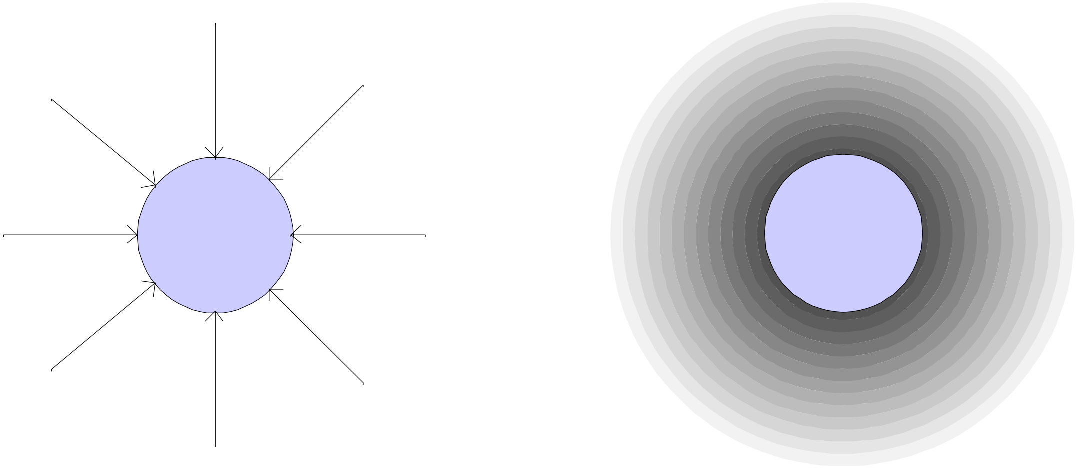
Fundamentals of Electrostatics II

In *Fundamentals of Electrostatics I* the construction of a simple electroscope was described along with a couple of experiments using a Lucite rod or plastic pocket comb. After experimenting with comb and an electroscope for a few minutes, the experimenter will notice that the leaves can be charged in steps - each touch of the comb causes more deflection. The leaves accumulate the bits of charge and the voltage builds. Bigger leaves will require more charge to achieve the same voltage because the electrons are spread over a larger area. If we coat the entire insides of the jar with aluminum, the capacity of the jar to hold charge will be much higher - and it will have a name! Named for its discovery by the physicist Pieter van Musschenbroek at the University of Leyden in the mid 1700s, the Leyden Jar is a simple form of capacitor that belongs in every electrostatics laboratory. The first Leyden Jar was simply a water-filled jar with a wire electrode dipped into the water (tap water is a conductor). A more modern version is made by lining the inside and coating the outside of the jar with aluminum foil forming two capacitor plates. The attraction of opposite charges between the two plates gives the foil Leyden Jar the capacity to store a significant charge. The outer foil is usually the "ground" plate and the inner foil is the "hot" plate. A connection is made to an electrode on the top of the jar and the foil with a wire or metal chain. The Leyden Jar in the photo was made from an empty spaghetti sauce jar, a rubber stopper, a long screw, aluminum foil, and black electrical tape. Spray adhesive was used to affix the foil to the inside surface and the electrode connection was made by tangling a long piece of bare wire with some of the foil on the inside and fastening the other end to the screw with electrical tape. A ground wire was fastened to the outer foil and the foil was wrapped with a coating of fiber tape for protection. A one inch gap was left at the top of the jar for insulation. This gap should be cleaned with alcohol and not touched thereafter. The jar top was wrapped with electrical tape to enhance the insulation. The jar may be tested by connecting it to an electroscope with a stiff wire that touches only the electrodes. A few discharges from a charged pocket comb or plastic rod should give an indication on the electroscope. If the electroscope discharges quickly, there is too much leakage - probably due to dirty glass or too much humidity in the air. Electrostatics experiments are best performed on dry days! If everything is working well, charge the Leyden jar with repeated comb charges then bring your finger near the electrode while holding the ground wire in the other hand. You should get a spark and the electroscope will indicate a sudden drop in charge.

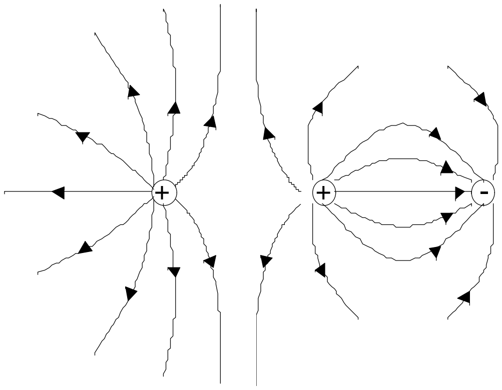
Electric Fields

The concept of a "force field" is quite familiar to us earth-bound humans. We spend our entire lives in the earth's gravitational field which seems perfectly uniform and constant. The lines of force are clearly "down" - the direction the fine china heads when it slips out of our hands. A line of force is simply an imaginary line that traces the path an object would take due to the influence of a field. For diagramming purposes, it might be useful to associate a line with a certain amount of force so that more lines indicate more force. A person standing on the moon would be "skewered" by fewer lines of force than a person standing on the earth since the moon's gravitational field is weaker than earth's. The force on an object is proportional to the line *density* - when the lines are far apart, the force is weak and when they are closely spaced, the force is high. Remember that the line density can be hard to gauge since only a few lines are usually drawn. Lines of force are most useful for showing the general shape of a field - only hinting at the field strength at any particular point. One could draw so many lines that they blur together giving shades of gray, darkening in areas of greater force, but it would be difficult to determine the direction of the force if the individual lines could not be seen. The following figure shows typical lines of force for an attracting body and a simulation of how the lines of force might look if thousands were drawn. As an object moves along a line of force it usually encounters a changing level of force.



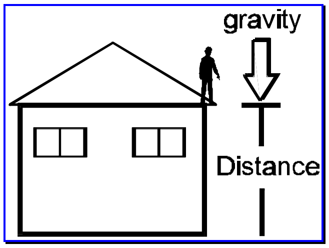
Lines of force are easily seen in the first figure and field strength is more easily seen in the second figure which simulates a very large number of lines.

The lines of electric force are defined to point in the direction a positive charge would move so electrons have lines of force pointing at them and protons have similar lines of force pointing away. When several charges are in close proximity, the line of force at a particular point will be the vector sum of the contributions from all of the charges. Unlike the gravitational field we know, electric fields have two polarities and can work together or oppose each other as the diagram shows. Notice how a positive particle precisely between the two positive charges would experience no force since the two fields cancel. But if the charge is slightly above or below the center line, it is squeezed out like a watermelon pit between wet fingers! Just like gravity, the electric field is a "force field" - the field intensity at any point is equal to the amount of mechanical force that would be applied to a charge of one coulomb positioned at the point. The units are newtons per coulomb. The electric field surrounding a charged Lucite rod can be detected by holding the rod near the electrode of a sensitive electroscope. Without the rod making contact, the leaves will spread due to the induced charge caused by the field.



Voltage

When you climb on the roof of your house the earth's gravity has the *potential* to do you great harm. In fact, your body has potential energy which is proportional to your height off the ground. If you slip, you will accelerate and accumulate kinetic energy until something stops you (hopefully, a soft pile of hay). The scientific term, work, is defined as force multiplied by distance. So gravity does work on you as you fall - the force is your weight (mass times gravity) and the distance is the height of the roof. The total amount of work done on you during your fall determines how hard you hit! Since gravity is uniform, you can gauge your safety by peeking over the edge of the roof to see how high you are. But if gravity were not uniform, you would need to multiply the height by the local gravity to determine how much potential energy your body has (which is equal to the kinetic energy it will have when it hits the ground).



When dealing with charges and electric fields, this product (meters times newtons per coulomb) is referred to as “voltage”. In a uniform electric field, the voltage between two points is the product of the magnitude of the electric field and the distance between the points. A charged particle will “fall” or accelerate in the electric field just as a body will accelerate in a gravitational field. The farther it falls, the more kinetic energy it accumulates. So the voltage between two points tells us how much energy will be imparted to a unit charge moving between them. Multiply the voltage by the amount of charge to get the energy (in newton-meters which are called joules). For example, a single electron with a charge of 1.6×10^{-19} coulombs will pick up 1.6×10^{-16} joules moving between two points with 1000 volts of potential difference:

$$1.6 \times 10^{-19} \text{ coul} \times 1000 \text{ volts} = 1.6 \times 10^{-16} \text{ joules}$$

Notice that we did not need to know the physical dimensions or the magnitude of the electric field - these quantities are already combined in the “voltage”. It is a handy combination since much of electronics involves the effects of potential differences regardless of the distances involved or the resulting electric fields. In your car, wiring carries the battery’s 12 volt potential difference to various motors, lamps and assorted gadgets in different locations and the electric field within the car is a complex three-dimensional structure. But the concept of voltage lets us greatly simplify the description of the functioning of the auto’s wiring without regard to the path the charges follow.

The electron that moved across the potential difference of 1000 volts in the above example gained 1000 “electron-volts” - a unit of energy commonly used in particle physics. One electron-volt is the amount of energy a single electron picks up when it accelerates through one volt. The electron-volt is a measure of energy - not voltage. Electron-volts and joules are the same type of quantity.

(1 ev = 1.6×10^{-19} joules)

You may have already noticed that the “units” of voltage are joules/coulomb which is an amount of energy (work) per unit charge. You could say, “my vehicle’s voltaic pile performs 12 joules of work moving one coulomb of charge from one terminal to the other.” Or you could say, “my car has a 12 volt battery.”

Now here is where it gets a little tricky! You have probably heard voltage referred to as “electromotive force” or emf and you have probably heard knowledgeable folk say that voltage is “how hard the electricity is pushing”. But from the previous discussion it would seem that the electric field is the force and that voltage is the amount of work done on (or energy imparted to) a charge moving in the electric field. So what is all this talk of emfs? Well, its a bit of a word shortcut. A higher voltage battery will generate higher electric fields in a given circuit and that is why more work will be done on

the charges moving in the circuit. Remember how the concept of voltage let us sidestep the difficulties of knowing the electric field distribution and the physical movement of the charges. We know that electric fields are doing the "pushing" and the charges are moving some distance but what really matters is how much real work is being done. Its a bit like saying someone hit the baseball with a lot of "force" instead of "energy".
Actually, the bat does deliver some number of joules per baseball depending upon the speed of the swing so a batter's "voltage" could be defined! And it would be natural to refer to this "work per baseball" as a "force" even though it isn't newtons.

Here is a silly analogy. Suppose one postal employee can toss one pound packages 15 feet and another can toss them 30 feet. The second employee could be said to have a higher "voltage" since he is doing more work per unit weight. We could say he has a parcel-motive force of 30. Actually, the force he applies to the package is not 30 and calling the work he did on the package a "force" is not entirely correct but the meaning is clear enough - he "motivates" the parcel 30 feet.

EXPERIMENT

Here is an interesting voltage experiment which will literally cost you a pile of money! Count Alessandro Volta's voltaic pile was the first and probably the simplest battery ever made. To duplicate this important invention you will need 10 pennies, some aluminum foil, some cardboard (try the back from a pad of paper), some saltwater and a voltmeter to measure the results.

Cut a one inch square of aluminum foil for the base of the pile. Place a penny in the center of the aluminum. Now cut 9 disks of cardboard and foil the same size as a penny. The foil disks may be all cut at one time by folding the aluminum foil until 10 layer are stacked. Squeeze the foil and a penny between a finger and thumb so that the penny may be used as a cutting guide. Soak the cardboard disks in the saltwater for several minutes then place one of these paper disks on top of the first penny. Now place a small disk of aluminum foil on top of the wet paper. Keep building the stack repeating the pattern: foil, penny, paper, foil, penny, paper, etc. (Note that the pennies go directly on the foil - no paper.) When you run out of money, stop with a penny on top.

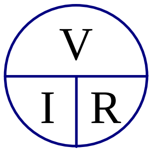
Touch one probe of your voltmeter to the foil square at the base and touch the other to the penny on top. If you made the pile carefully, you should measure about 4 volts. The penny will be the negative terminal. Try touching an LED across this "battery" in a dimly lit room.

The concept of *resistance* can be readily demonstrated using this voltaic pile. Measure the voltage across the pile when the LED is connected and feebly glowing. Notice that the voltage is much lower than the voltage with only the voltmeter connected. Every electrical source has internal resistance - this pile has a pile of resistance! (Pun intended!) The resistance of the pile limits the current that can flow in the LED. The mathematical relationship between voltage, current, and resistance is known as Ohm's law:

$$V = I \times R$$

"V equals I times R" may be manipulated to give equally simple equations for resistance and current:

$$I = V / R \quad \text{and} \quad R = V / I$$



This Ohm’s Law Circle is an easy way to remember the three formulas. The voltage, V is I times R since they are next to each other. R is “V over I” and I is “V over R”. Ohm’s Law is intuitive: One would expect more current to flow in a resistor if the voltage across it is increased and one would expect less current to flow if the resistance is increased for a given voltage. The penny battery has an electromotive force of about 4 volts and it will supply about 1 milliampere. Try switching your voltmeter to the current scale to see for yourself. How much internal resistance does the pile have? Modern batteries have much lower internal resistance and can supply far more current due to their superior “chemistry” and physical design.

You now have the tools to begin your study of basic circuits.

Charge: A Coulomb is a unit of charge.

Current: An Ampere is a flow rate of one Coulomb per second.

Voltage: A Volt is a potential of one Joule of energy (or work) per Coulomb of charge.

Resistance: An Ohm is a ratio of Volts per Ampere. (Or volt-seconds per coulomb.)

In day-to-day electronics work, voltage is simply thought of as an “electromotive force” without much attention paid to the units of “joule per coulomb”. The units of current are sometimes contemplated when calculating the coulomb charge on a capacitor but it is more common to refer to the energy stored in the capacitor rather than the coulombs. (In electrochemistry the coulomb has a high visibility since the products of electrolysis are directly proportional to the total charge that flows.) The fundamental units of the ohm are rarely mentioned in polite company (joule-second per coulomb ^{2}). In fact, the electric field strength is usually referred to in units of “volts per meter” instead of “newtons per coulomb” because voltage is readily measured whereas the force on a charge is rather difficult to measure. So, in most instances, “volts”, “amps”, and “ohms” are among the units of choice when discussing electronic circuitry.

Philosophical Point: You should work through the fundamental units several times until you achieve the emotion of full understanding - at which point you may forget them without suffering the debilitating effects of the emotion of not understanding. You can always look ‘em up to “refresh” your memory. Ask a few electronics engineers to tell you the units of voltage. Be polite. Just say, “Oh, I see”.