SCATTERING PROPERTIES OF A STATISTICALLY ROUGH INTERFACE INSIDE A PIECEWISE HOMOGENEOUS STRATIFIED MEDIUM

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Abstract: The problem of electromagnetic wave scattering from a statistically rough interface in a multi-layered medium is considered. The backscattering coefficients are found in the framework of small perturbation method combined with the Green functions approach. The influence of the stratification of the medium on the scattered field is analysed numerically.

INTRODUCTION

Analysis of scattering of electromagnetic waves from an air-ground interface is a topic of practical interest for many applications, among them for a ground penetrating radar (GPR). Theoretical study of the problem has been initiated by microwave remote sensing of ice and snow covers and has been under consideration more than 20 years. The simulation of wave propagation in a stratified medium with rough interfaces has been done for the first time in the framework of the radiative transfer theory by Fung et al.[1]. Inside this theory a field propagated inside a medium has a narrow angular spectrum and all layers of a medium are considered as very thick in comparison with wavelength. Scattering properties of corrugated interfaces are calculated for the case of a rough interface between two homogeneous halfspaces and are inserted in the radiative transfer equation. To some extent a similar procedure has been used in the paper by Elachi et al.[2]. In both approaches the influence of medium’s stratification on scattering properties of interfaces was not considered.

In this paper the above mentioned problem is examined by means of the Green function approach in combination with the small perturbation method. A three-layer model is chosen for the ground. The upper layer is thought to be vegetation or snow, the middle layer and the homogeneous halfspace below it represent the ground. A vegetation-ground interface is considered as rough with statistically homogeneous corrugation while the two other interfaces are plane. Roughness of an air-ground interface and the vertical stratification of the ground are two main features that should be considered in any theoretical model of GPR operation.

FORMULATION OF THE PROBLEM AND ANALYTICAL CONSIDERATION

We assume that the whole medium occupies the half-space $z \le b \, (b \ge 0)$ and consists of two homogeneous layers $0 < z < b$ and $-a < z < 0, (a > 0)$ lying on a uniform half-space $z < -a$. The dielectric permittivity of the medium in the upper layer equals to the complex-valued constant e_1, in the bottom layer - e_2, and in the substrate - e_s. The magnetic permeability of the medium equals 1. The half-space $z > b$ is free space. In the presence of the statistical roughness, the interface between two layers is assumed to be specified by the equation $z = V(\vec{r})$, where $\vec{r}$ is a two-dimensional vector lying in the plane $x0y$, and $V(\vec{r})$ is a random function with zero mean value and correlation function $\left\langle V(\vec{r})V(\vec{r}\,')\right\rangle = \mathrm{B}(\vec{r} - \vec{r}\,')$. The brackets $\langle \ \ldots \ \rangle$ mean averaging over the realisations of the random function V. We assume that roughness is small $(s \ / \ l_0 << 1)$ and mildly sloping $(s \ / \ l << 1)$; here s is the root mean square height of the roughness, l_0 is the wavelength in free space, and l is the characteristic horizontal scale of the roughness.

The scattering problem is solved by means of the small perturbation method (Bass et al[3]) combined with the Green functions approach. The analytical expressions for the radar backscattering cross section $s_j(a, j)$ on vertically $(j = v)$ and horizontal $(j = h)$ polarization in a general case of arbitrary stratified medium have derived in Yarovoy[4]. In the particular case of the three interface structure these expressions are

$$
\begin{aligned}
s_v &= 64k_0^4\left|e_2^2\,(e_2-e_1)/e_1\right|^2 \tilde{B}(-2\vec{k})\cos^4 a\, \left|g_1\left(e_s g_2\cos(g_2 a)-ie_2 g_s\sin(g_2 a)\right)\right|^4 \times \\
&\left\{\left|\sin^2 a/e_2 + Z_v{}^2(k)\right|^2 / \left|\cos a + Z_v^b(k)\right|^4\right\} / \\
&\left\{\left|\exp(-ig_1 b)\left(\left(e_2 g_1 e_s g_2 + e_1 g_2 e_2 g_s\right)\cos(g_2 a) - i\left(e_2 g_1 e_2 g_s + e_1 g_2 e_s g_2\right)\sin(g_2 a)\right) + \right.\right. \\
&\left.\left.\exp(+ig_1 b)\left(\left(e_2 g_1 e_s g_2 - e_1 g_2 e_2 g_s\right)\cos(g_2 a) - i\left(e_2 g_1 e_2 g_s - e_1 g_2 e_s g_2\right)\sin(g_2 a)\right)\right|^4\right\},
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
s_h &= 64k_0^4\left|e_2-e_1\right|^2\tilde{B}(-2\vec{k})\cos^4 a\,\left|Z_h^b(k)\right|^4\,\left|g_1\left(g_2\cos(g_2 a)-ig_s\sin(g_2 a)\right)\right|^4 / \\
&\left\{\ \left|1+\cos a\ \ Z_h^b(k)\right|^4\left|\exp(-ig_1 b)\left(g_2\left(g_1+g_s\right)\cos(g_2 a) - i\left(g_1 g_s + g_2{}^2\right)\sin(g_2 a)\right) + \right.\right. \\
&\left.\left.\exp(+ig_1 b)\left(g_2\left(g_1-g_s\right)\cos(g_2 a) - i\left(g_1 g_s - g_2{}^2\right)\sin(g_2 a)\right)\right|^4\ \right\}.
\end{aligned}
\tag{2}
$$

Here $k_0 = 2p/l_0$ is the wave number in free space; $\vec{k} = k_0\sin a\,(\vec{x}_0\cos j - \vec{y}_0\sin j)$ is the tangential (to the mean interface position) component of the incident wave vector; the spatial spectrum of the interface's corrugations $\tilde{B}$ is defined as $\tilde{B}(\vec{r}) = (2p)^{-2}\int d\vec{r}\,B(\vec{r})\exp(-i\vec{r}\cdot\vec{r})$; functions Z_j, Z_j^b are input impedances of the media in halfspaces $z<0$ and $z<b$ correspondingly for j-polarized $(j=v,h)$ plane waves (see e.g. Zhuk et al.[5]); and finally functions g_n, $n=1,2,s$ are defined as $g_n = \sqrt{k_0^2 e_n - k^2}$.

The influence of a stratification of the surrounding medium on the interface corrugation's scattering characteristics is analysed using the ratio F of the backscattering coefficient for the above defined (testing) medium and a reference medium. The latter consists of two homogeneous halfspaces with the same values of permittivity as in the testing medium just above plane $z=b$ and below the rough interface. The interface between these halfspaces has the same corrugations as the testing medium. The backscattering coefficient of the reference medium is well-known (see e.g. Bass[3]).The quantity F measures the effect of the stratification of the testing medium on the scattering properties of the corrugated interface, but does not depend on properties of corrugation. Furthermore, it does not depend on the azimuth angle of propagation of the incident wave because this dependence in the backscattering coefficients are concerned with the spatial spectrum of the corrugation. The relations $F>1$ $(F<1)$ imply enhancement (weakening - correspondingly) of the scattering compared with the case of a uniform halfspace.

NUMERICAL ANALYSIS AND DISCUSSION

Two cases has been analysed numerically. In the first case, the dielectric permittivity of the upper layer equals 1. Physically this case corresponds to the absence of snow or vegetation over a ground with a rough surface. By means of this model we investigate the influence of the subsurface on the backscattering coefficient. The typical dependencies for this case are plotted on the Fig.1 for $e_1 = 1$, $e_2 = 2+0.02i$, $e_s = 20+2i$. In the second case, the dielectric permittivity of a bottom layer equals the one of a homogeneous halfspace below it. This case describes a homogeneous ground with a rough surface covered by snow or vegetation and allows us to study the influence of the cover layer on the backscattering coefficient. The typical dependencies for this case are plotted on the Fig.2 for $e_1 = 2+0.02i$, $e_2 = e_s = 20+2i$.

A comprehensive numerical simulation has been performed for layers with loss tangent less than 0.1. The real part of the dielectric permittivity of a layer is assumed to be less than the permittivity of an underlying halfspace. It was shown that in both cases for horizontally polarised waves the stratification of the medium decreases the backscattering on arbitrary angle for underlying (or covering) layers with D<0.5 and increases it for layers with 1.0<D<2.4, where D is an optical thickness of the layer. For vertically polarised waves the stratification of the medium decreases the backscattering for an incident angle smaller than 45 degrees and an optical thickness of a layer less than 0.5. Increasing of the backscattering has been observed for layers with

1.0<D<2.2 and incident angles less than 30 degrees. For small ohmic losses and grazing angles the alternation of backscattering due to stratification of a medium can reach 20dB.

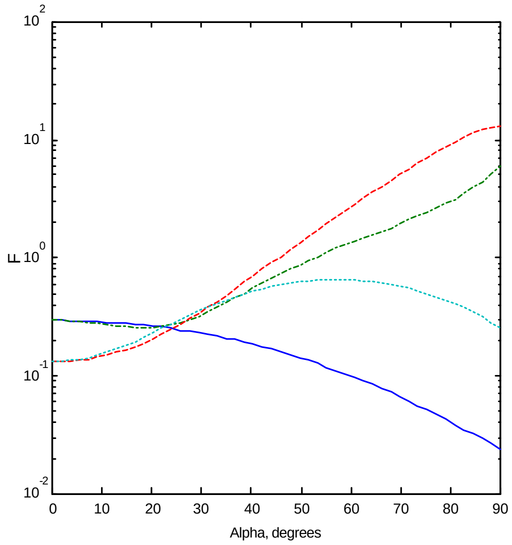

Figure 1.

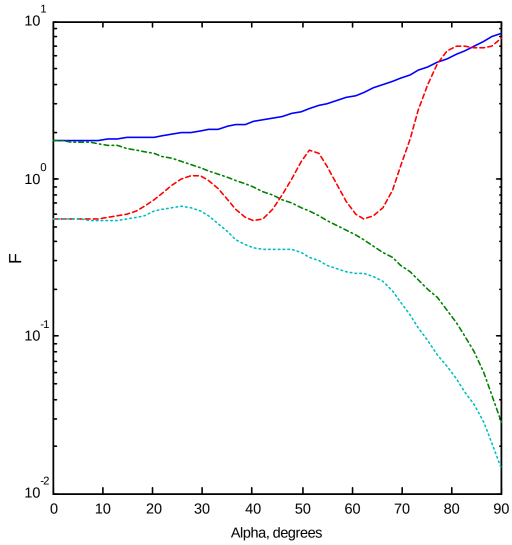

Figure 2.

Fig. 1. *Backscattering dependence from the incident angle in the first case: solid line -* $k_0 a = 0.3,\ j = h$ *;dashdot line -* $k_0 a = 0.3,\ j = v$ *; dashed line -* $k_0 a = 2.0,\ j = h$ *; dotted line -* $k_0 a = 2.0,\ j = v$

Fig. 2. *Backscattering dependence from the incident angle in the second case: solid line -* $k_0 b = 1.0,\ j = h$ *;dashdot line -* $k_0 b = 1.0,\ j = v$ *; dashed line -* $k_0 b = 20.0,\ j = h$ *; dotted line -* $k_0 b = 20.0,\ j = v$

The results obtained can be explained as follows. The piece-wise homogeneous stratification of a medium leads to interference of waves reflected from its interfaces. This phenomenon results in an alternation of the total power of an incident wave transmitted into a ground. In the framework of the small perturbation technique the magnitude of a field scattered from an interface corrugation is proportional to this transmitted power for horizontally polarised waves. For vertically polarised waves it is proportional to a weighted sum of tangential and normal components of the electric field in square. Therefore interference phenomena in a stratified structure cause an alternation of backscattering from its interfaces.

CONCLUSION

The paper shows that scattering properties of an interfaces in a stratified medium not only depend on corrugations characteristics but also on vertical permittivity profile. This effect must be taken into account in theoretical models for GPR especially in the inverse scattering models. Additional investigation by means of more precise approaches than the small perturbation technique are needed in order to clarify the role of the vertical gradient of the dielectric permittivity of the medium.

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