_1}{a_1} y \\ y &= \frac{c_2}{b_2} - \frac{a_2}{b_2} x \end{aligned} \tag{8-38}$$

A summing amplifier solves each equation, Fig. 8-21. The output of the top amplifier is a voltage analogous to x and the output of the lower amplifier is a voltage analogous to y. A fraction b_1/a_1 of y is taken from the output of the lower amplifier by means of a variable potentiometer and summed with a voltage $-c_1/a_1$ derived from a potentiometer and a potential source. Applying Eq. (8-33) to the upper amplifier,

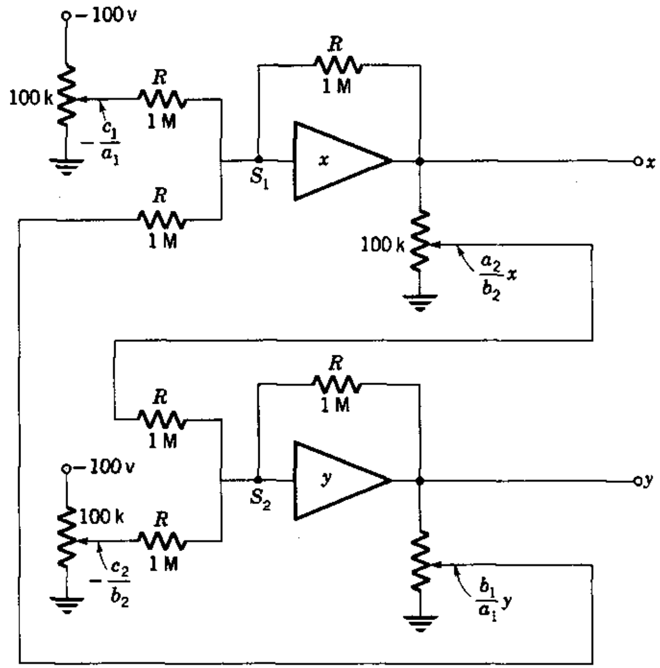


FIGURE 8-21 Analog-computer circuit to solve simultaneous linear equations in Eqs. (8-38).

$$x = -R\left(\frac{-c_1/a_1}{R} + \frac{b_1/a_1}{R} y\right) = \frac{c_1}{a_1} - \frac{b_1}{a_1} y \tag{8-39}$$

which is just the upper equation in (8-38). A similar expression applies to the lower equation in (8-38) and the corresponding amplifier.

This analog computer thus presents a continuous solution for x and y of Eqs. (8-37). Desired values of the coefficients are selected by adjusting the potentiometers in the circuit, and the solutions are determined by measuring the voltages x and y with, say, two VTVMs. If, for example, c_1 varies with time, a voltage signal of the proper waveform is supplied to the summing point. In this case the solutions x and y are time-varying and can be measured with an oscilloscope or a chart recorder.

Actually, the simple analog circuit devised here is not convenient since each potentiometer involves two constants. By means of a slightly more complicated circuit than Fig. 8-21, it is possible to make each potentiometer relate to only one constant. This is useful since the parameters can then be adjusted individually with greater ease.

Many types of differential equations can also be solved by analog computers. Consider the mechanical vibrations of a body of mass m on the end of a spring of force constant k in the presence of viscous damping described by the damping constant b and driven by an arbitrary force $F(t)$. The differential equation for the position x of the body is

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F(t) \tag{8-40}$$

Rearranging,

$$\frac{d^2x}{dt^2} = -\frac{b}{m}\frac{dx}{dt} - \frac{k}{m}x + \frac{1}{m}F(t) \tag{8-41}$$

The design of an analog computer to solve this equation begins by assuming a voltage signal corresponding to d^2x/dt^2 is available. This is integrated to yield $-dx/dt$, where for convenience the RC time constant in Eq. (8-34) is made equal to unity. Next, $-dx/dt$ is integrated again to obtain x. A fraction b/m of $-dx/dt$ is obtained from a voltage-divider potentiometer across the output of the first integrator; this is inverted and added to the fraction k/m of x from the output of the second integrator and to a voltage signal corresponding to $(1/m)F(t)$ (Fig. 8-22). The sum is equal to d^2x/dt^2, according to Eq. (8-41), and is returned to the input, where the second derivative signal was assumed to be originally. This computer, therefore, continuously solves the original differential equation (8-40) and voltages corresponding to x and, if desired, dx/dt can be measured at appropriate points in the circuit.

It is necessary to set voltages corresponding to dx/dt, and x for their initial values at the time when the solution begins, as in solving any differential equation. This is most effectively accomplished by opening switches s_1 and s_2 at $t = 0$. The voltages across the inte-

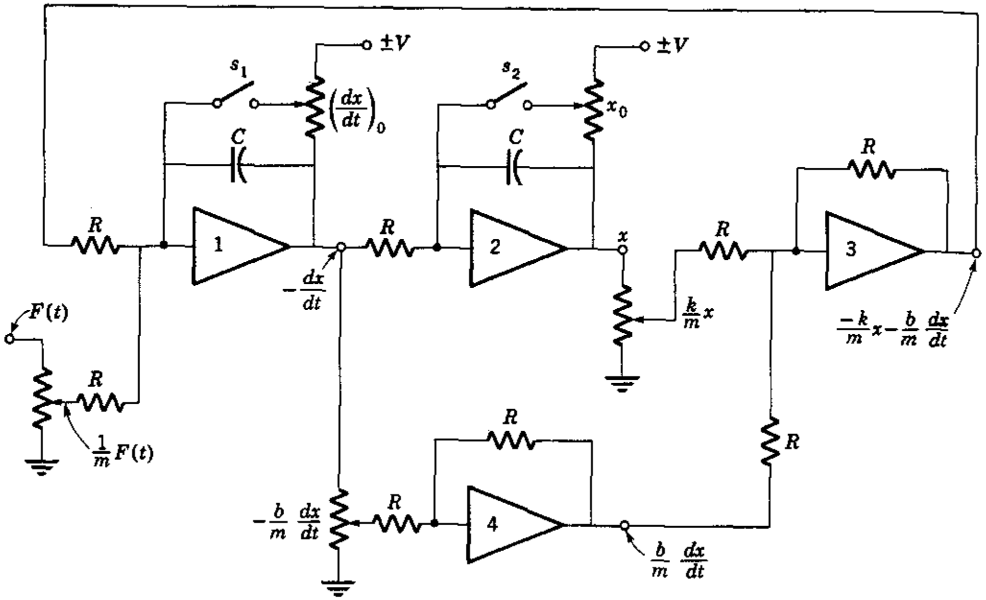


FIGURE 8-22 *Analog computer for vibrating-mass problem.*

grating capacitors represent the velocity and displacement at all times, as can be seen by the fact that the summing points are at ground potential. These switches must be opened simultaneously with the beginning of $F(t)$ and in practice this is most often done with electronic switches such as diodes.

Note that Eq. (8-40) has the same form as Eq. (3-28) for the current in an *RLC* circuit. This means that the analog computer in Fig. 8-22 also can be used to investigate, say, resonance effects in this simple circuit. Although the computer is much more complicated than the circuit it simulates, study of the circuit is facilitated by the ease with which circuit parameters can be altered. Furthermore, the computer can be rewired to simulate other circuits as the need arises.

8-7 Measurement and control

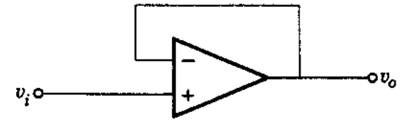
The operational amplifier is a useful tool in measurement and control of electrical quantities. For example, according to Eq. (8-30), the output voltage of an amplifier with operational feedback is

$$v_o = -R_f i_i \tag{8-42}$$

where i_i is the input current. Thus by eliminating the input resistor R in Fig. 8-17 the operational amplifier becomes a very sensitive ammeter. If R_f is 1 MΩ, Eq. (8-42) shows that the output voltage is 1 volt for each microampere of input current. Also the resistance introduced into the circuit is the impedance between S and ground, which may be of the order of 10 Ω, according to Eq. (8-31). Currents as low as 10^{-11} amp can be measured with very large values of R_f although care must be exercised to take into account stray leakage currents at this sensitivity.

A modified operational feedback circuit which uses both inputs, Fig. 8-23, is known as a *voltage follower* because the output voltage

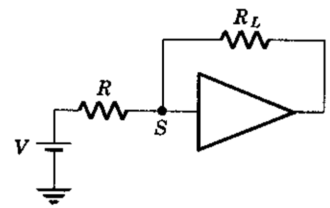
FIGURE 8-23 *Voltage follower.*



is equal to the input signal. Since the full output voltage is returned to the input, the feedback ratio β is -1 and the gain equals, from Eq. (8-4), $a/(1 + a) \cong 1$. The voltage follower has very high input impedance and very low output impedance, of the order of 100 MΩ and 0.1 Ω, respectively, for a typical vacuum-tube amplifier. Obviously, very effective isolation between the voltage source and output circuit is achieved.

Conventional operational feedback can control the current in an impedance (Fig. 8-24). According to Eq. (8-28) the current in the

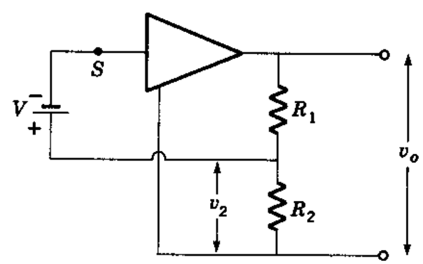
FIGURE 8-24 *Current regulator.*



arbitrary impedance R_L is equal to V/R. In this way the current in R_L can be kept constant independent of changes in R_L or can be adjusted by altering V or R.

The high-gain dc operational amplifier can be used in a conventional voltage feedback circuit, Fig. 8-25, where the ground con-

FIGURE 8-25 *Voltage stabilizer.*



nection is shown explicitly to enhance comparison with Fig. 8-1. Since the feedback ratio is $-R_2/(R_1+R_2)$, the output voltage is controlled by the amplifier to be, from Eq. (8-5),

$$v_o = -\frac{R_1+R_2}{R_2} V \tag{8-43}$$

Suppose V is a constant reference voltage, such as a standard battery or the breakdown voltage of a Zener diode. Then the output voltage is controlled at the value given by Eq. (8-43) independent of the current in a load connected to the output terminals. Practical circuits of this type are considered in greater detail in the next section.

Note also that the feedback voltage v_2 is

$$v_2 = \frac{R_2}{R_1+R_2} v_o = \frac{R_2}{R_1+R_2}\left(-\frac{R_1+R_2}{R_2} V\right) = -V \tag{8-44}$$

This means that the voltage across R_2 is controlled to be equal to the standard voltage, independent of the values of R_1 and R_2. Therefore, the voltage at any point in a complex network can be maintained at the reference potential regardless of the other circuit connections to R_2.

VOLTAGE REGULATORS

It is often desirable to stabilize the voltage output of a rectifier power supply to a greater extent than is possible using the simple Zener-diode or *VR*-tube circuits described in Chap. 4. Feedback amplifiers are widely used in this application. Control over the dc supply voltage is maintained in the face of changes in load and also in ac line voltage. Such *voltage regulators* are essential in power supplies for operational amplifiers to minimize drifts and in other similar critical circuits.

8-8 Series regulator

The output voltage of a power supply is conveniently controlled by introducing a power transistor in series with the rectifier-filter and the load, Fig. 8-26. This circuit is equivalent to that of Fig. 8-25 with the addition of the power transistor, which enables the regulator to control larger currents. Actually, the combination of the dc amplifier and the control transistor may be considered to be an operational amplifier equivalent to Fig. 8-25. The output voltage is stabilized at $V_o = -V(R_1+R_2)/R_2$, according to Eq. (8-43). In effect, the voltage-regulator circuit compares a fraction of the dc output voltage with the reference voltage V and adjusts the control transistor to maintain V_o constant.

The stabilizing effect of the regulator is determined by analyzing the control transistor separately from the dc amplifier. Variations in the supply voltage are assumed to arise from an ac generator v_i

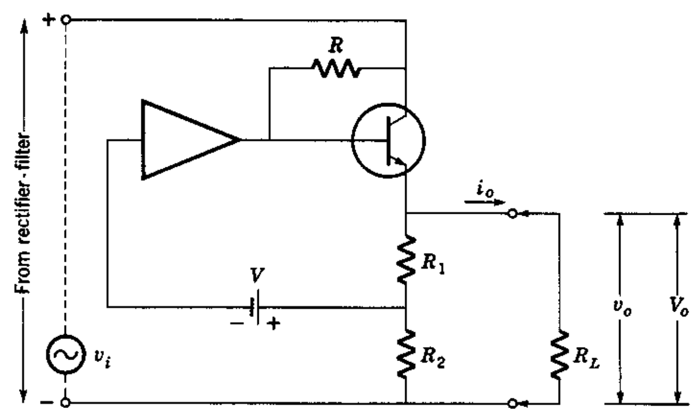


FIGURE 8-26 Series voltage regulator.

in Fig. 8-26 and result in a variation v_o in the output voltage. The control transistor is essentially an emitter-follower amplifier and the appropriate equivalent circuit (using Fig. 6-17*b*) is shown in Fig. 8-27. Since we want to determine the output voltage conditions,

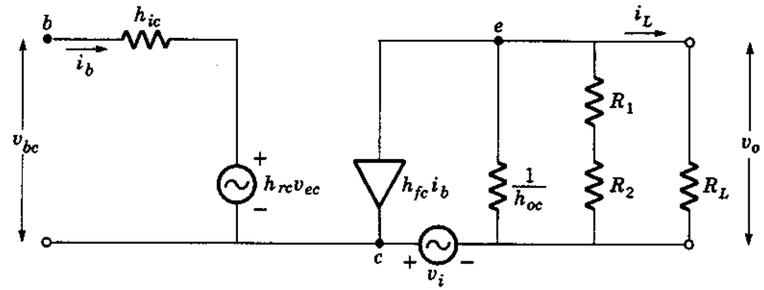


FIGURE 8-27 Equivalent circuit of control transistor in Fig. 8-26.

begin by writing an expression for the output current. Considering the currents at point e and neglecting the minor current drain in the feedback divider R_1R_2,

$$h_{fc}i_b + i_L + \frac{v_o - v_i}{1/h_{oc}} = 0 \tag{8-45}$$

where the third term is the current in $1/h_{oc}$. The base current is found by considering the voltages in the input circuit,

$$i_b = \frac{v_{bc} - h_{rc}v_{ec}}{h_{ic}} = -\frac{a\beta v_o + v_i + h_{rc}v_{ec}}{h_{ic}} \tag{8-46}$$

where the input voltage v_{bc} is v_i less the amplified feedback voltage

$-a\beta v_o$, as may be best seen from Fig. 8-26. Finally, v_{ec} is determined by writing Kirchhoff's rule around the output circuit,

$$v_{ec} - v_o + v_i = 0 \tag{8-47}$$

Equations (8-47) and (8-46) are substituted into Eq. (8-45),

$$i_L = v_i \left[h_{oc} + \frac{h_{fc}(1 - h_{rc})}{h_{ic}} \right] - v_o \left[h_{oc} - \frac{h_{fc}(h_{rc} + a)}{h_{ic}} \right]$$

Solving for the output voltage,

$$v_o = v_i \frac{h_{ic}h_{oc} + h_{fc}(1 - h_{rc})}{h_{ic}h_{oc} - h_{fc}(h_{rc} + a\beta)} - i_L \frac{h_{ic}}{h_{ic}h_{oc} - h_{fc}(h_{rc} + a\beta)} \tag{8-48}$$

This expression has the form of the Thévenin equivalent of the regulated power supply in that the first term is the internal voltage generator and the second term is the voltage drop resulting from the load current in the equivalent internal resistance.

Before considering Eq. (8-48) further, it is useful to simplify the expression by considering the relative magnitudes of the h parameters. When this is done (Exercise 8-14), Eq. (8-48) reduces to

$$v_o = v_i \frac{h_{ic}}{r_c(1 + a\beta)} - i_L \frac{h_{ic}(1 - \alpha)}{1 + a\beta} \tag{8-49}$$

which is sufficiently accurate for all practical purposes. According to Eq. (8-49) input voltage variations are reduced by a large factor through the feedback action of the regulator. Typical values of the power-transistor parameters in Eq. (8-49) are $h_{ic} =$ 1000 Ω, $r_c = 10{,}000$ Ω, $a = 50$, and $\beta = 0.3$, so that input voltage changes are reduced by a factor of 160 at the output terminals.

Additionally, the effective internal impedance of the regulated power supply, which is the coefficient of i_L in Eq. (8-49), is very small. Taking $1 - \alpha = 0.05$, the internal resistance is only 3.1 Ω. This means that changes in load current cause only minor changes in the regulated supply voltage. It is apparent that this simple regulator is effective against both input-voltage and output-current variations.

The control transistor in this regulator circuit must be capable of withstanding the full supply voltage without collector breakdown. Higher voltage regulators conventionally employ vacuum tubes as the control element because of their higher operating voltages. The circuit is identical to Fig. 8-26 with the transistor replaced by a triode or pentode. Analysis of the vacuum-tube regulator proceeds the same as that for the transistor version (see Exercise 8-15). The expression corresponding to Eq. (8-48) for the variation in output voltage is

$$v_o = \frac{v_i}{1 + \mu(1 + a\beta)} - i_L \frac{r_p}{1 + \mu(1 + a\beta)} \tag{8-50}$$

Typical values for the control tube in a regulator are $\mu = 10$ and $r_p = 2000$ Ω. Therefore, input voltage variations are reduced by a factor of 160 and the equivalent internal resistance is 12.5 Ω, if the amplifier gain and feedback ratio are the same as in the transistor circuit.

Note that the factor by which input voltage variations are reduced at the output terminals applies to ripple voltages as well as to other voltage changes. Therefore output voltage ripple is very small in a regulated power supply. Both the reduction factor and the effective internal resistance can be improved by increasing the gain of the dc amplifier, according to Eqs. (8-49) and (8-50). In many critical applications the gain is made large enough that residual variations in the regulated voltage reflect the stability of the reference potential.

The series control transistor or tube must be capable of dissipating the heat generated by the entire load current without overheating. Power tubes and transistors are often used. It is feasible to employ two or more identical units connected in parallel, if this is necessary to carry the maximum load current.

Although the series regulator is most popular, it is also possible to connect the control transistor in parallel with the load, much as in the case of a simple Zener-diode regulator, Fig. 4-26. The current through the transistor is controlled so that the voltage drop across the series resistor is the same independent of load-current changes. Effective voltage stabilization is obtained by such a *shunt regulator*, but the circuit suffers from the additional power loss in the series resistor. On the other hand, it is only necessary for the control transistor to carry a fraction of the full load current, which is a considerable advantage in high-current power supplies (see Exercise 8-18).

8-9 Practical circuits

A complete regulated power supply is illustrated in Fig. 8-28. The dc amplifier is a single grounded-emitter transistor stage and the reference voltage is provided by an 18-volt Zener diode. The Zener diode is placed in the emitter circuit of the amplifier, rather than in series with the feedback signal, so that one terminal of the diode can be grounded. The 33-kΩ resistor provides reverse potential for the diode. Note that the feedback ratio is adjustable. This enables the output voltage to be set at any desired value, within limits dictated by the operating conditions of the transistors. In this circuit, the output voltage can be adjusted over the range 40 to 50 volts, while maintaining good regulation. The 1500-μf capacitor across the output terminals provides a very low effective internal impedance for ac signals.

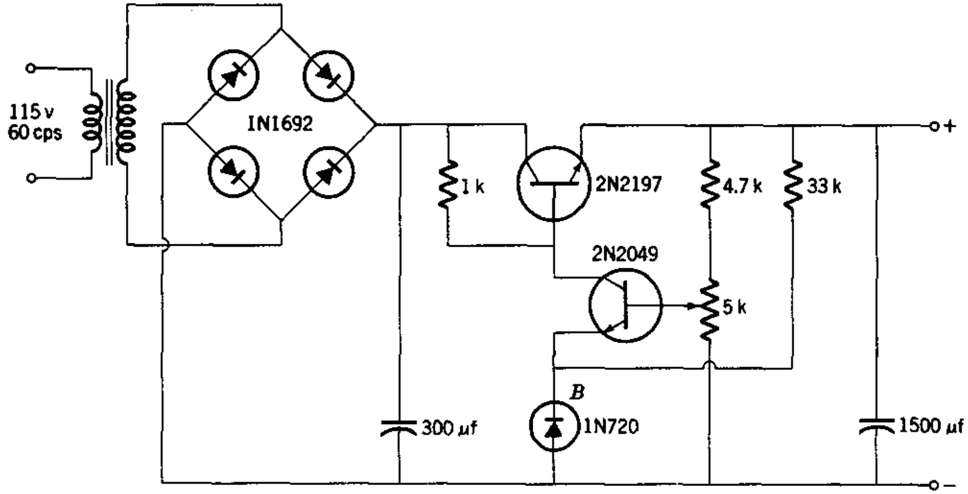


FIGURE 8-28 *Complete 40-volt 500-ma regulated power supply.*

A very precisely regulated voltage is obtained with the more elaborate vacuum-tube version in Fig. 8-29. A two-stage dc amplifier is used to increase the factor a in Eq. (8-50), and the pentode

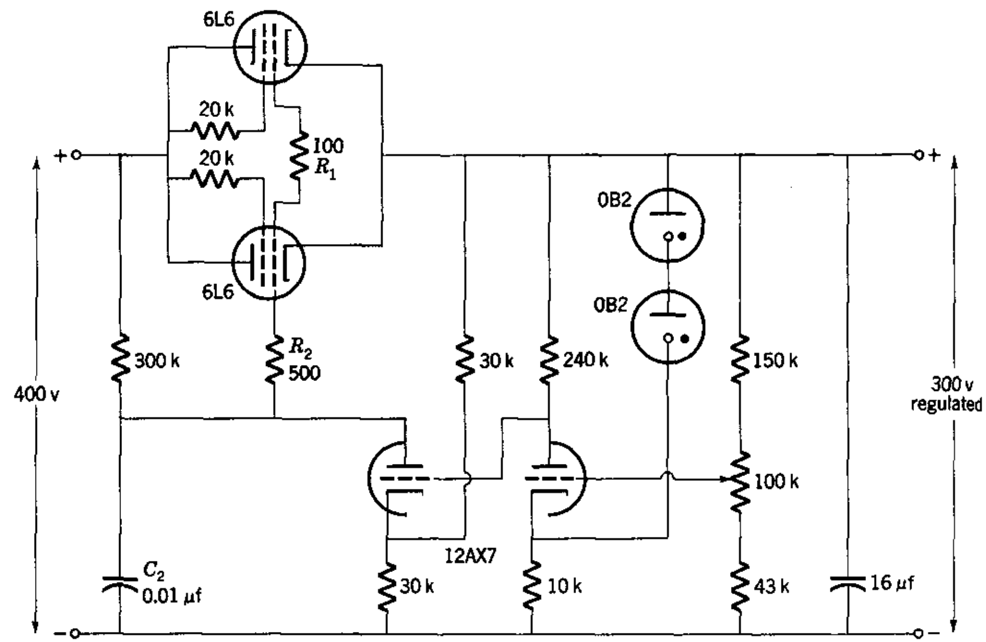


FIGURE 8-29 *Vacuum-tube voltage regulator.*

regulator tubes have a large μ. Both improvements result in excellent regulation (see Exercise 8-17). Two regulator tubes are used in parallel in order to permit a greater current to be drawn from the power supply without exceeding the rated current of the type 6L6 tube.

Note that the reference voltage is developed across two *VR* tubes in series. The difference between the regulated voltage and the reference potential is introduced into the cathode circuit of the dc amplifier. This is done to achieve negative feedback since the output of a two-stage amplifier is in phase with the input. The purpose of R_2 and C_2 is to reduce the amplifier gain at high frequencies and thus eliminate the possibility of positive feedback. Similarly, resistor R_1 is included to reduce the tendency of parallel-connected tubes to oscillate as a result of stray capacitances and inductances.

SERVOS

It is often useful to control the mechanical position or motion of a device in accordance with an electric signal. To assure that the desired motion takes place, a voltage feedback signal corresponding to the mechanical motion is developed. This voltage is compared with the input signal. When the two are equal, mechanical motion ceases since the mechanism has then responded to the actuating signal. Feedback systems incorporating a mechanical link in the feedback network are called *servomechanisms*, or *servos*, from the Latin word for "slave," since the mechanical motion is the slave of the input signal. Servos are widely used in control applications, from laboratory chart recorders to ballistic-missile guidance systems.

8-10 Mechanical feedback

Consider the servo system sketched in block-diagram form in Fig. 8-30. The amplifier drives a mechanical actuator as, for example, a dc motor. Suppose the motor moves a mechanical arm to a position x and that the feedback voltage is proportional to

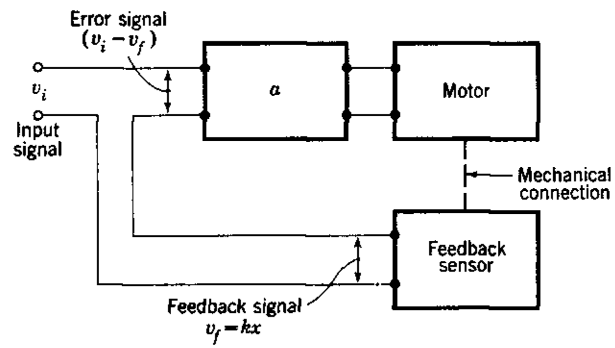


FIGURE 8-30 *Block diagram of servo system.*

the position of the arm and equal to kx. So long as the *error signal*, which is the difference between the input and feedback voltages, is not equal to zero, the amplified error signal continues to drive the motor. When the feedback signal equals the actuating signal, the error is zero, and motion ceases. Thus

$$v_i - v_f = v_i - kx_0 = 0 \tag{8-51}$$

so that

$$x_0 = \frac{v_i}{k} \tag{8-52}$$

According to Eq. (8-52), the equilibrium position of the mechanical motion x_0 is directly proportional to the input signal.

The servo cannot respond instantaneously to rapid changes in the input signal because of mechanical inertia and friction. According to the above discussion, the motive force from the motor is proportional to the error signal. This force is opposed by the inertial and frictional forces (which are assumed to be proportional to the velocity), so that

$$Ka(v_i - v_f) = m\frac{d^2x}{dt^2} + b\frac{dx}{dt} \tag{8-53}$$

where K is the motor constant, a is the amplifier gain, m is the mass of the moving parts, and b is a friction constant. Inserting Eq. (8-52), the differential equation for the response of the system is

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + (Kak)x = (Kak)x_0 \tag{8-54}$$

This equation can be solved by the methods used in Chap. 3 or by analog-computer methods (compare Eq. 8-40). In fact, the equation is identical in form to the one for an RLC circuit, Eq. (3-28). Based on the discussion in Chap. 3, a ringing effect is anticipated. Actually, the solutions are of two general kinds. If the system is *underdamped*, i.e., the frictional force is small compared with the driving force, a damped oscillation results, Fig. 8-31. On the other hand, if the frictional force is large, the system is *overdamped* and requires an excessively long time to reach the equilibrium position. Most servo systems are designed so they are slightly underdamped because this minimizes the response time. This results in a small *overshoot*, Fig. 8-31. If desirable, additional damping may be introduced by including an RC filter in the input circuit. Thus, the servo is never subjected to a signal change faster than the time constant of the filter.

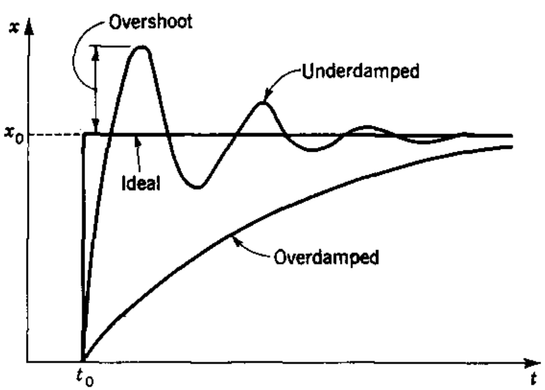


FIGURE 8-31 *Characteristic responses of servo to sudden input signal.*

8-11 The recording potentiometer

A common laboratory instrument based on servo principles is the *recording potentiometer*. This is a self-balancing dc potentiometer in which the balance position is marked on a moving paper strip or chart producing a continuous record of the input voltage. The heart of the system, Fig. 8-32, is the slide-wire R_S, in which there is a calibrated current supplied by the battery V_S and variable resistor R_A. The current is standardized by comparison with a standard battery, as described in Chap. 1. If less accuracy is satis-

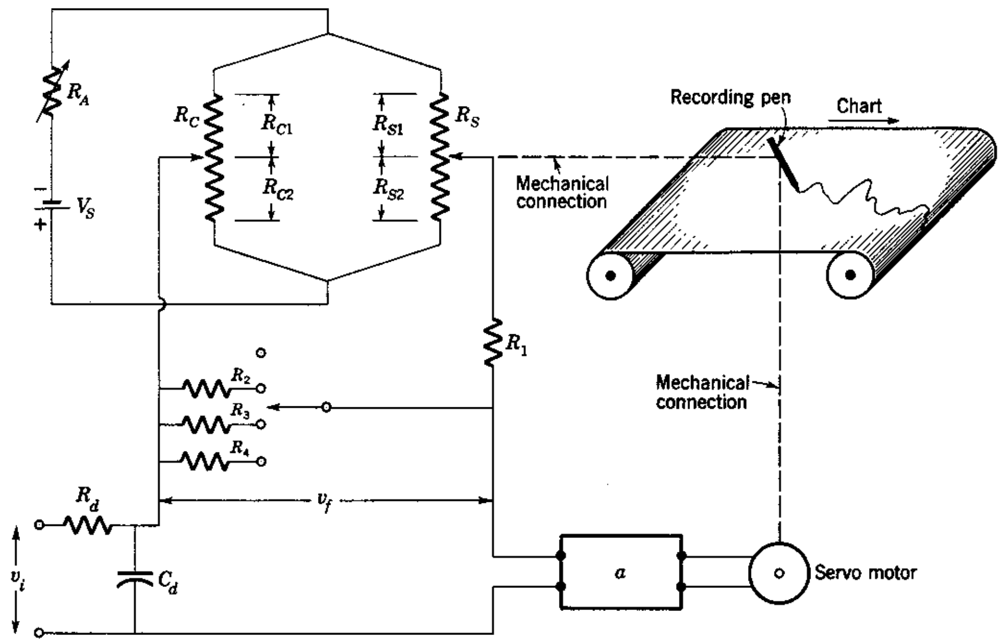


FIGURE 8-32 *Recording-potentiometer servo.*

factory, V_S may be a mercury battery which supplies a relatively constant current.

The potentiometer in parallel with the slide-wire resistance forms a bridge circuit which is balanced when, from Eq. (1-69),

$$\frac{R_{C1}}{R_{C2}} = \frac{R_{S1}}{R_{S2}} \tag{8-55}$$

Under this condition the feedback voltage v_f is zero. In the absence of an input signal, the adjustable contact on the slide-wire is moved by the servomotor until balance is achieved. The recording pen mechanically attached to the slide-wire contact continuously records the balance position on the chart. This means that the zero position ($v_i = 0$) may be placed anywhere on the chart by adjusting R_C.

When an input voltage is applied, an initial signal $v_i - v_f$ is fed to the amplifier. The servomotor drives the slide-wire contact to a new equilibrium position where the unbalanced bridge voltage equals the input voltage, and $v_i - v_f = 0$. The displacement of the sliding contact is a linear function of the input voltage, as can be shown in the following way. Using Eq. (1-81), the unbalance voltage from the bridge is

$$v_f = V\left(\frac{R_{C1}}{R_{C1} + R_{C2}} - \frac{R_{S1}}{R_{S1} + R_{S2}}\right) \tag{8-56}$$

where V is the voltage across the bridge. If the slide-wire is uniform, and the zero position, $x = 0$, is taken to be at the center,

$$\begin{aligned} R_{S1} &= \frac{R_S}{2} - mx \\ R_{S2} &= \frac{R_S}{2} + mx \end{aligned} \tag{8-57}$$

where m is a constant of the slide-wire. Introducing Eqs. (8-57) into Eq. (8-56) and noting that $R_{C1} = R_{C2}$ since the zero position is at the center,

$$\begin{aligned} v_f &= V\left(\frac{1}{2} - \frac{R_S/2 - mx}{R_S/2 - mx + R_S/2 + mx}\right) \\ &= V\left(\frac{1}{2} - \frac{1}{2} + \frac{mx}{R_S}\right) = \frac{V}{R_S}\,mx \end{aligned} \tag{8-58}$$

Therefore, according to Eq. (8-52),

$$x_0 = \frac{1}{mI_S}\,v_i \tag{8-59}$$

where $I_S = V/R_S$ is the standardized current in the slide-wire. Equation (8-59) shows that the deflection of the slider, and hence of the recording pen, is proportional to the input signal. It is common practice to select m and the standard current I_S such that the full chart width is an integral voltage such as 100 mv.

Resistors R_1 through R_4 in Fig. 8-32 constitute a voltage divider for changing the sensitivity of the potentiometer. When the divider is in the circuit, only a fraction of the unbalance voltage is available as the feedback signal and the sensitivity is increased correspondingly. The combination of R_d and C_d is a damping filter at the input to optimize the response time of the recorder.

The recording potentiometer is a very versatile instrument in the laboratory. It combines high sensitivity for dc signals and the inherently high input impedance of a potentiometer circuit with automatic operation and a permanent record. It is widely used as part of other instruments, such as optical spectrometers, X-ray diffractometers, and gas chromatographs, or as a separate unit.

8-12 Servo amplifiers

Amplifiers used in servo systems are of several types, depending upon the application and the kind of servomotor used. Simple systems may use a dc motor, so that a dc amplifier and power amplifier are required. This approach is subject to the drift problems of dc amplifiers and requires that the entire power of the servomotor come from the amplifier. Most often, two-phase ac motors are used in which one phase is supplied by the ac line voltage. An ac power amplifier controlled by the servo amplifier feeds the second-phase winding of the motor. Thus only a fraction of the total power of the motor is required from the amplifier.

In this case a chopper amplifier develops an ac voltage corresponding to the dc input signal. This is followed by a conventional ac amplifier and a push-pull class-B power amplifier to drive the motor. Chopping frequencies of 60 and 400 cps are standard, although the latter is preferred as it permits a faster response.

Since the amplifier need only handle signals of the chopping frequency, it is permissible to employ a rectified but unfiltered collector supply voltage of the chopping frequency, Fig. 8-33. The efficiency of the power amplifier is greater than that of a conventional class-B amplifier using a dc collector supply voltage. The reason for this is that the pulsating collector supply reduces the average power dissipation in each transistor. It is conventional to drive the servomotor directly without an output transformer, although this requires a center-tapped motor winding. The other phase winding of the motor is fed directly from the ac line through a phase-shifting capacitor C_2. This provides a 90° phase shift between the currents in the two windings, as required by a two-phase motor. Capacitor C_1 resonates the motor winding at the chopping frequency so that the motor presents a resistive load to the amplifier. The circuit of Fig. 8-33 is a grounded-collector push-pull amplifier using diodes for temperature compensation of the base bias, as described in the previous chapter. Grounded-emitter and common-base circuits are also used for servo amplifiers.

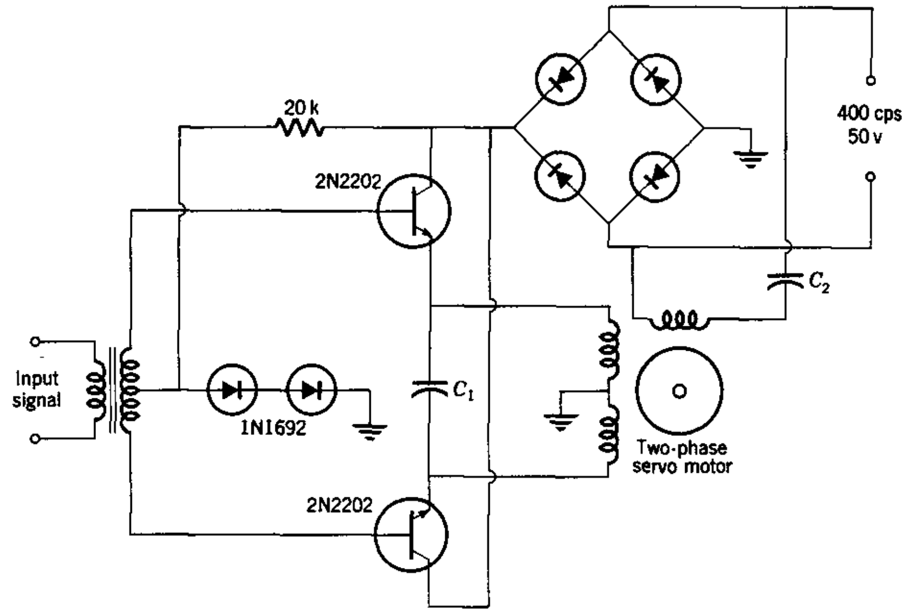
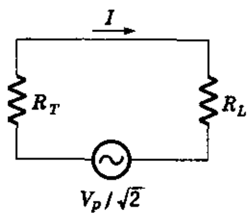


FIGURE 8-33 Push-pull servo power amplifier.

The increased efficiency accompanying an unfiltered collector supply is analyzed in the following way. Each transistor is considered a switch with internal resistance R_T which depends upon the magnitude of the input signal. The internal resistance is very large for a small input signal and reaches a lower limit called the collector saturation resistance for large values of input signal. The internal resistance of the transistors determines the magnitude of the supply voltage that is delivered to the load. This analysis is epitomized by an equivalent circuit consisting of R_T in series with the load resistance R_L and a voltage generator $V_p/\sqrt{2}$ for each tran-

FIGURE 8-34



sistor. V_p is the peak value of the rectified collector supply potential, Fig. 8-34.

Using this equivalent circuit, the average power delivered to the load by each transistor is

$$P_L = \frac{1}{2} I^2 R_L = \frac{V_p^2 R_L}{4(R_L + R_T)^2} \tag{8-60}$$

where the factor 1/2 results from the fact that each transistor is active for only one-half of the input cycle because of the class-B operation of the power amplifier. Similarly, the power dissipated in each transistor is

$$P_T = \frac{1}{2} I^2 R_T = \frac{V_p^2 R_T}{4(R_L + R_T)^2} = \frac{V_p^2}{4R_L} \frac{R_T/R_L}{(1 + R_T/R_L)^2} \tag{8-61}$$

The power efficiency is, from Eqs. (8-60) and (8-61),

$$\eta = \frac{P_L}{P_L + P_T} = \frac{R_L}{R_L + R_T} = \frac{1}{1 + R_T/R_L} \tag{8-62}$$

which indicates that the efficiency approaches 100 percent when $R_T \ll R_L$. Although this ideal is not achieved in practical amplifiers, efficiencies greater than those of conventional class-B stages can be attained.

Note that according to Eq. (8-61) the power dissipated in each transistor is small when R_T is large, that is, when the input signal is zero. Transistor dissipation is also small when maximum power is delivered to the load ($R_T \ll R_L$) corresponding to a large input signal. This means that the power lost in the power amplifier is smallest not only when the servo is quiescent, but also when the servo is striving to attain a new stable position corresponding to a change in the input signal. These two conditions are predominant in normal servo applications. This analysis shows that deleterious heating effects caused by transistor dissipation are minimized by the use of an unfiltered collector supply voltage.