Transmission of Electromagnetic Fields Through an Air-Ground Interface in the Presence of Statistical Roughness

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Abstract: The transmission coefficient for a vacuum-dielectric interface with small (compared to the wavelength) statistically homogeneous roughness is analyzed. The analytical expressions for the transmission coefficients for both vertically and horizontally polarised plane waves are derived. For simplicity only terms quadratic in the roughness height are taken into account. For typical GPR scenarios the energy transmitted into the ground for both vertically and horizontally polarised plane waves is calculated as functions of dielectric permittivity, operating frequency, incident angle and corrugation's parameters.

INTRODUCTION

Electromagnetic wave scattering from natural surfaces has been treated intensively during several last decades. Mainly radar and bistatic cross-sections of the surfaces have been investigated [1]. Recently, Ground Penetrating Radar (GPR) received a large attention and new GPR technologies are being developed. In GPR, different physical characteristics of natural surfaces are of interest, namely input impedance and transmission coefficient. In this paper the transmission coefficient is analyzed for a vacuum-dielectric interface with small (compared to the wavelength) statistically homogeneous roughness. The typical air-ground corrugations satisfy this limitation for low frequencies (up to 300MHz), which are used in GPR for deep sensing [2].

THEORETICAL CONSIDERATION

Let the region $-\infty<z<\zeta(\mathbf{r})$, $\mathbf{r}=(x,y,0)$ of a three-dimensional space be filled with a dissipative dielectric medium with a complex permittivity profile $\varepsilon(z)$, and let the region adjacent to it be free. The rectangular Cartesian coordinates x,y,z, $-\infty<x,y,z<\infty$ with the z axis directed upward is used. The random function $\zeta(\mathbf{r})$ describes the gently sloping irregularities of the boundary $z=0$ with a zero mean, a variance of σ, and a correlation function $<\zeta(\mathbf{r})\zeta(\mathbf{r}')>\equiv B(\mathbf{r}-\mathbf{r}')$. The angle brackets denote the averaging over the ensemble of realization of $\zeta(\mathbf{r})$. We assume that the characteristic height of an individual surface is considerably less than the minimal wavelength of the illuminating electromagnetic field. To avoid additional complexity we consider here only a homogeneous medium (the function $\varepsilon(z)$ equals ε). The magnetic permeability of the medium equals to 1 everywhere.

The problem is treated by means of the statistical perturbation theory [3]. The original boundary problem of electrodynamics for geometry with a rough interface between vacuum and dielectric is transformed to an equivalent problem in the same region but with plane interface and perturbed boundary conditions. A statistical averaging is applied to resulting problem and equivalent boundary conditions are derived for the averaged field at the plane interface. The boundary conditions take into account multiple scattering from roughness. The range of their validity is far beyond the usual small perturbation technique.

Let us consider the incident plane wave in the vector form

$$\mathbf{E}^i(\mathbf{R})=\left[-\mathbf{z}_0\times\mathbf{n}E_0+(\mathbf{n}\gamma+\mathbf{z}_0\kappa)H_0\right]\exp(i(\boldsymbol{\kappa}\cdot\mathbf{r}-\gamma z)),$$
$$\mathbf{H}^i(\mathbf{R})=\left[(\mathbf{n}\gamma-\mathbf{z}_0\kappa)E_0+\mathbf{z}_0\times\mathbf{n}H_0\right]\exp(i(\boldsymbol{\kappa}\cdot\mathbf{r}-\gamma z)),\quad (1)$$

where E_0, H_0 are the magnitudes of the horizontal- and vertical-polarised component of the wave; $\mathbf{z}_0$ is the unit vector along the z-axis; $\boldsymbol{\kappa}=\kappa\mathbf{n}$, the vector $\mathbf{n}$ is a tangential component of the wave vector and $\kappa=k_0\sin\theta$; θ is the angle of incidence and k_0 is the wave number in free space, $\gamma=\sqrt{k_0^2-\kappa^2}$.

The mean (averaged) component of the field after transmission through the rough interface into the ground can be presented as follows

$$\mathbf{E}^t(\mathbf{R})=\left[-\mathbf{z}_0\times\mathbf{n}T_{HH}E_0+(\mathbf{n}\Gamma+\mathbf{z}_0\kappa)T_{VV}H_0/(k_0\varepsilon)\right]\times \exp(i(\boldsymbol{\kappa}\cdot\mathbf{r}-\Gamma z)),$$
$$\mathbf{H}^t(\mathbf{R})=\left[(\mathbf{n}\Gamma-\mathbf{z}_0\kappa)T_{HH}E_0+\mathbf{z}_0\times\mathbf{n}T_{VV}H_0/(k_0\varepsilon)\right]\times \exp(i(\boldsymbol{\kappa}\cdot\mathbf{r}-\Gamma z)),\quad (2)$$

where $\Gamma=\sqrt{k_0^2\varepsilon-\kappa^2}$, the functions $T_{VV}\equiv T_{VV}(\boldsymbol{\kappa})$ and $T_{HH}\equiv T_{HH}(\boldsymbol{\kappa})$ are the transmission coefficients for vertically (T_{VV}) and horizontally (T_{HH}) polarised plane waves. In a general case of the anisotropic spectrum of the roughness they depend from the azimuthal angle of the incident wave propagation. The analytical expressions for them were derived from the equivalent boundary conditions. For simplicity only terms quadratic in the roughness height are taken into account and thus the effect of the depolarisation in the transmitted wave is out of consideration in (2). In this case we have

$$T_{VV}(\boldsymbol{\kappa})=2\gamma(1-\tau_V(\boldsymbol{\kappa}))/(\gamma+k_0 Z_{Ve}(\boldsymbol{\kappa})),$$
$$T_{HH}(\boldsymbol{\kappa})=2\gamma Z_H(\kappa)(1-\tau_H(\boldsymbol{\kappa}))/(\gamma Z_{He}(\boldsymbol{\kappa})+k_0). \quad (3)$$

Here the input impedance of the medium with a smooth (σ=0) surface for vertically and horizontally polarized waves are

$$Z_V(\kappa)=\Gamma(\kappa)/(\varepsilon k_0), \quad Z_H(\kappa)=k_0/\Gamma(\kappa), \quad (4)$$

Functions $Z_{Ve}(\boldsymbol{\kappa})$ and $Z_{He}(\boldsymbol{\kappa})$ are the components of the equivalent input impedance of the medium with a rough ($\sigma\neq0$) surface. Explicit expressions for them in the case of an arbitrary stratified magneto-dielectric anisotropic medium can be found in [3]. In our case they can be simplified as

$$Z_{Ve}(\boldsymbol{\kappa})=Z_V(\kappa)+(k_0\sigma)^2 Z_V(\kappa)\left[(1/\nu-\nu)(\kappa/k_0)^2+(\nu-1)\right]+$$
$$k_0^2\int\Big\{k_0\Big(((1/\nu-1)\kappa\kappa'/k_0^2)^2+$$
$$(1-\nu)Z_V(\kappa)(\gamma(\kappa')-k_0 Z_V(\kappa'))\mathbf{n}\cdot\mathbf{n}'(1/\nu-1)\kappa\kappa'/k_0^2-$$
$$(1-\nu)^2 Z_V^2(\kappa)\gamma(\kappa')Z_V(\kappa')(\mathbf{n}\cdot\mathbf{n}')^2\Big)/\Delta_V-$$
$$(\mathbf{n}\times\mathbf{n}')^2(\nu-1)^2 k_0 Z_V^2(\kappa)Z_H(\kappa')/\Delta_H\Big\}\tilde{B}(\boldsymbol{\kappa}-\boldsymbol{\kappa}')\,d\boldsymbol{\kappa}',$$

$$Z_{He}(\boldsymbol{\kappa})=Z_H(\kappa)+(k_0\sigma)^2(\nu-1)Z_H(\kappa)-$$
$$k_0^2(1-\nu)^2 Z_H^2(\kappa)\int\Big\{k_0(\mathbf{n}\cdot\mathbf{n}')^2 Z_H(\kappa')/\Delta_H+ \quad (5)$$
$$(\mathbf{n}\times\mathbf{n}')^2\gamma(\kappa')Z_V(\kappa')/\Delta_V\Big\}\tilde{B}(\boldsymbol{\kappa}-\boldsymbol{\kappa}')\,d\boldsymbol{\kappa}',$$

The explicit expressions for the functions τ are as follows:

$$\tau_V(\boldsymbol{\kappa})=(k_0\sigma)^2\left[(\nu-1)(\kappa/k_0)^2-(\nu-1)/2\right]+$$
$$k_0^2\int\Big\{(\nu-1)\gamma(\kappa')\mathbf{n}\cdot\mathbf{n}'((1/\nu-1)\kappa\kappa'/k_0^2-$$
$$(1-\nu)Z_V(\kappa)Z_V(\kappa')\mathbf{n}\cdot\mathbf{n}')/\Delta_V+$$
$$(\mathbf{n}\times\mathbf{n}')^2(\nu-1)^2 k_0 Z_V(\kappa)Z_H(\kappa')/\Delta_H\Big\}\tilde{B}(\boldsymbol{\kappa}-\boldsymbol{\kappa}')\,d\boldsymbol{\kappa}',$$
$$\tau_H(\boldsymbol{\kappa})=-(k_0\sigma)^2(\nu-1)/2+k_0^2(\nu-1)^2 Z_H(\kappa)\times$$
$$\int\Big\{k_0(\mathbf{n}\cdot\mathbf{n}')^2 Z_H(\kappa')/\Delta_H+ \quad (6)$$
$$(\mathbf{n}\times\mathbf{n}')^2\gamma(\kappa')Z_V(\kappa')/\Delta_V\Big\}\tilde{B}(\boldsymbol{\kappa}-\boldsymbol{\kappa}')\,d\boldsymbol{\kappa}',$$

$$\Delta_V=\gamma(\kappa')+k_0 Z_V(\kappa'), \quad \Delta_H=k_0+\gamma(\kappa')Z_H(\kappa'),$$

and finally the spatial spectrum of the roughness is defined as

$$\tilde{B}(\boldsymbol{\kappa})=(2p)^{-2}\int B(\mathbf{r})\exp(i\boldsymbol{\kappa}\cdot\mathbf{r})d\boldsymbol{\kappa}. \quad (7)$$

NUMERICAL ANALYSIS AND DISCUSSION

The equations (2-7) make up the analytic foundation of the algorithm, which has been developed for numerical simulation of typical GPR scenarios are. Because the typical correlation functions for the natural surfaces are not known exactly, an isotropic Gaussian correlation function has been used. It means that $B(\mathbf{r})\equiv B(r)=\sigma^2\exp(-r^2/l^2)$, where l is the correlation interval of the surface roughness. The requirements of small slope and small magnitude are satisfied if $\sigma/l \ll 1$, $k_0\sigma \ll 1$.

The expression for the mean value of the energy transmitted into the ground for both vertically and horizontally polarised plane waves can be derived from (2)

$$P_V(\boldsymbol{\kappa})\equiv \mathrm{Re}\{\gamma/(\varepsilon\Gamma)\}|T_{VV}(\boldsymbol{\kappa})|^2, \quad P_H(\boldsymbol{\kappa})\equiv \mathrm{Re}\{\Gamma/\gamma\}|T_{HH}(\boldsymbol{\kappa})|^2.$$

Here the value are normalized to the z-component of the total energy flux of the incident wave.

The distortion of the total transmitted energy ΔP (i.e. the difference between the values of P for the medium with a rough ($\sigma\neq0$) and a smooth (σ=0) surfaces) are calculated as functions of dielectric permittivity, incident angle and operating frequency. The typical results are shown on Fig.1-5.

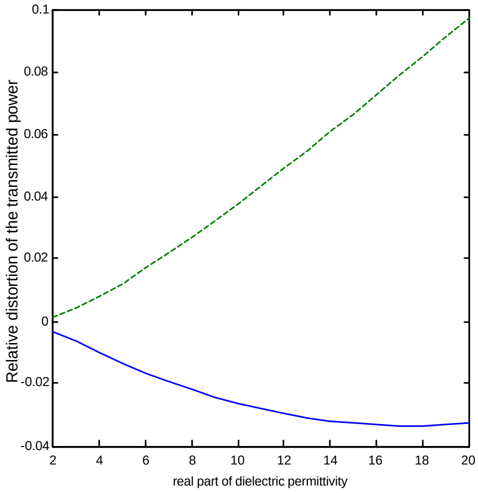

Fig.1. $\Delta P/P$ versus real part of the dielectric permittivity of the ground for normal incidence ($k_0\sigma$=0.09, Im(ε)/Re(ε)=0.1): $k_0 l$=0.3 (solid curve), $k_0 l$=5.0 (dash curve).

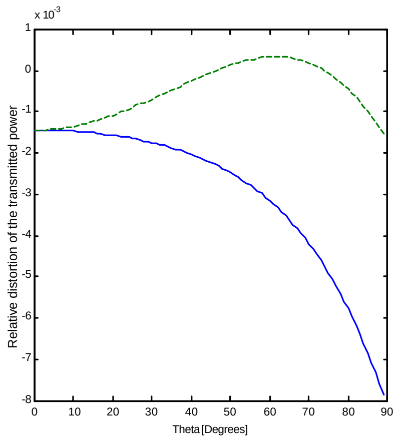

Fig.2. $\Delta P/P$ versus the angle of incidence ($k_0\sigma$=0.09, $k_0 l$=0.3, ε=2+0.02i): horizontal (solid curve) and vertical (dash curve) polarization.

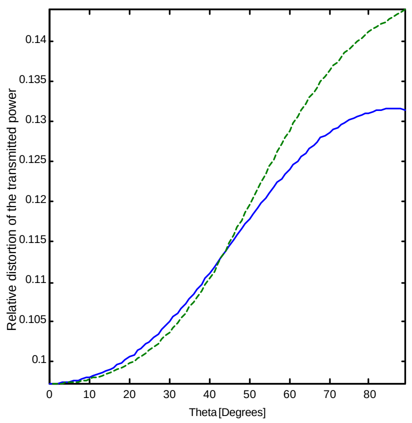

Fig.3. $\Delta P/P$ versus the angle of incidence ($k_0\sigma$=0.09, $k_0 l$=5.0, ε=20+2i): horizontal (solid curve) and vertical (dash curve) polarization.

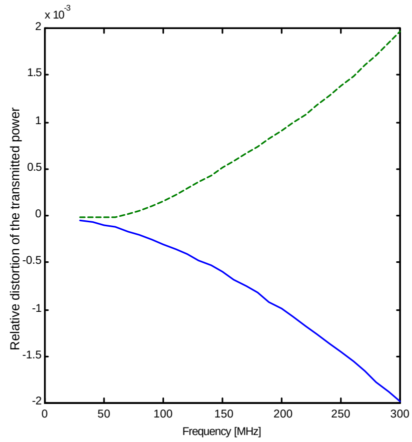

Fig.4. $\Delta P/P$ versus frequency for the normal incidence (σ=1.7cm, ε=2+0.02i): l=5cm (solid curve), l=85cm (dash curve).

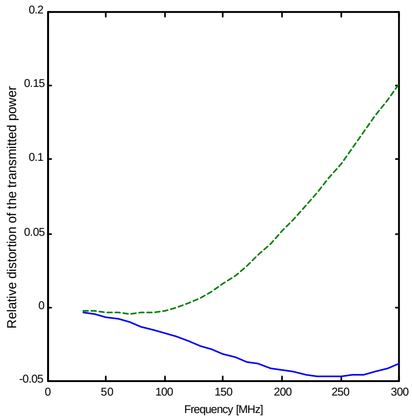

Fig.5. $\Delta P/P$ versus frequency for the normal incidence (σ=1.7cm, ε=20+2i): l=5cm (solid curve), l=17cm (dash curve).

It was observed that the ΔP can reach the value up to 15% of P. The absolute value of ΔP increases with the real part of the dielectric permittivity of the ground. On horizontal polarization the absolute value of ΔP increases with the incident angle (in the range from 0 till 60 degrees). The short correlated roughness decreases the power transmitted into the ground, while the long correlated roughness increases it. The possible explanation of this phenomena is that short correlated roughness scatters the incident wave mainly into a noncoherent component, thus decreasing both coherently reflected and transmitted components. In the contrary, long correlated roughness of the interface leads to redirection of the power from reflected to transmitted component. So, the main danger for GPR is a short correlated roughness, which decreases the transmitted into the ground power and causes additional cross-talk between transmitting and receiving antennas.

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