Chapter 20 Quasi-Resonant Converters

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The resonant switch concept

A quite general idea:

- 1. PWM switch network is replaced by a resonant switch network
- 2. This leads to a quasi-resonant version of the original PWM converter

Example: realization of the switch cell in the buck converter



Two quasi-resonant switch cells



Insert either of the above switch cells into the buck converter, to obtain a ZCS quasi-resonant version of the buck converter. L_r and C_r are small in value, and their resonant frequency f_0 is greater than the switching frequency f_s .

$$f_0 = \frac{1}{2\pi \sqrt{L_r C_r}} = \frac{\omega_0}{2\pi}$$

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20.1 The zero-current-switching quasi-resonant switch cell

Tank inductor L_r in series with transistor: transistor switches at zero crossings of inductor current waveform

Tank capacitor C_r in parallel with diode D_2 : diode switches at zero crossings of capacitor voltage waveform

Two-quadrant switch is required:

Half-wave: Q_1 and D_1 in series, transistor turns off at first zero crossing of current waveform

Full-wave: Q_1 and D_1 in parallel, transistor turns off at second zero crossing of current waveform

Performances of half-wave and full-wave cells differ significantly.



Averaged switch modeling of ZCS cells

It is assumed that the converter filter elements are large, such that their switching ripples are small. Hence, we can make the small ripple approximation as usual, for these elements:

$$i_{2}(t) \approx \left\langle i_{2}(t) \right\rangle_{T_{s}}$$
$$v_{1}(t) \approx \left\langle v_{1}(t) \right\rangle_{T_{s}}$$

In steady state, we can further approximate these quantities by their dc values: $i_{i}(t) \approx L$

$$v_2(t) \approx V_2$$
$$v_1(t) \approx V_1$$

Modeling objective: find the average values of the terminal waveforms

 $\langle v_2(t) \rangle_{T_s}$ and $\langle i_1(t) \rangle_{T_s}$

The switch conversion ratio μ

A generalization of the duty cycle d(t)

The switch conversion ratio μ is the ratio of the average terminal voltages of the switch network. It can be applied to non-PWM switch networks. For the CCM PWM case, $\mu = d$.

If $V/V_g = M(d)$ for a PWM CCM converter, then $V/V_g = M(\mu)$ for the same converter with a switch network having conversion ratio μ .

Generalized switch averaging, and μ , are defined and discussed in Section 10.3.



$$\frac{i_{2}(t) \approx \left\langle i_{2}(t) \right\rangle_{T_{s}}}{v_{1}(t) \approx \left\langle v_{1}(t) \right\rangle_{T_{s}}} \qquad \mu = \frac{\left\langle v_{2}(t) \right\rangle_{T_{s}}}{\left\langle v_{1r}(t) \right\rangle_{T_{s}}} = \frac{\left\langle i_{1}(t) \right\rangle_{T_{s}}}{\left\langle i_{2r}(t) \right\rangle_{T_{s}}}$$

In steady state:

$$i_2(t) \approx I_2 \qquad \qquad \mu = \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

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20.1.1 Waveforms of the half-wave ZCS quasi-resonant switch cell



Subinterval 1

Diode D_2 is initially conducting the filter inductor current I_2 . Transistor Q_1 turns on, and the tank inductor current i_1 starts to increase. So all semiconductor devices conduct during this subinterval, and the circuit reduces to:



Circuit equations: $\frac{di_{1}(t)}{dt} = \frac{V_{1}}{L_{r}} \quad \text{with } i_{1}(0) = 0$ Solution: $i_{1}(t) = \frac{V_{1}}{L_{r}} t = \omega_{0} t \frac{V_{1}}{R_{0}}$ where $R_{0} = \sqrt{\frac{L_{r}}{C_{r}}}$

This subinterval ends when diode D_2 becomes reverse-biased. This occurs at time $\omega_0 t = \alpha$, when $i_1(t) = I_2$.

$$i_1(\alpha) = \alpha \frac{V_1}{R_0} = I_2 \qquad \alpha = \frac{I_2 R_0}{V_1}$$

Subinterval 2

Diode D_2 is off. Transistor Q_1 conducts, and the tank inductor and tank capacitor ring sinusoidally. The circuit reduces to:



The circuit equations are

$$L_r \frac{di_1(\omega_0 t)}{dt} = V_1 - v_2(\omega_0 t) \qquad v_2(\alpha) = 0$$

$$C_r \frac{dv_2(\omega_0 t)}{dt} = i_1(\omega_0 t) - I_2 \qquad i_1(\alpha) = I_2$$

The solution is

$$i_1(\omega_0 t) = I_2 + \frac{V_1}{R_0} \sin\left(\omega_0 t - \alpha\right)$$
$$v_2(\omega_0 t) = V_1 \left(1 - \cos\left(\omega_0 t - \alpha\right)\right)$$

The dc components of these waveforms are the dc solution of the circuit, while the sinusoidal components have magnitudes that depend on the initial conditions and on the characteristic impedance R_0 .

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Subinterval 2 continued

 $i_{1}(\omega_{0}t) = I_{2} + \frac{V_{1}}{R_{0}} \sin\left(\omega_{0}t - \alpha\right)$ $v_{2}(\omega_{0}t) = V_{1}\left(1 - \cos\left(\omega_{0}t - \alpha\right)\right)$ Peak inductor current: $I_{1pk} = I_2 + \frac{V_1}{R_2}$ I_2 2 $\theta = \omega_0 t$ Subinterval: 3 This subinterval ends at the first zero δ crossing of $i_1(t)$. Define β = angular length of subinterval 2. Then $i_1(\alpha + \beta) = I_2 + \frac{V_1}{R_2} \sin(\beta) = 0$ Hence $\beta = \pi + \sin^{-1} \left(\frac{I_2 R_0}{V_1} \right)$ $\sin\left(\beta\right) = -\frac{I_2 R_0}{V}$ $-\frac{\pi}{2} < \sin^{-1}\left(x\right) \le \frac{\pi}{2}$ Must use care to select the correct branch of the arcsine function. Note (from the $i_1(t)$ waveform) that $\beta > \pi$. $I_2 < \frac{V_1}{R}$

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Boundary of zero current switching

If the requirement

$$I_2 < \frac{V_1}{R_0}$$

is violated, then the inductor current never reaches zero. In consequence, the transistor cannot switch off at zero current.

The resonant switch operates with zero current switching only for load currents less than the above value. The characteristic impedance must be sufficiently small, so that the ringing component of the current is greater than the dc load current.

Capacitor voltage at the end of subinterval 2 is

$$v_2(\alpha + \beta) = V_{c1} = V_1 \left(1 + \sqrt{1 - \left(\frac{I_2 R_0}{V_1}\right)^2} \right)$$

Subinterval 3

All semiconductor devices are off. The circuit reduces to:



The circuit equations are

$$C_r \frac{dv_2(\omega_0 t)}{dt} = -I_2$$
$$v_2(\alpha + \beta) = V_{c1}$$

The solution is

$$v_2(\omega_0 t) = V_{c1} - I_2 R_0 \left(\omega_0 t - \alpha - \beta \right)$$

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Subinterval 3 ends when the tank capacitor voltage reaches zero, and diode D_2 becomes forward-biased. Define δ = angular length of subinterval 3. Then

$$v_2(\alpha + \beta + \delta) = V_{c1} - I_2 R_0 \delta = 0$$

$$\delta = \frac{V_{c1}}{I_2 R_0} = \frac{V_1}{I_2 R_0} \left(1 - \sqrt{1 - \left(\frac{I_2 R_0}{V_1}\right)^2} \right)$$

Subinterval 4

Subinterval 4, of angular length ξ , is identical to the diode conduction interval of the conventional PWM switch network.

Diode D_2 conducts the filter inductor current I_2

The tank capacitor voltage $v_2(t)$ is equal to zero.

Transistor Q_1 is off, and the input current $i_1(t)$ is equal to zero.

The length of subinterval 4 can be used as a control variable. Increasing the length of this interval reduces the average output voltage.

Maximum switching frequency

The length of the fourth subinterval cannot be negative, and the switching period must be at least long enough for the tank current and voltage to return to zero by the end of the switching period.

The angular length of the switching period is

$$\omega_0 T_s = \alpha + \beta + \delta + \xi = \frac{2\pi f_0}{f_s} = \frac{2\pi}{F}$$

where the normalized switching frequency F is defined as

$$F = \frac{f_s}{f_0}$$

So the minimum switching period is

$$\omega_0 T_s \ge \alpha + \beta + \delta$$

Substitute previous solutions for subinterval lengths:

$$\frac{2\pi}{F} \ge \frac{I_2 R_0}{V_1} + \pi + \sin^{-1} \left(\frac{I_2 R_0}{V_1} \right) + \frac{V_1}{I_2 R_0} \left(1 - \sqrt{1 - \left(\frac{I_2 R_0}{V_1} \right)^2} \right)$$

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20.1.2 The average terminal waveforms

Averaged switch modeling: we need to determine the average values of $i_1(t)$ and $v_2(t)$. The average switch input current is given by

$$\left\langle i_{1}(t) \right\rangle_{T_{s}} = \frac{1}{T_{s}} \int_{t}^{t+T_{s}} i_{1}(t) dt = \frac{q_{1}+q_{2}}{T_{s}}$$

 q_1 and q_2 are the areas under the current waveform during subintervals 1 and 2. q_1 is given by the triangle area formula:

$$q_1 = \int_0^{\frac{\alpha}{\omega_0}} i_1(t) dt = \frac{1}{2} \left(\frac{\alpha}{\omega_0} \right) \left(I_2 \right)$$

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Charge arguments: computation of q_2

$$q_2 = \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha+\beta}{\omega_0}} i_1(t) dt$$

Node equation for subinterval 2:

$$i_1(t) = i_C(t) + I_2$$

Substitute:

$$q_{2} = \int_{\frac{\alpha}{\omega_{0}}}^{\frac{\alpha+\beta}{\omega_{0}}} i_{C}(t)dt + \int_{\frac{\alpha}{\omega_{0}}}^{\frac{\alpha+\beta}{\omega_{0}}} I_{2}dt$$

Second term is integral of constant I_2 :

$$\int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha+\beta}{\omega_0}} I_2 dt = I_2 \frac{\beta}{\omega_0}$$



Circuit during subinterval 2

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Charge arguments continued

$$q_{2} = \int_{\underline{\alpha}_{0}}^{\underline{\alpha}+\underline{\beta}} i_{C}(t)dt + \int_{\underline{\alpha}_{0}}^{\underline{\alpha}+\underline{\beta}} I_{2}dt$$

First term: integral of the capacitor current over subinterval 2. This can be related to the change in capacitor voltage :

$$\int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha+\beta}{\omega_0}} i_C(t) dt = C \left(v_2 \left(\frac{\alpha+\beta}{\omega_0} \right) - v_2 \left(\frac{\alpha}{\omega_0} \right) \right)$$

$$\int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha+\beta}{\omega_0}} i_C(t) dt = C \left(V_{c1} - 0 \right) = C V_{c1}$$



Substitute results for the two integrals:

$$q_2 = CV_{c1} + I_2 \frac{\beta}{\omega_0}$$

Substitute into expression for average switch input current:

$$\left\langle i_1(t) \right\rangle_{T_s} = \frac{\alpha I_2}{2\omega_0 T_s} + \frac{CV_{c1}}{T_s} + \frac{\beta I_2}{\omega_0 T_s}$$

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Switch conversion ratio µ

$$\mu = \frac{\left\langle i_1(t) \right\rangle_{T_s}}{I_2} = \frac{\alpha}{2\omega_0 T_s} + \frac{CV_{c1}}{I_2 T_s} + \frac{\beta}{\omega_0 T_s}$$

Eliminate α , β , V_{c1} using previous results:

$$\mu = F \frac{1}{2\pi} \left[\frac{1}{2} J_s + \pi + \sin^{-1}(J_s) + \frac{1}{J_s} \left(1 + \sqrt{1 - J_s^2} \right) \right]$$

where

$$J_s = \frac{I_2 R_0}{V_1}$$

Analysis result: switch conversion ratio μ



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Characteristics of the half-wave ZCS resonant switch



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Buck converter containing half-wave ZCS quasi-resonant switch

Conversion ratio of the buck converter is (from inductor volt-second balance):

$$M = \frac{V}{V_g} = \mu$$

For the buck converter,

$$J_s = \frac{IR_0}{V_g}$$

ZCS occurs when

$$I \leq \frac{V_g}{R_0}$$

Output voltage varies over the range $0 \le V \le V_g - \frac{FIR_0}{4\pi}$



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 J_{s}

Boost converter example



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20.1.3 The full-wave ZCS quasi-resonant switch cell



Analysis: full-wave ZCS

Analysis in the full-wave case is nearly the same as in the half-wave case. The second subinterval ends at the second zero crossing of the tank inductor current waveform. The following quantities differ:

$$\beta = \begin{cases} \pi + \sin^{-1} \left(J_s \right) & \text{(half wave)} \\ 2\pi - \sin^{-1} \left(J_s \right) & \text{(full wave)} \end{cases}$$
$$V_{c1} = \begin{cases} V_1 \left(1 + \sqrt{1 - J_s^2} \right) & \text{(half wave)} \\ V_1 \left(1 - \sqrt{1 - J_s^2} \right) & \text{(full wave)} \end{cases}$$

In either case, μ is given by

$$\mu = \frac{\left\langle i_1(t) \right\rangle_{T_s}}{I_2} = \frac{\alpha}{2\omega_0 T_s} + \frac{CV_{c1}}{I_2 T_s} + \frac{\beta}{\omega_0 T_s}$$

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Full-wave cell: switch conversion ratio µ



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20.2 Resonant switch topologies



- Voltage-bidirectional two-quadrant switch for half-wave cell
- Current-bidirectional two-quadrant switch for full-wave cell

Connection of resonant elements:

Can be connected in other ways that preserve high-frequency components of tank waveforms

Connection of tank capacitor

Connection of tank capacitor to two other points at ac ground.

This simply changes the dc component of tank capacitor voltage.

The ac highfrequency components of the tank waveforms are unchanged.



A test to determine the topology of a resonant switch network

Replace converter elements by their high-frequency equivalents:

- Independent voltage source V_g : short circuit
- Filter capacitors: short circuits
- Filter inductors: open circuits

The resonant switch network remains.

If the converter contains a ZCS quasi-resonant switch, then the result of these operations is



Zero-current and zero-voltage switching

ZCS quasi-resonant switch:

- Tank inductor is in series with switch; hence *SW* switches at zero current
- Tank capacitor is in parallel with diode *D*₂; hence *D*₂ switches at zero voltage



Discussion

- Zero voltage switching of D_2 eliminates switching loss arising from D_2 stored charge.
- Zero current switching of SW: device Q_1 and D_1 output capacitances lead to switching loss. In full-wave case, stored charge of diode D_1 leads to switching loss.
- Peak transistor current is $(1 + J_s) V_g/R_0$, or more than twice the PWM value.

20.2.1 The zero-voltage-switching quasi-resonant switch cell



ZVS quasi-resonant switch cell



20.2.2 The ZVS multiresonant switch



20.2.3 Quasi-square-wave resonant switches



A quasi-square-wave ZCS buck with input filter



- The basic ZCS QSW switch cell is restricted to $0 \le \mu \le 0.5$
- Peak transistor current is equal to peak transistor current of PWM cell
- Peak transistor voltage is increased
- Zero-current switching in all semiconductor devices

A quasi-square-wave ZVS buck



- The basic ZVS QSW switch cell is restricted to $0.5 \le \mu \le 1$
- Peak transistor voltage is equal to peak transistor voltage of PWM cell
- Peak transistor current is increased
- Zero-voltage switching in all semiconductor devices

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20.3 Ac modeling of quasi-resonant converters

Use averaged switch modeling technique: apply averaged PWM model, with d replaced by μ

Buck example with full-wave ZCS quasi-resonant cell:



Small-signal ac model



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Low-frequency model

Tank dynamics occur only at frequency near or greater than switching frequency —discard tank elements



-same as PWM buck, with d replaced by F

Example 2: Half-wave ZCS quasi-resonant buck



Small-signal modeling

Perturbation and linearization of $\mu(v_{1r}, i_{2r}, f_s)$:

$$\hat{\mu}(t) = K_v \hat{v}_{1r}(t) + K_i \hat{i}_{2r}(t) + K_c \hat{f}_s(t)$$
with
$$K_v = -\frac{\partial \mu}{\partial j_s} \frac{R_0 I_2}{V_1^2}$$

$$K_i = -\frac{\partial \mu}{\partial j_s} \frac{R_0}{V_1}$$

$$\frac{\partial \mu}{\partial j_s} = \frac{F_s}{2\pi f_0} \left(\frac{1}{2} - \frac{1 + \sqrt{1 - J_s^2}}{J_s^2}\right)$$

$$K_c = \frac{\mu_0}{F_s}$$

Linearized terminal equations of switch network:

$$\hat{i}_{1}(t) = \hat{\mu}(t) I_{2} + \hat{i}_{2r}(t) \mu_{0}$$
$$\hat{v}_{2}(t) = \mu_{0} \hat{v}_{1r}(t) + \hat{\mu}(t) V_{1}$$

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Equivalent circuit model



Low frequency model: set tank elements to zero



Predicted small-signal transfer functions Half-wave ZCS buck

$$G_{vg}(s) = G_{g0} \frac{1}{1 + \frac{1}{Q} \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$
$$G_{vc}(s) = G_{c0} \frac{1}{1 + \frac{1}{Q} \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

Full-wave: poles and zeroes are same as PWM

Half-wave: effective feedback reduces Q-factor and dc gains



20.4 Summary of key points

- 1. In a resonant switch converter, the switch network of a PWM converter is replaced by a switch network containing resonant elements. The resulting hybrid converter combines the properties of the resonant switch network and the parent PWM converter.
- 2. Analysis of a resonant switch cell involves determination of the switch conversion ratio μ . The resonant switch waveforms are determined, and are then averaged. The switch conversion ratio μ is a generalization of the PWM CCM duty cycle *d*. The results of the averaged analysis of PWM converters operating in CCM can be directly adapted to the related resonant switch converter, simply by replacing *d* with μ .
- 3. In the zero-current-switching quasi-resonant switch, diode D_2 operates with zero-voltage switching, while transistor Q_1 and diode D_1 operate with zero-current switching.

- 4. In the zero-voltage-switching quasi-resonant switch, the transistor Q_1 and diode D_1 operate with zero-voltage switching, while diode D_2 operates with zero-current switching.
- 5. Full-wave versions of the quasi-resonant switches exhibit very simple control characteristics: the conversion ratio μ is essentially independent of load current. However, these converters exhibit reduced efficiency at light load, due to the large circulating currents. In addition, significant switching loss is incurred due to the recovered charge of diode D_1 .
- 6. Half-wave versions of the quasi-resonant switch exhibit conversion ratios that are strongly dependent on the load current. These converters typically operate with wide variations of switching frequency.
- 7. In the zero-voltage-switching multiresonant switch, all semiconductor devices operate with zero-voltage switching. In consequence, very low switching loss is observed.

- 8. In the quasi-square-wave zero-voltage-switching resonant switches, all semiconductor devices operate with zero-voltage switching, and with peak voltages equal to those of the parent PWM converter. The switch conversion ratio is restricted to the range $0.5 \le \mu \le 1$.
- 9. The small-signal ac models of converters containing resonant switches are similar to the small-signal models of their parent PWM converters. The averaged switch modeling approach can be employed to show that the quantity d(t) is simply replaced by $\mu(t)$.
- 10. In the case of full-wave quasi-resonant switches, μ depends only on the switching frequency, and therefore the transfer function poles and zeroes are identical to those of the parent PWM converter.
- 11. In the case of half-wave quasi-resonant switches, as well as other types of resonant switches, the conversion ratio μ is a strong function of the switch terminal quantities v_1 and i_2 . This leads to effective feedback, which modifies the poles, zeroes, and gains of the transfer functions.