## Chapter 20 <br> Quasi-Resonant Converters

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## The resonant switch concept

A quite general idea:

1. PWM switch network is replaced by a resonant switch network
2. This leads to a quasi-resonant version of the original PWM converter

Example: realization of the switch cell in the buck converter


## Two quasi-resonant switch cells



Insert either of the above switch cells into the buck converter, to obtain a ZCS quasi-resonant version of the buck converter. $L_{r}$ and $C_{r}$ are small in value, and their resonant frequency $f_{0}$ is greater than the switching frequency $f_{s}$.

$$
f_{0}=\frac{1}{2 \pi \sqrt{L_{r} C_{r}}}=\frac{\omega_{0}}{2 \pi}
$$



### 20.1 The zero-current-switching quasi-resonant switch cell

Tank inductor $L_{r}$ in series with transistor: transistor switches at zero crossings of inductor current waveform

Tank capacitor $C_{r}$ in parallel with diode $D_{2}$ : diode switches at zero crossings of capacitor voltage waveform

Two-quadrant switch is required:
Half-wave: $Q_{1}$ and $D_{1}$ in series, transistor turns off at first zero crossing of current waveform

Full-wave: $Q_{1}$ and $D_{1}$ in parallel, transistor turns off at second zero crossing of current waveform

Performances of half-wave and full-wave cells differ significantly.


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## Averaged switch modeling of ZCS cells

It is assumed that the converter filter elements are large, such that their switching ripples are small. Hence, we can make the small ripple approximation as usual, for these elements:

$$
\begin{aligned}
& i_{2}(t) \approx\left\langle i_{2}(t)\right\rangle_{T_{s}} \\
& v_{1}(t) \approx\left\langle v_{1}(t)\right\rangle_{T_{s}}
\end{aligned}
$$

In steady state, we can further approximate these quantities by their dc values:

$$
\begin{aligned}
& i_{2}(t) \approx I_{2} \\
& v_{1}(t) \approx V_{1}
\end{aligned}
$$

Modeling objective: find the average values of the terminal waveforms

$$
\left\langle v_{2}(t)\right\rangle_{T_{s}} \text { and }\left\langle i_{1}(t)\right\rangle_{T_{s}}
$$

## The switch conversion ratio $\mu$

A generalization of the duty cycle $d(t)$

The switch conversion ratio $\mu$ is the ratio of the average terminal voltages of the switch network. It can be applied to non-PWM switch networks. For the CCM PWM case, $\mu=d$.

If $V / V_{g}=M(d)$ for a PWM CCM converter, then $V / V_{g}=M(\mu)$ for the same converter with a switch network having conversion ratio $\mu$.
Generalized switch averaging, and $\mu$, are defined and discussed in Section 10.3.


$$
\begin{aligned}
& i_{2}(t) \approx\left\langle i_{2}(t)\right\rangle_{T_{s}} \\
& v_{1}(t) \approx\left\langle v_{1}(t)\right\rangle_{T_{s}}
\end{aligned} \quad \mu=\frac{\left\langle v_{2}(t)\right\rangle_{T_{s}}}{\left\langle v_{1 r}(t)\right\rangle_{T_{s}}}=\frac{\left\langle i_{1}(t)\right\rangle_{T_{s}}}{\left\langle i_{2 r}(t)\right\rangle_{T_{s}}}
$$

In steady state:

$$
\begin{aligned}
& i_{2}(t) \approx I_{2} \\
& v_{1}(t) \approx V_{1}
\end{aligned} \quad \mu=\frac{V_{2}}{V_{1}}=\frac{I_{1}}{I_{2}}
$$

### 20.1.1 Waveforms of the half-wave ZCS quasi-resonant switch cell

The half-wave ZCS quasi-resonant switch cell, driven by the terminal quantities $\left\langle v_{1}(t)\right\rangle_{T s}$ and $\left\langle i_{2}(t)\right\rangle_{T s}$.


Each switching period contains four subintervals

Waveforms:


## Subinterval 1

Diode $D_{2}$ is initially conducting the filter inductor current $I_{2}$. Transistor $Q_{1}$ turns on, and the tank inductor current $i_{1}$ starts to increase. So all semiconductor devices conduct during this subinterval, and the circuit reduces to:


Circuit equations:

$$
\frac{d i_{1}(t)}{d t}=\frac{V_{1}}{L_{r}} \quad \text { with } i_{1}(0)=0
$$

Solution: $\quad i_{1}(t)=\frac{V_{1}}{L_{r}} t=\omega_{0} t \frac{V_{1}}{R_{0}}$
where $\quad R_{0}=\sqrt{\frac{L_{r}}{C_{r}}}$

This subinterval ends when diode $D_{2}$ becomes reverse-biased. This occurs at time $\omega_{0} t=\alpha$, when $i_{1}(t)=I_{2}$.

$$
i_{1}(\alpha)=\alpha \frac{V_{1}}{R_{0}}=I_{2} \quad \alpha=\frac{I_{2} R_{0}}{V_{1}}
$$

## Subinterval 2

Diode $D_{2}$ is off. Transistor $Q_{1}$ conducts, and the tank inductor and tank capacitor ring sinusoidally. The circuit reduces to:


The circuit equations are

$$
\begin{array}{ll}
L_{r} \frac{d i_{1}\left(\omega_{0} t\right)}{d t}=V_{1}-v_{2}\left(\omega_{0} t\right) & v_{2}(\alpha)=0 \\
C_{r} \frac{d v_{2}\left(\omega_{0} t\right)}{d t}=i_{1}\left(\omega_{0} t\right)-I_{2} & i_{1}(\alpha)=I_{2}
\end{array}
$$

The solution is

$$
\begin{aligned}
& i_{1}\left(\omega_{0} t\right)=I_{2}+\frac{V_{1}}{R_{0}} \sin \left(\omega_{0} t-\alpha\right) \\
& v_{2}\left(\omega_{0} t\right)=V_{1}\left(1-\cos \left(\omega_{0} t-\alpha\right)\right)
\end{aligned}
$$

The dc components of these waveforms are the dc solution of the circuit, while the sinusoidal components have magnitudes that depend on the initial conditions and on the characteristic impedance $R_{0}$.

## Subinterval 2

## continued

Peak inductor current:

$$
I_{1 p k}=I_{2}+\frac{V_{1}}{R_{0}}
$$

$$
\begin{aligned}
& i_{1}\left(\omega_{0} t\right)=I_{2}+\frac{V_{1}}{R_{0}} \sin \left(\omega_{0} t-\alpha\right) \\
& v_{2}\left(\omega_{0} t\right)=V_{1}\left(1-\cos \left(\omega_{0} t-\alpha\right)\right)
\end{aligned}
$$

This subinterval ends at the first zero crossing of $i_{1}(t)$. Define $\beta=$ angular length of subinterval 2. Then

$$
\begin{aligned}
& i_{1}(\alpha+\beta)=I_{2}+\frac{V_{1}}{R_{0}} \sin (\beta)=0 \\
& \sin (\beta)=-\frac{I_{2} R_{0}}{V_{1}}
\end{aligned}
$$

Must use care to select the correct branch of the arcsine function. Note (from the $i_{1}(t)$ waveform) that $\beta>\pi$.


Hence

$$
\begin{aligned}
& \beta=\pi+\sin ^{-1}\left(\frac{I_{2} R_{0}}{V_{1}}\right) \\
& -\frac{\pi}{2}<\sin ^{-1}(x) \leq \frac{\pi}{2} \\
& I_{2}<\frac{V_{1}}{R_{0}}
\end{aligned}
$$

## Boundary of zero current switching

If the requirement

$$
I_{2}<\frac{V_{1}}{R_{0}}
$$

is violated, then the inductor current never reaches zero. In consequence, the transistor cannot switch off at zero current.

The resonant switch operates with zero current switching only for load currents less than the above value. The characteristic impedance must be sufficiently small, so that the ringing component of the current is greater than the dc load current.

Capacitor voltage at the end of subinterval 2 is

$$
v_{2}(\alpha+\beta)=V_{c 1}=V_{1}\left(1+\sqrt{1-\left(\frac{I_{2} R_{0}}{V_{1}}\right)^{2}}\right)
$$

## Subinterval 3

All semiconductor devices are off. The circuit reduces to:


The circuit equations are

$$
\begin{aligned}
& C_{r} \frac{d v_{2}\left(\omega_{0} t\right)}{d t}=-I_{2} \\
& v_{2}(\alpha+\beta)=V_{c 1}
\end{aligned}
$$

Subinterval 3 ends when the tank capacitor voltage reaches zero, and diode $D_{2}$ becomes forward-biased.
Define $\delta=$ angular length of subinterval 3. Then

$$
v_{2}(\alpha+\beta+\delta)=V_{c 1}-I_{2} R_{0} \delta=0
$$

$$
\delta=\frac{V_{c 1}}{I_{2} R_{0}}=\frac{V_{1}}{I_{2} R_{0}}\left(1-\sqrt{1-\left(\frac{I_{2} R_{0}}{V_{1}}\right)^{2}}\right)
$$

The solution is

$$
v_{2}\left(\omega_{0} t\right)=V_{c 1}-I_{2} R_{0}\left(\omega_{0} t-\alpha-\beta\right)
$$

## Subinterval 4

Subinterval 4, of angular length $\xi$, is identical to the diode conduction interval of the conventional PWM switch network.

Diode $D_{2}$ conducts the filter inductor current $I_{2}$
The tank capacitor voltage $v_{2}(t)$ is equal to zero.
Transistor $Q_{1}$ is off, and the input current $i_{1}(t)$ is equal to zero.
The length of subinterval 4 can be used as a control variable. Increasing the length of this interval reduces the average output voltage.

## Maximum switching frequency

The length of the fourth subinterval cannot be negative, and the switching period must be at least long enough for the tank current and voltage to return to zero by the end of the switching period.

The angular length of the switching period is

$$
\omega_{0} T_{s}=\alpha+\beta+\delta+\xi=\frac{2 \pi f_{0}}{f_{s}}=\frac{2 \pi}{F}
$$

where the normalized switching frequency $F$ is defined as

$$
F=\frac{f_{s}}{f_{0}}
$$

So the minimum switching period is

$$
\omega_{0} T_{s} \geq \alpha+\beta+\delta
$$

Substitute previous solutions for subinterval lengths:

$$
\frac{2 \pi}{F} \geq \frac{I_{2} R_{0}}{V_{1}}+\pi+\sin ^{-1}\left(\frac{I_{2} R_{0}}{V_{1}}\right)+\frac{V_{1}}{I_{2} R_{0}}\left(1-\sqrt{1-\left(\frac{I_{2} R_{0}}{V_{1}}\right)^{2}}\right)
$$

### 20.1.2 The average terminal waveforms

Averaged switch modeling: we need to determine the average values of $i_{1}(t)$ and $v_{2}(t)$. The average switch input current is given by

$$
\left\langle i_{1}(t)\right\rangle_{T_{s}}=\frac{1}{T_{s}} \int_{t}^{t+T_{s}} i_{1}(t) d t=\frac{q_{1}+q_{2}}{T_{s}}
$$


$q_{1}$ and $q_{2}$ are the areas under the current waveform during subintervals 1 and 2. $q_{1}$ is given by the triangle area formula:

$$
q_{1}=\int_{0}^{\frac{\alpha}{\omega_{0}}} i_{1}(t) d t=\frac{1}{2}\left(\frac{\alpha}{\omega_{0}}\right)\left(I_{2}\right)
$$



## Charge arguments: computation of $q_{2}$

$$
q_{2}=\int_{\frac{\alpha}{\omega_{0}}}^{\frac{\alpha+\beta}{\omega_{0}}} i_{1}(t) d t
$$

Node equation for subinterval 2 :

$$
i_{1}(t)=i_{C}(t)+I_{2}
$$

Substitute:


$$
q_{2}=\int_{\frac{\alpha}{\omega_{0}}}^{\frac{\alpha+\beta}{\omega_{0}}} i_{C}(t) d t+\int_{\frac{\alpha}{\omega_{0}}}^{\frac{\alpha+\beta}{\omega_{0}}} I_{2} d t
$$

Second term is integral of constant $I_{2}$ :

$$
\int_{\frac{\alpha}{\omega_{0}}}^{\frac{\alpha+\beta}{\omega_{0}}} I_{2} d t=I_{2} \frac{\beta}{\omega_{0}}
$$



Circuit during subinterval 2

## Charge arguments <br> continued

$$
q_{2}=\int_{\frac{\alpha}{\omega_{0}}}^{\frac{\alpha+\beta}{\omega_{0}}} i_{C}(t) d t+\int_{\frac{\alpha}{\omega_{0}}}^{\frac{\alpha+\beta}{\omega_{0}}} I_{2} d t
$$

First term: integral of the capacitor current over subinterval 2 . This can be related to the change in capacitor voltage :

$$
\begin{aligned}
& \int_{\frac{\alpha}{\omega_{0}}}^{\frac{\alpha+\beta}{\omega_{0}}} i_{C}(t) d t=C\left(v_{2}\left(\frac{\alpha+\beta}{\omega_{0}}\right)-v_{2}\left(\frac{\alpha}{\omega_{0}}\right)\right) \\
& \int_{\frac{\alpha}{\omega_{0}}}^{\frac{\alpha+\beta}{\omega_{0}}} i_{C}(t) d t=C\left(V_{c 1}-0\right)=C V_{c 1}
\end{aligned}
$$



Substitute results for the two integrals:

$$
q_{2}=C V_{c 1}+I_{2} \frac{\beta}{\omega_{0}}
$$

Substitute into expression for average switch input current:

$$
\left\langle i_{1}(t)\right\rangle_{T_{s}}=\frac{\alpha I_{2}}{2 \omega_{0} T_{s}}+\frac{C V_{c 1}}{T_{s}}+\frac{\beta I_{2}}{\omega_{0} T_{s}}
$$

## Switch conversion ratio $\mu$

$$
\mu=\frac{\left\langle i_{1}(t)\right\rangle_{T_{s}}}{I_{2}}=\frac{\alpha}{2 \omega_{0} T_{s}}+\frac{C V_{c 1}}{I_{2} T_{s}}+\frac{\beta}{\omega_{0} T_{s}}
$$

Eliminate $\alpha, \beta, V_{c 1}$ using previous results:

$$
\mu=F \frac{1}{2 \pi}\left[\left.\frac{1}{2} J_{s}+\pi+\sin ^{-1}\left(J_{s}\right)+\frac{1}{J_{s}}\left(1+\sqrt{1-J_{s}^{2}}\right) \right\rvert\,\right.
$$

where

$$
J_{s}=\frac{I_{2} R_{0}}{V_{1}}
$$

## Analysis result: switch conversion ratio $\mu$

Switch conversion ratio: $\quad \mu=F \frac{1}{2 \pi}\left[\frac{1}{2} J_{s}+\pi+\sin ^{-1}\left(J_{s}\right)+\frac{1}{J_{s}}\left(1+\sqrt{1-J_{s}^{2}}\right)\right]$

$$
\text { with } J_{s}=\frac{I_{2} R_{0}}{V_{1}}
$$

This is of the form

$$
\begin{gathered}
\mu=F P_{\frac{1}{2}}\left(J_{s}\right) \\
P_{\frac{1}{2}}\left(J_{s}\right)=\frac{1}{2 \pi}\left|\frac{1}{2} J_{s}+\pi+\sin ^{-1}\left(J_{s}\right)+\frac{1}{J_{s}}\left(1+\sqrt{1-J_{s}^{2}}\right)\right|
\end{gathered}
$$

## Characteristics of the half-wave ZCS resonant switch



Switch
characteristics:

$$
\mu=F P_{\frac{1}{2}}\left(J_{s}\right)
$$

Mode boundary:

$$
\begin{aligned}
& J_{s} \leq 1 \\
& \mu \leq 1-\frac{J_{s} F}{4 \pi}
\end{aligned}
$$

Buck converter containing half-wave ZCS quasi-resonant switch

Conversion ratio of the buck converter is (from inductor volt-second balance):

$$
M=\frac{V}{V_{g}}=\mu
$$

For the buck converter,

$$
J_{s}=\frac{I R_{0}}{V_{g}}
$$

ZCS occurs when

$$
I \leq \frac{V_{g}}{R_{0}}
$$

Output voltage varies over the range

$$
0 \leq V \leq V_{g}-\frac{F I R_{0}}{4 \pi}
$$



## Boost converter example

For the boost converter,

$$
\begin{aligned}
M & =\frac{V}{V_{g}}=\frac{1}{1-\mu} \\
J_{s} & =\frac{I_{2} R_{0}}{V_{1}}=\frac{I_{g} R_{0}}{V} \\
I_{g} & =\frac{I}{1-\mu}
\end{aligned}
$$

Half-wave ZCS equations:


$$
\begin{aligned}
& \mu=F P_{\frac{1}{2}}\left(J_{s}\right) \\
& P_{\frac{1}{2}}\left(J_{s}\right)=\frac{1}{2 \pi}\left|\frac{1}{2} J_{s}+\pi+\sin ^{-1}\left(J_{s}\right)+\frac{1}{J_{s}}\left(1+\sqrt{1-J_{s}^{2}}\right)\right|
\end{aligned}
$$

### 20.1.3 The full-wave ZCS quasi-resonant switch cell



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## Analysis: full-wave ZCS

Analysis in the full-wave case is nearly the same as in the half-wave case. The second subinterval ends at the second zero crossing of the tank inductor current waveform. The following quantities differ:

$$
\begin{aligned}
& \beta= \begin{cases}\pi+\sin ^{-1}\left(J_{s}\right) & \text { (half wave) } \\
2 \pi-\sin ^{-1}\left(J_{s}\right) & \text { (full wave) }\end{cases} \\
& V_{c 1}= \begin{cases}V_{1}\left(1+\sqrt{1-J_{s}^{2}}\right) & \text { (half wave) } \\
V_{1}\left(1-\sqrt{1-J_{s}^{2}}\right) & \text { (full wave) }\end{cases}
\end{aligned}
$$

In either case, $\mu$ is given by

$$
\mu=\frac{\left\langle i_{1}(t)\right\rangle_{T_{s}}}{I_{2}}=\frac{\alpha}{2 \omega_{0} T_{s}}+\frac{C V_{c 1}}{I_{2} T_{s}}+\frac{\beta}{\omega_{0} T_{s}}
$$

## Full-wave cell: switch conversion ratio $\mu$

$$
\begin{aligned}
& P_{1}\left(J_{s}\right)=\frac{1}{2 \pi}\left|\frac{1}{2} J_{s}+2 \pi-\sin ^{-1}\left(J_{s}\right)+\frac{1}{J_{s}}\left(1-\sqrt{1-J_{s}^{2}}\right)\right| \\
& \mu=F P_{1}\left(J_{s}\right) \\
& \text { case: } P_{1} \text { can be } \\
& \text { ted as } \\
& P_{1}\left(J_{s}\right) \approx 1 \\
& \mu \approx F=\frac{1}{f_{s}} \\
& f_{0}
\end{aligned}
$$

Full-wave case: $P_{1}$ can be approximated as
so

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### 20.2 Resonant switch topologies

Basic ZCS switch cell:

SPST switch $S W$ :


- Voltage-bidirectional two-quadrant switch for half-wave cell
- Current-bidirectional two-quadrant switch for full-wave cell

Connection of resonant elements:
Can be connected in other ways that preserve high-frequency components of tank waveforms

## Connection of tank capacitor

Connection of tank capacitor to two other points at ac ground.
This simply changes the dc component of tank capacitor voltage.

The ac highfrequency components of the tank waveforms are unchanged.


## A test to determine the topology of a resonant switch network

Replace converter elements by their high-frequency equivalents:

- Independent voltage source $V_{g}$ : short circuit
- Filter capacitors: short circuits
- Filter inductors: open circuits

The resonant switch network remains.

If the converter contains a ZCS quasi-resonant switch, then the result of these operations is


## Zero-current and zero-voltage switching

ZCS quasi-resonant switch:

- Tank inductor is in series with switch; hence $S W$ switches at zero current
- Tank capacitor is in parallel with diode $D_{2}$; hence $D_{2}$ switches at zero voltage


Discussion

- Zero voltage switching of $D_{2}$ eliminates switching loss arising from $D_{2}$ stored charge.
- Zero current switching of SW: device $Q_{1}$ and $D_{1}$ output capacitances lead to switching loss. In full-wave case, stored charge of diode $D_{1}$ leads to switching loss.
- Peak transistor current is $\left(1+J_{s}\right) V_{g} / R_{0}$, or more than twice the PWM value.


### 20.2.1 The zero-voltage-switching quasi-resonant switch cell

When the previously-described operations are followed, then the converter reduces to

A full-wave version based on the PWM buck converter:


## ZVS quasi-resonant switch cell

Switch conversion ratio

$$
\begin{array}{ll}
\mu=1-F P_{\frac{1}{2}}\left(\frac{1}{J_{s}}\right) & \text { half-wave } \\
\mu=1-F P_{1}\left(\frac{1}{J_{s}}\right) & \text { full-wave }
\end{array}
$$

ZVS boundary

$$
J_{s} \geq 1
$$

peak transistor voltage $V_{c r, p k}=\left(1+J_{s}\right) V_{1}$
A problem with the quasi-resonant ZVS switch cell: peak transistor voltage becomes very large when zero voltage switching is required for a large range of load currents.

### 20.2.2 The ZVS multiresonant switch

When the previously-described operations are followed, then the converter reduces to

A half-wave version based on the
 PWM buck converter:


### 20.2.3 Quasi-square-wave resonant switches

When the previouslydescribed operations are followed, then the converter reduces to

ZCS


ZVS


## A quasi-square-wave ZCS buck with input filter



- The basic ZCS QSW switch cell is restricted to $0 \leq \mu \leq 0.5$
- Peak transistor current is equal to peak transistor current of PWM cell
- Peak transistor voltage is increased
- Zero-current switching in all semiconductor devices


## A quasi-square-wave ZVS buck



- The basic ZVS QSW switch cell is restricted to $0.5 \leq \mu \leq 1$
- Peak transistor voltage is equal to peak transistor voltage of PWM cell
- Peak transistor current is increased
- Zero-voltage switching in all semiconductor devices


### 20.3 Ac modeling of quasi-resonant converters

Use averaged switch modeling technique: apply averaged PWM model, with d replaced by $\mu$

Buck example with full-wave ZCS quasi-resonant cell:


## Small-signal ac model

Averaged switch equations:

$$
\begin{aligned}
& \left\langle v_{2}(t)\right\rangle_{T_{s}}=\mu\left\langle v_{1 r}(t)\right\rangle_{T_{s}} \\
& \left\langle i_{1}(t)\right\rangle_{T_{s}}=\mu\left\langle i_{2 r}(t)\right\rangle_{T_{s}}
\end{aligned}
$$

Resulting ac model:


## Low-frequency model

Tank dynamics occur only at frequency near or greater than switching frequency -discard tank elements

—same as PWM buck, with d replaced by F

## Example 2: Half-wave ZCS quasi-resonant buck

Now, $\mu$ depends on $j_{s}: \quad \mu(t)=\frac{f_{s}(t)}{f_{0}} P_{\frac{1}{2}}\left(j_{s}(t)\right) \quad j_{s}(t)=R_{0} \frac{\left\langle i_{2 r}(t)\right\rangle_{T_{s}}}{\left\langle v_{1 r}(t)\right\rangle_{T_{s}}}$


## Small-signal modeling

Perturbation and linearization of $\mu\left(v_{1 r}, i_{2 r}, f_{s}\right)$ :

$$
\mathfrak{\mu}(t)=K_{v} \hat{v}_{1 r}(t)+K_{i} \hat{i}_{2 r}(t)+K_{c} \hat{f}_{s}(t)
$$

with $\quad K_{v}=-\frac{\partial \mu}{\partial j_{s}} \frac{R_{0} I_{2}}{V_{1}^{2}}$

$$
\begin{aligned}
K_{i} & =-\frac{\partial \mu}{\partial j_{s}} \frac{R_{0}}{V_{1}} \\
K_{c} & =\frac{\mu_{0}}{F_{s}}
\end{aligned}
$$

Linearized terminal equations of switch network:

$$
\begin{aligned}
& \hat{i}_{1}(t)=\hat{\mu}(t) I_{2}+\hat{i}_{2 r}(t) \mu_{0} \\
& \hat{v}_{2}(t)=\mu_{0} \hat{v}_{1 r}(t)+\hat{\mu}(t) V_{1}
\end{aligned}
$$

## Equivalent circuit model



## Low frequency model: set tank elements to zero



## Predicted small-signal transfer functions

Half-wave ZCS buck

$$
\begin{aligned}
& G_{v g}(s)=G_{g 0} \frac{1}{1+\frac{1}{Q} \frac{s}{\omega_{0}}+\left(\frac{s}{\omega_{0}}\right)^{2}} \\
& G_{v c}(s)=G_{c 0} \frac{1}{1+\frac{1}{Q} \frac{s}{\omega_{0}}+\left(\frac{s}{\omega_{0}}\right)^{2}}
\end{aligned}
$$

Full-wave: poles and zeroes are same as PWM

Half-wave: effective feedback reduces Q-factor and dc gains

$$
\begin{aligned}
G_{g 0} & =\frac{\mu_{0}+K_{v} V_{g}}{1+\frac{K_{i} V_{g}}{R}} \\
G_{c 0} & =\frac{K_{c} V_{g}}{1+\frac{K_{i} V_{g}}{R}} \\
\omega_{0} & =\sqrt{\frac{1+\frac{K_{i} V_{g}}{R}}{L_{r} C_{r}}} \\
Q & =\frac{\sqrt{1+\frac{K_{i} V_{g}}{R}}}{\frac{R_{0}}{R}+K_{i} V_{g} \frac{R}{R_{0}}} \\
R_{0} & =\sqrt{\frac{L_{r}}{C_{r}}}
\end{aligned}
$$

### 20.4 Summary of key points

1. In a resonant switch converter, the switch network of a PWM converter is replaced by a switch network containing resonant elements. The resulting hybrid converter combines the properties of the resonant switch network and the parent PWM converter.
2. Analysis of a resonant switch cell involves determination of the switch conversion ratio $\mu$. The resonant switch waveforms are determined, and are then averaged. The switch conversion ratio $\mu$ is a generalization of the PWM CCM duty cycle $d$. The results of the averaged analysis of PWM converters operating in CCM can be directly adapted to the related resonant switch converter, simply by replacing $d$ with $\mu$.
3. In the zero-current-switching quasi-resonant switch, diode $D_{2}$ operates with zero-voltage switching, while transistor $Q_{1}$ and diode $D_{1}$ operate with zero-current switching.

## Summary of key points

4. In the zero-voltage-switching quasi-resonant switch, the transistor $Q_{1}$ and diode $D_{1}$ operate with zero-voltage switching, while diode $D_{2}$ operates with zero-current switching.
5. Full-wave versions of the quasi-resonant switches exhibit very simple control characteristics: the conversion ratio $\mu$ is essentially independent of load current. However, these converters exhibit reduced efficiency at light load, due to the large circulating currents. In addition, significant switching loss is incurred due to the recovered charge of diode $D_{1}$.
6. Half-wave versions of the quasi-resonant switch exhibit conversion ratios that are strongly dependent on the load current. These converters typically operate with wide variations of switching frequency.
7. In the zero-voltage-switching multiresonant switch, all semiconductor devices operate with zero-voltage switching. In consequence, very low switching loss is observed.

## Summary of key points

8. In the quasi-square-wave zero-voltage-switching resonant switches, all semiconductor devices operate with zero-voltage switching, and with peak voltages equal to those of the parent PWM converter. The switch conversion ratio is restricted to the range $0.5 \leq \mu \leq 1$.
9. The small-signal ac models of converters containing resonant switches are similar to the small-signal models of their parent PWM converters. The averaged switch modeling approach can be employed to show that the quantity $d(t)$ is simply replaced by $\mu(t)$.
10. In the case of full-wave quasi-resonant switches, $\mu$ depends only on the switching frequency, and therefore the transfer function poles and zeroes are identical to those of the parent PWM converter.
11. In the case of half-wave quasi-resonant switches, as well as other types of resonant switches, the conversion ratio $\mu$ is a strong function of the switch terminal quantities $v_{1}$ and $i_{2}$. This leads to effective feedback, which modifies the poles, zeroes, and gains of the transfer functions.
