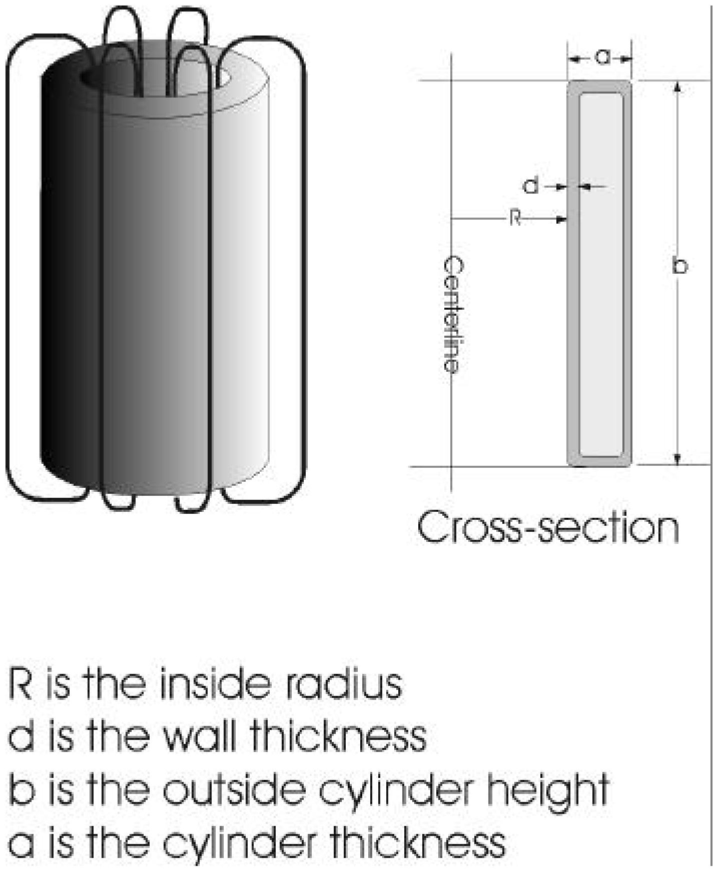
Calculation of EMF for a cylindrical toroid proton-precession magnetometer



The magnetization produced by the polarization field inside a toroidal form is:

$$M = \chi B_p$$

where B_p is the polarization field which is a function of radius. We have discussed the valuation of χ earlier. B_p is given by:

$$B_p = \mu_0 \frac{nI_p}{2\pi r}$$

where I_p is the polarization current and n is the total number of turns. The total flux inside the toroid is given by the integral:

$$\phi = \int M dA$$

where dA is the elemental area in a strip of constant r inside the cylindrical toroid. Thus ϕ becomes:

$$f = \int cB_p (b - 2d)dr$$

integrated over r from R+d to R+(a-d). Putting in the value for B_p, we get:

$$f = \frac{cm_0 (b - 2d) nI_p}{2p} \int_{R+d}^{R+a-d} \frac{dr}{r}$$

which is evaluated as:

$$f = \frac{cm_0 (b - 2d) nI_p}{2p} \ln\left(\frac{R + a - d}{R + d} \right)$$

Again, assuming that the precession is sinusoidal, the generated voltage can be shown to be:

$$e_s = \frac{cm_0 w}{\sqrt{2}} n^2 I_p \frac{(b - 2d)}{2p} \ln\left(\frac{R + a - d}{R + d} \right)$$

This equation is similar to those derived earlier and can be expressed as a general term followed by a geometric constant:

$$e_s = \frac{cm_0 w}{\sqrt{2}} n^2 I_p G_c$$

where G_c is the geometric constant for this configuration and has a value of :

$$G_c = \frac{(b - 2d)}{2p} \ln\left(\frac{R + a - d}{R + d} \right)$$

I have created a special spreadsheet named cyl.xls which calculates generated voltage and S/N for this geometry.