Fundamentals of Electrostatics I

For centuries people pondered lightning, lodestones, static electricity and other electromagnetic phenomenon without developing even rudimentary explanations or models. It is hard to imagine anything as dramatic as lightning going unexplained in our modern technical world but electricity is usually invisible and, without electronic gadgets, it is rather elusive. Fortunately, today's electronics experimenter will have no shortage of electronic gadgets and electricity is available "on tap" at the nearest electrical outlet! Even a poorly financed experimenter can acquire precision equipment that would have served proudly in a modern laboratory only a few decades ago and junk yards have mountains of scrap electronic components that would have been worth a fortune to that same lab. The concepts are simple and materials are at hand, so it is time to begin the adventure into electronics.

Electrical interactions are surprisingly simple and a few basic laws of electricity model the real world with stunning precision. Quantum physics has shown that Faraday, Maxwell, and other scientists of the nineteenth century were working with a few misconceptions but the fundamental mathematical relationships they discovered are the workhorses of modern technology.

Lets start our discussion with a quantum concept! As far as anyone knows there is a minimum quantity of charge - an amount of charge that cannot be subdivided. This is the amount of charge that an electron and proton possess. This packet of charge has a magnitude denoted by e and comes in two "polarities" denoted by the plus and minus signs (positive and negative). All electrical charges are made up of these tiny packets and charges with the same polarity physically repel each other and opposite charges attract each other. The electron charge is defined as the negative charge. A little more that 6,240,000,000,000,000,000 electrons are needed to make the common unit of charge called the coulomb. The unit of current, the ampere, is defined as the flow of one coulomb per second. Clearly, the charge on one electron is rather small. If the electric charge on the electron were represented by a tiny drop of water, one coulomb would fill a lake more than 30 miles across and over 100 feet deep. The current flowing in a one amp flashlight bulb is one such lake per second. Despite this incredible flow rate, an electron entering a short wire at the battery might take several minutes to reach the bulb. Obviously, there are an even more incredible number of free electrons in the wire! Most electronic circuits deal with huge numbers of electrons and the discrete nature of the charge carriers is insignificant.

Since there are 6.24×10^{18} electrons in a coulomb the charge on a single electron is $1/(6.24 \times 10^{18})$ coulombs or 1.6×10^{-19} coulombs.

R.A. Millikan and associates are credited with being the first to accurately measure this charge using an ingenious apparatus in what is now known as the Millikan oil-drop experiment. Tiny drops of oil from an atomizer were injected between two metal plates with an adjustable voltage between them. The resulting electric field would attract one polarity of charge to the top plate and if the voltage was just right, the force of gravity could be perfectly balanced, freezing the particle in mid-air. At this point, gravity times the drop's mass equals the charge times the electric field. Millikan discovered that the random charge on the droplets was always an integer multiple of 1.6×10^{-19} coulombs. The methods used to calibrate the apparatus are detailed in many physics texts and are well worth reading! Pay special attention to the technique used to determine the drop's mass and ponder whether you would have had confidence in the measurement!

Its time for a little lab work of your own:

1. Obtain a clear Lucite rod or a black plastic pocket comb - depending on your scientific tastes!

2. Rub the rod on your hair, a piece of fur, or silk cloth with rapid strokes.

3. Hold the rod near some tiny pieces of paper or sawdust and observe the results. Watch closely!

You should have observed the expected attraction of the neutral particles to the charged plastic. The careful observer will also notice that many of the attracted particles will suddenly fly away from the rod after a minute or two - as if repelled. The reaction is nearly immediate if the little pieces of paper are a slightly damp. Try dropping a damp piece of paper or a tiny piece of aluminum foil past the charged plastic. It will be attracted but as soon as it touches it will fly away. The rapid, erratic motion is hard to follow - so watch carefully!

The charge on the rod attracts the “neutral” particle but as soon as contact is made the particle picks up some of the charge and the like charges repel each other. Dry particles do not pick up the charge as quickly because they are less CONDUCTIVE and remain attracted to the rod for a longer period.

If you have never done these experiments then it is important that you do them now. No amount of study replaces real-world experiences. Millikan’s amazing experiment was the result of sound scientific reasoning combined with an engineering common-sense that comes only from tinkering. You have charged rubber balloons to stick them to the wall, no doubt! But did you ever determine if the charge on the balloon is the same polarity as the change on your pocket comb? How about a glass rod rubbed with silk? See if you can find out through experimentation.
Hint: Like charges repel - opposites attract.
Another Hint: Glass and silk are positively attractive whereas cheap plastic and fake fur have negative connotations! (Puns intended!)

The force between two tiny particles with charges q_1 and q_2 is given by Coulomb’s Law:

$$F_{elec} = K \frac{q_1 q_2}{r^2}$$

where K is the “dielectric constant” and r is the distance between the charges. This force is a vector that points along the line between the two charges. This simple equation can give one food for thought! If the force is proportional to the product of two charges then why does the charged plastic rod pick up neutral particles? Since one of the charge terms is zero shouldn’t the force be zero? The answer is not obvious and may leave one wondering if the equation has any “real world” relevance. When the charged rod is held near a small object, a charge is *induced* in the object. If the object is a conductor, the like charges will be repelled to the far side of the object and the opposite charges will be attracted to the side nearest the charged rod. Since the charges are segregated at different distances, their contribution to the total force will be different (note the $1/r^2$ term). The charges in an insulating material are not free to move about as in a metal but the charges can redistribute on a microscopic level resulting in a somewhat weaker attraction to the charged rod. If a conductive object is touching a conductive surface, it can accumulate a net charge since like charges can leave the object entirely and opposite charges can accumulate - the resulting attraction can be quite strong.

Coulomb’s Law assumes infinitely small particles and induced charge cannot form since there is nowhere for the charges to go! So, it may seem that Coulomb’s Law is only useful for particles that don’t exist! Or perhaps the law is incorrect! To find out if Coulomb’s Law is possibly true and might have any real-world application you may wish to do another experiment. This experiment will require the construction of a unique differential electroscope.

The traditional electroscope consists of two metal leaves hanging freely from a wire like a sheet draped over a clothes line. The leaves are usually placed inside a glass jar to block the wind. The wire protrudes through an insulated top to allow for the deposition of charge.

A differential electroscope has two isolated supports for the metal leaves allowing different charges to be applied to each leaf. For the purposes of this experiment, construction can be quite simple requiring only a small block of Styrofoam, two large nails and a couple of strips of aluminum foil about 4 inches long and 1/2 inch wide. Push the two nails through the foam about 1 1/2 inches apart forming two hangers for the foil strips. Bend the ends of the foil strips around the nails so that the strips hang freely.

Collect some charge on your plastic rod or pocket comb and transfer the charge to one of the leaves by dragging the charged rod across the head of the nail on the back side of the foam. Do not touch the other nail. Notice that only a little attraction or perhaps a slight repulsion is noted between the strips. Now touch the head of the nail supporting the uncharged strip. The two strips slam together! Because the strips are very thin compared to the distance between them the amount of force due to the induced charge is small. Whether the charge is on one face of the leaf or the other it is almost the same distance from the other leaf so $1/r^2$ is about the same. Since the net charge on the neutral leaf is zero, the force between the leaves is nearly zero. But when you touch the neutral leaf, the induced charge can skyrocket since there is now a place for the like charge to go and a plentiful source of opposite charge (your body).

So Coulomb’s Law is looking good and the physics of what happens when a comb picks up little particles involves induced charge - not simply different levels of charge. Attraction only occurs if the charged rod can induce a significant opposite charge or polarization in the otherwise neutral objects. It is an important distinction - one would be wasting time trying to attract neutrons with an electric field even though a charged rod seemingly attracts “neutral” particles!

A coulomb is one amp for one second, a seemingly modest amount of charge but actually a coulomb is an extremely large electrostatic charge. According to Coulomb’s Law, two plates one yard apart each with one coulomb of charge will attract or repel each other with billions of pounds of force! Obviously, charged bodies usually contain much less than a coulomb!

By the way, most of the world and all of science uses one of two metric-based systems for calculations, the mks and the cgs systems. Mks stands for meters, kilograms, seconds and cgs stands for centimeters, grams and seconds. For those of us more familiar with the British engineering system of feet, pounds and seconds the metric system is becoming more and more familiar. But a troublesome unit is the newton which is the unit of *force* in the mks system. One newton is 0.225 pounds. The kilogram is a unit of *mass* and on the earth a kilogram will weigh (experience a *force* of) 2.21 pounds due to gravity. Kilograms are commonly used as a weight instead of a mass since most of us spend our entire lives in the earth’s gravitational field. But pounds (weight) and kilograms (mass) are not really interchangeable. A person will still have his kilograms in the weightlessness of space but his bathroom scale will indicate zero pounds of force. Weight scales that indicate kilograms only function properly in the earth's gravity - they assume earth's gravity when determining the kilogram mass. This careless mixing of weight and mass units can make scientific calculations a bit confusing at times! As a memory aid, imagine Isaac Newton sitting under the apple tree pondering his famous equation, F = MA. The force, F, is appropriately in "newtons"! Cover your ears when you hear an engineer discussing grams of force. The acceleration of gravity is 9.81 m/sec ^{2} so the number of newtons is 9.81 times greater than the number of "kilograms of force". That is pretty close to 10 so if a weapons engineer tells you that it takes 2 kilograms of force to detonate his new mine you can estimate the required force to be 2 x 10 or 20 newtons.

The dielectric constant of space in the mks system is 9 x 10 ^{9} nm^2/coul^2. In the above example of two, one coulomb charges separated by one meter, Coulomb's Law predicts 9 x 10 ^{9} newtons (a couple of billion pounds.):

$$F = \frac{9 \times 10^9 \text{ n m}^2 / \text{coul}^2 \times 1 \text{ coul} \times 1 \text{ coul}}{1 \text{ m}^2} = 9 \times 10^9 \text{ n} = 2.025 \times 10^9 \text{ lb.}$$

The Electric Field

The acceleration term in Newton's equation, F = MA, is also the gravitational field intensity when calculating the effects of gravity on masses. In fact, Einstein showed that acceleration and gravity were indistinguishable! The concept of a gravitational field is familiar to most people so the concept of an electric field is readily described. We can define an electric field as we would define a gravitational field - the gravitational field strength is the ratio of an objects weight to its mass. The electric field is defined as the ratio of the force to the charge causing the force. The equivalent equation to F = MA is F = qE where E is the electric field intensity. Electric fields act just like gravitational fields with one interesting exception! The force on a charge due to an electric field can have two polarities but gravity only attracts (so far as we know).

Here is an exercise:

Suppose that the distance between an electron and the p roton in a hydrogen atom is 5 x 10 $^{-11}$ meters. What is the force attracting the electron? This is an easy problem! You know the charges of the particles (the smallest charge that exists), the distance and the dielectric constant of the space in between! See if you can calculate about 8.5 x 10 $^{-8}$ newtons.

Hobby Project: Building Classic Electroscopes

Although the electroscope is a bit obsolete, they still possess a charm and elegant simplicity. The classic electroscope is constructed in a large clear glass jar or bottle with an attractive brass wire hanger for the foil leaves and a polished brass knob at the top for the electrode. Laboratory grade electroscopes used extremely thin gold foil to minimize the weight and therefore maximize the deflection. Similar voltage sensitivities may be achieved with aluminum foil if very long leaves are used. The "capacity" will be higher so more charge will be necessary but the sensitivity to voltage can be quite good. Here is a picture of an electroscope built into a large, clear wine bottle with 11 inch leaves. They are fastened to a bolt which protrudes through the cork. This electroscope will visibly respond to potentials under 100 volts! It is usually desirable to choose a wider jar than the wine bottle so that the leaves do not become attracted to the glass when highly charged. Various chemistry beakers make attractive electroscopes with necks suitable for stoppers and wide chambers for the leaves. Hanging short leaves can be a bit tricky since they must swing independently. Give one leaf arms for hanging like a person hanging from a bar. The other leaf has a single narrow arm in the middle like a monkey hanging by its tail! The two leaves can move away from each other without the pivot points interfering with each other. The long leaves in the picture were simply attached to the screw on the top since they are long enough to flex. Smooth the leaves by gently rubbing them with a finger against a flat surface. If you wish to make an electroscope with a scale, replace one of the leaves with a stiff strip of metal or copper-clad circuit board. Fasten the flexible leaf to the stiff leaf at the top and place a paper scale behind the leaves. If the flexible leaf is at least 1/2” wide it will be stiff enough to not twist and stick to the scale. Enclose the scope in a large, clear plastic box with square sides - the type sold for decorative purposes will do nicely. A colored Lucite rod mounted in a wooden handle makes a nice accessory for this unique conversation piece.