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BY MODIFYING FILTER-DESIGN EQUATIONS TO IMPLEMENT

EXACT COMPONENT VALUES, YOU CAN MINIMIZE FILTER-

RESPONSE DISTORTION AND STATISTICALLY PREDICT

BEHAVIOR.

Designing RC active filters with standard-component values

ACTIVE FILTERS ARE UBIQUITOUS in electronic design today, performing signal-frequency manipulation and conditioning for audio, IF (intermediate frequency), and digital-signal processing. The success of active filters is due primarily to integration capability and the extensive body of theoretical knowledge. Although DSPs can outperform active filters in dynamic range, active filters can achieve good performance with significantly lower power demands.

Engineers have many vendor and product options when implementing an active filter. With the advent of switching technology, you can use continuous-time filters with good accuracy in applications up to video frequencies (**Reference 1**). Semiconductor process improvements have made it possible and perhaps even economically mandatory to implement entire analog-signal-processing blocks, including filters, on silicon.

Despite the availability of active-filter ICs, most engineers still resort to RC (resistor/capacitor) active implementations consisting of operational amplifiers, resistors, and capacitors. Indeed, the popularity of RC active filters has not diminished since their heyday in the 1970s. The fact that active-filter development is mature helps engineers; active-filter textbooks and "cookbooks" abound. Furthermore, op-amp technology has advanced to meet the seemingly contradictory demands of high speed and low power.

Op amps are often the most critical elements in any RC-active-filter design and deserve much attention (see **sidebar** "Critical op-amp parameters"). Voltage-feedback op amps are still the mainstay of RC active filters, but current-feedback topologies offer gain that's independent of bandwidth, as well as good current outputs at high frequencies (**Reference 2**). With more designs concentrating on portable systems, low-voltage CMOS op amps offer rail-to-rail voltage-handling capability down to power supply voltages of 3V or less (**Reference 3**).

Op amps are not the only critical aspects of an active filter, however. Filter performance is directly related to the various available realization forms available (**Reference 4**). These forms are cascade, multiple feedback, and direct. In cascade realization, biquad structures (filters implementing complex poles or zeros) are cascaded together with independent, per-stage tuning. In multiple-feedback, or "multiloop," "follow-the-leader," or "state-variable" realization, biquad structures and gain blocks sum at different points in the circuit path to achieve the desired response. You determine parameters using a computer to solve a system of linear but underconstrained equations, and you can reduce overall sensitivities through subsequent optimization.

Direct realization is a realization of RLC (resistor/inductor/capacitor)-transfer functions in which you replace certain elements by synthetic elements composed of one or more op amps and other elements. The simulated inductor, for example, replaces

Figure 1
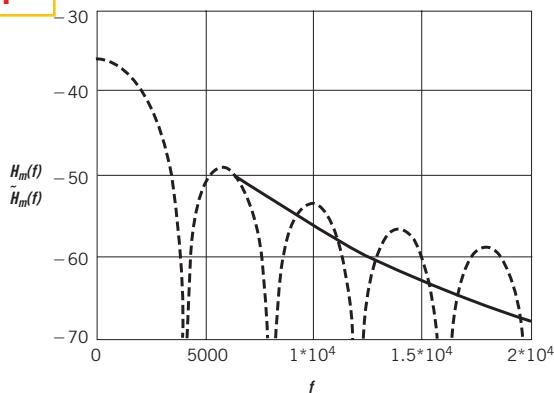
The ideal matched-filter response (dotted line) and the approximating filter response (solid line) out to 20 kHz for a 2-kbps data rate shows good correspondence between ideal and approximating filtering response out to the first side lobe.

the inductor by a combination of a gyrator and a capacitor or an impedance converter and a resistor. An alternative is to replace certain elements by frequency-dependent negative resistors.

The cascade realization is by far the easiest to implement and finds extensive use in most applications. However, it suffers from poor sensitivity, due to the isolation between blocks whereby different stages cannot provide compensation for each other. You should consider cascade and
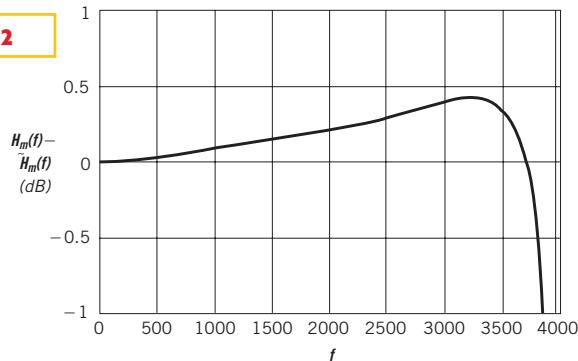
The deviation of the approximating filter from the ideal filter in the main lobe, containing 92% of the energy, is well within 1 dB almost to the notch frequency.

multiple-feedback alternatives when the requirements call for high-Q, low-drift filters.

Of the variety of available biquad topologies for cascade implementation, only a few select configurations have found favor with engineers. Among the most popular are Sallen-Key filters, in which the time constants are decoupled, meaning that the filter has zero active-frequency sensitivity (**Reference 5**). However,

CRITICAL OP-AMP PARAMETERS

Op amps are the most important elements of an active RC filter. Fortunately, manufacturers have provided extensive documentation of the electrical characteristics of op amps. However, the amount of information is often considerable, and engineers must concentrate on the most important characteristics of the design. This article considers the critical op-amp parameters that are useful for high-frequency designs. In addition, assume that the design is implementing voltage-feedback op amps.

GBW (gain-bandwidth) product is typically the first figure of merit that engineers consider in specifying an op amp. GBW is the product of the open-loop gain of the op amp and the open-loop bandwidth. The purpose of GBW is to determine whether the op amp can meet a frequency requirement at the specified gain: The higher the gain required, the less the bandwidth the op amp can support. The advantage of implementing high gains is the sensitivity reduction of the filter gain to amplifier gain.

GBW is only a partial indicator of the amplifier's capability to handle high-frequency signals. An equally important parameter to determine the potential speed of an amplifier is its slew rate. Slew rate is the maximum rate of change of output voltage under large-signal conditions. If you write the output voltage, v_o, of the op amp as $v_o = V_p \cos\omega t$, so that $dv_o/dt = \omega V_p \sin\omega t$, then the slew rate equals

$$S = dv_o/dt\bigg|_{max} = \omega V_p$$

The maximum output voltage, V_p, at the frequency, ω, for an op amp slew rate S then equals:

$$V_p = S/\omega = S/2\pi f$$

Figure A provides a nomograph for slew rates. It can relate frequency, peak voltage, and slew rate. Specifying any two parameters determines the third. You generally

measure slew rate under unity-gain conditions that yield a worst-case value.

Peak output voltage and current are also important parameters for high-frequency applications. One critical item to note is that peak output voltage depends upon capacitive load. Large capacitive loads reduce the peak output-voltage swing and therefore reduce slew rate. Also, large capacitive loads can introduce oscillations. For high-speed op amps, vendors sometimes specify full-power bandwidth. This parameter is also an indicator for the suitability of an op amp in a given high-frequency application.

Input offset voltage and current are the result of the physical device. Ideal op amps have zero output for zero input. However, the op-amp input stage may depend on the technology (bipolar or FET) and match. Input offset current is the difference of the input bias currents, and input offset voltage is the voltage that you must apply to the input to obtain zero output voltage. To alleviate the effect of the input bias currents, it is good practice to make sure that the dc path resistance from the positive and negative inputs of the op amp are equal. For inverting infinite-gain topologies, you should place a resistor between the noninverting terminal and ground with a value equal to the dc resistance that the inverting terminal sees.
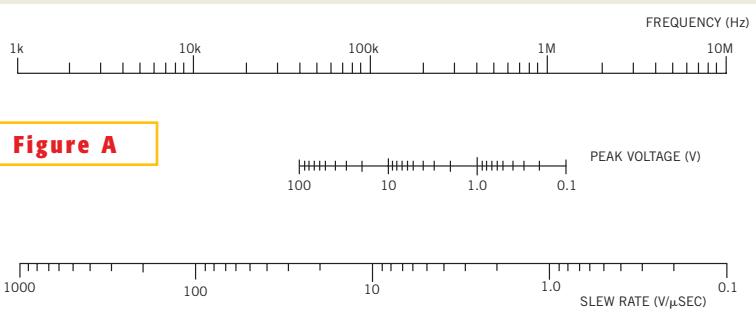
This nomograph for slew rates can relate frequency, peak voltage, and slew rate.

Sallen-Key filters have potentially high active-Q sensitivity, so you must typically make a gain-versus-sensitivity design compromise.

Other available biquad topologies allow trade-offs in required loop gain, active- and passive-Q sensitivity, and active-frequency sensitivity. Two such basic topologies are the single-amplifier and the multiamplifier biquads. In both cases, you can modify the filters' design equations so that the design resolves to standard component values. Specifically, you can solve for nearly identical resistor values, treating the capacitors as variables. By tabulating standard value ratios, you can establish common standard values for the capacitors in the design. The following examples use this approach in a matched-filter design for variable rates. This approach presents a unique design opportunity to highlight the power of the proposed method.

SINGLE- AND MULTIPLE-AMPLIFIER BIQUADS

The **sidebar** "Lowpass, single-amplifier, biquad design" contains both original

LOWPASS, SINGLE-AMPLIFIER, BIQUAD DESIGN

The lowpass, single-amplifier, biquad (**Figure A**) is from **Reference A** with the following design equations: Set $C_2 = KC_1$. Given that ω_{Lp}, ζ_{Lp}, $-H_0$ (dc gain), choose C_1, K, and calculate:

$$C_2 = KC_1 ,$$
$$R_2 = \frac{F}{\omega_{Lp} C_2} ,$$
$$R_1 = \frac{R_2}{-H_0} ,$$
and
$$R_3 = \frac{1}{F \omega_{Lp} C_1} ,$$

where

$$F = \zeta_{Lp} \pm [\zeta_{Lp}^2 - K(1 - H_0)]^{1/2} .$$

You want the term in the square root to be positive; hence the implicit design requirement is:

$$K \leq \frac{\zeta_{Lp}^2}{1 - H_0} .$$

The sensitivities are:

$$S_{R_2}^{\omega_{Lp}} = S_{R_3}^{\omega_{Lp}} = S_{C_1}^{\omega_{Lp}} = S_{C_2}^{\omega_{Lp}} ,$$
$$S_{R_1}^{\zeta} = -H_0 K/2\zeta ,$$
$$S_{R_2}^{\zeta} = 1/2 - K/2\zeta F ,$$
and
$$S_{R_3}^{\zeta} = 1/2 - F/\zeta .$$

You can now modify the design per the matched-filter requirements. In the modified design equations, you set $C_2 = KC_1$, $H_0 = -0.09 \approx -1$, so that $K \leq 0.3738$.

Given that ω_{Lp}, ζ_{Lp}, choose K, R_2 and calculate:

$$C_2 = \frac{F}{\omega_{Lp} R_2} , \quad F = \zeta_{Lp} \pm [\zeta_{Lp}^2 - 1.99K]^{1/2} ,$$

$$R_1 = R_2 ,$$
$$R_3 = \frac{K}{F^2} R_2 ,$$
and
$$C_1 = \frac{1}{F \omega_{Lp} R_3} = \frac{C_2}{K} .$$

For variable data rate f_r, you can calculate the capacitances as:

$$C_2 = \frac{0.108}{2f_r \times \text{factor}} ,$$
and
$$C_1 = 3C_2$$

The sensitivities are:

$$S_{R_1}^{\zeta} = -H_0 K/2\zeta F \approx K = 0.3334 ,$$
$$S_{R_2}^{\zeta} = 1/2 - K/2\zeta F \approx 1/2 - K = 0.1660 ,$$
and
$$S_{R_3}^{\zeta} = 1/2 - F/\zeta = -0.1707 ,$$

REFERENCE
A. Huelsman, LP, JG Graeme, and GE Tobey, _Operational Amplifiers_, McGraw-Hill, New York, NY, 1971.
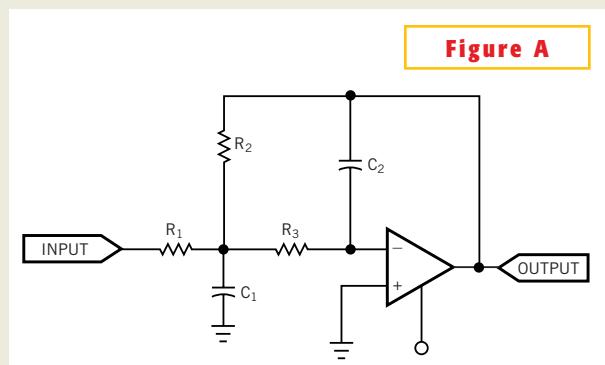
Figure A

You can use the equations to implement a lowpass, single-amplifier biquad.

and modified equations. From the original design equations, the term $\zeta = 1/2Q$ defines the peaking of the response (**Reference 6**). Also, because the variable F includes a square-root term, you must satisfy the implicit design requirement so F is not complex. From the sensitivities, you realize that this filter has zero active sensitivity because it uses the infinite gain of the op amp. By carefully choosing an appropriate ratio K and dc gain H_0 it is possible to manipulate F for small sensitivities. You should consider this in the actual design.

The **sidebar** "Notch, multiple-amplifier, biquad design" also contains both original and modified equations. As with the single-amplifier biquad, the infinite gain of the op amp achieves zero active sensitivities. Some passive sensitivities also approach zero. This notch imple-

NOTCH, MULTIPLE-AMPLIFIER, BIQUAD DESIGN

The notch, multiple-amplifier, biquad design (**Figure A**) is from **Reference A** with the following design equations. Set $R_8 = R_{10} = R$. Given that ω_{Np}, $Q_{Np} = 1/2\zeta_{Np}$, ω_{Nz}, $-H_0(\infty)$, choose C_3, C_4, R, and calculate:

$$R_6 = \frac{Q_{Np}}{\omega_{Np}C_4},$$

$$R_{11} = \frac{1}{\omega_{Np}C_3},$$

$$R_5 = \frac{C_3}{C_4}R_{11},$$

$$R_7 = \frac{R_6}{-H_0},$$

$$R_4 = \frac{\omega_{Np}/(-H_0\omega_{Nz})}{\omega_{Nz}C_3},$$

$$R_9 = \frac{R}{-H_0},$$

$$S_{R_{10}}^{\omega_{Np}} = 1/2,$$

$$S_{R_5}^{\omega_{Np}} = S_{R_8}^{\omega_{Np}} = S_{R_{11}}^{\omega_{Np}} = S_{C_3}^{\omega_{Np}} = S_{C_4}^{\omega_{Np}} = -1/2,$$

$$S_{R_9}^{\omega_{Nz}} = 1/2,$$

$$S_{R_4}^{\omega_{Nz}} = S_{R_5}^{\omega_{Nz}} = S_{R_8}^{\omega_{Nz}} = S_{C_3}^{\omega_{Nz}} = S_{C_4}^{\omega_{Nz}} = -1/2,$$

$$S_{R_6}^{\zeta_{Np}} = -1,$$

$$S_{R_{10}}^{\zeta_{Np}} = S_{C_3}^{\zeta_{Np}} = -1/2,$$

$$S_{R_5}^{\zeta_{Np}} = S_{R_8}^{\zeta_{Np}} = S_{R_{11}}^{\zeta_{Np}} = S_{C_4}^{\zeta_{Np}} = 1/2,$$

In the modified design equations, you set $R_8 = R_{10} = R$. Given that ω_{Np}, $Q_{Np} = 1/2\zeta_{Np}$, ω_{Nz}, choose R, R_{11}. Make $R_6 = Q_{Np}R_{11}$ to force $C_3 = C_4$. Now, calculate:
Including the selectable-rate feature with rate f_r, the capacitances are:

$$C_3 = C_4 = \frac{1}{\omega_{Np}R_{11}},$$

$$R_4 = 1.12^2 R_{11},$$

$$R_5 = \frac{C_3}{C_4}R_{11} = R_{11},$$

$$R_7 = \frac{R_6}{-H_0} = R_6,$$

and

$$R_9 = \frac{R}{-H_0} = R.$$

The sensitivities remain the same.

$$C_3 = C_4 = \frac{0.133}{2f_r \times \text{factor}}.$$

REFERENCE
A. Fleischer, PE, and J Tow, "Design formulas for biquad active filters using three operational amplifiers," *Proceedings of the IEEE*, Volume 61, pg 662, May 1973.
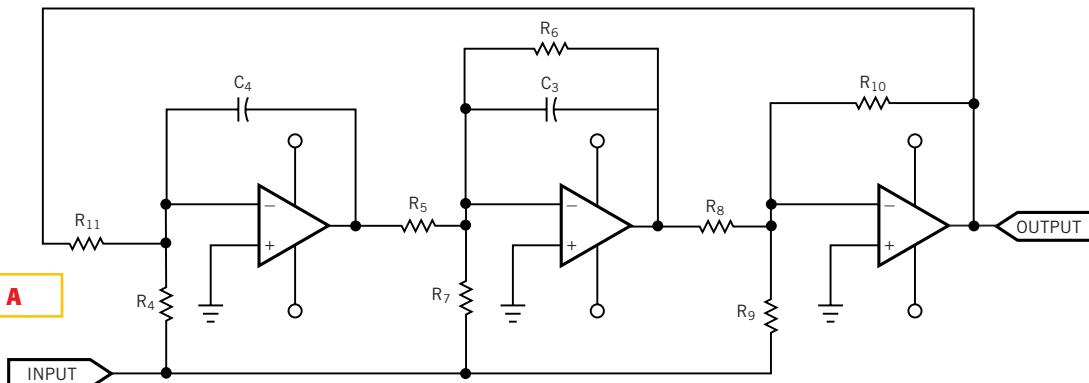
Figure A

You can use the equations to implement this notch, multiple-amplifier biquad.

ments a true zero of transmission so that the Q of the zero is infinite. Although you cannot achieve this figure in practice, a good op-amp selection lets your design achieve more than 50-dB rejection. Although this biquad requires three op amps, eight resistors, and two capacitors, this topology offers excellent sensitivity. The most critical sensitivity is the pole peaking with respect to R_6. Therefore, you should choose R_6 to have low tolerance.

You can apply a lowpass, single-amplifier biquad and a notch, multiple-amplifier biquad in a matched-filter design. As you will see, the matched filter consists of a lowpass section followed by a notch at the desired frequency rate. You can extend the design-equation modifi-

cations to any filter as long as you follow the same basic rules.

MATCHED-FILTER-DESIGN EXAMPLE

Matched filters find use in data-recovery systems to optimize the SNR by matching their frequency response to the spectrum of the input signal. Filters with data rates higher than 10 Mbps come in analog form with expensive hybrid operational amplifiers and tunable inductors and capacitors. Filters with typical data rates lower than 1 Mbps can benefit from an active-RC implementation with its inherent benefits. One such benefit is the ability to exploit the filter-design equations to obtain exact component values and superior sensitivity.

For pulse data such as BPSK (binary

phase-shift keying) or QPSK (quadrature phase-shift keying), the matched-filter response must match the pulse spectrum, given by the relation:

$$H_m(f) = \frac{1}{f_r} \frac{\sin^2(\pi f/f_r)}{(\pi f/f_r)^2} \quad (1)$$

where $f_r = 1/T_r$ is the pulse rate and f is the frequency (**Reference 7**). **Equation 1** is normalized by f_r so that the energy is unity; that is,

$$E = \int_{-\infty}^{\infty} H_m(f)df = 1 \quad (2)$$

The simplest implementation of the above matched filter is a second-order, lowpass transfer function in cascade with

MATCHED-FILTER ANALYSIS

You can use this Mathcad file as a template for analyzing matched-filter responses. The following example defines all variables and performs normalization to generate the required parameters. It compares the ideal matched-filter response with an ideal brick-wall filter to demonstrate the deviation in response over the entire spectrum. R=4000, and the data rate is:

$$f := \frac{R}{100}, \frac{2 \cdot R}{100} .. 5 \cdot R$$

frequencies for analysis.

$$F(f) := \text{if}\left[f \approx 0, \frac{1}{R}, \frac{1}{R} \cdot \frac{\sin\left(\frac{\pi \cdot f}{R}\right)^2}{\left(\frac{\pi \cdot f}{R}\right)^2}\right]$$

is the matched-filter response you want to approximate. You perform

normalization by multiplying by 1/R, the data rate. This operation produces a normalized power of unity as shown.

$$Pow := \int_{-10^6}^{10^6} F(f)df \ ,$$

where Pow=1. Rate normalizes power.
$F(0) = 2.5 \times 10^{-4}$.
At dc, F is 1/R=1/4000, the normalized value. Converting to decibels produces the classic sinX/X response.
F dB(f)=10×log(F(f)), and F dB(0) =−36.021 (**Figure A**).

To calculate the SNR of the filter over the spectrum, you calculate the ratio of two integrals. The numerator is the product of the root of the responses, and the denominator is the ideal response. Due to the performed normalization, any deviation from an ideal response shows up as a negative SNR.

(a)

(b)
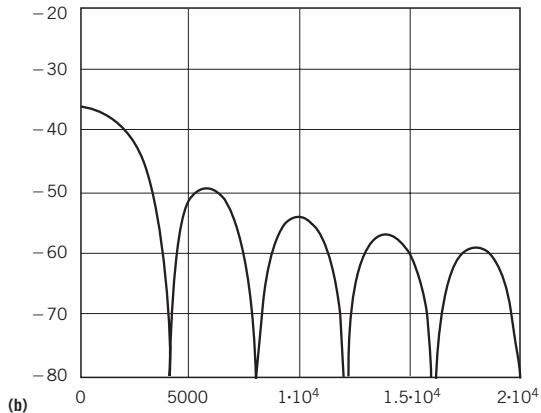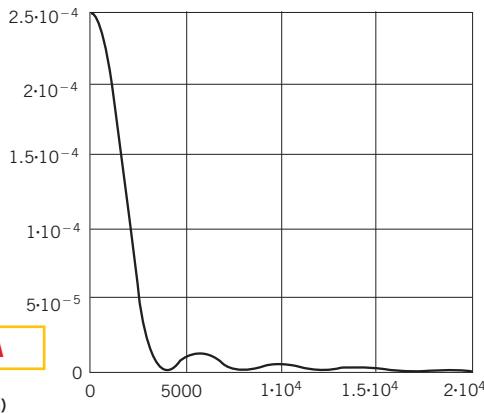
Converting the numeric response of sinX/X (a) to decibels produces the classic sinX/X response (b).

a first-order, notch transfer function. The use of two biquadratic equations allows us to fine-tune the transfer function through the filter Q's (or peaking) in addition to frequency. The second-order, lowpass section also ensures a zero at infinity; that is, the filter's asymptotic roll-off is −20 dB/decade, and you reject high-frequency terms.

The approximating matched-filter transfer function,

$$\tilde{H}_m(s) = H_L(s)H_N(s) \, , \qquad (3)$$

is composed of $H_L(s)$, the lowpass biquad section, and $H_N(s)$, the notch biquad section. The following **equation** yields the lowpass transfer function:

$$H_L(s) = K_L \frac{\omega_{Lp}^2}{s^2 + 2\zeta_{Lp}\omega_{Lp}s + \omega_{Lp}^2} \, , \quad (4)$$

where

$$\omega_{Lp} = 0.71\omega_r \, , \qquad (5)$$

K_L = gain of lowpass stage, $\qquad (6)$
and

$$2\zeta_{Lp}\omega_{Lp} = 1.2247\omega_r \Rightarrow \zeta_{Lp} = 0.8625 \quad (7)$$

with $\omega_r = 2\pi f_r$, and the subscript p denotes pole. The following **equation** gives the notch transfer function.

$$H_N(s) = K_N \frac{s^2 + 2\zeta_{Nz}s + \omega_{Nz}^2}{s^2 + 2\zeta_{Np}\omega_{Np}s + \omega_{Np}^2} \, , \quad (8)$$

where

$$\omega_{Nz} = \omega_r \, , \qquad (9)$$

$$\omega_{Np} = 1.12\,\omega_r \, , \qquad (10)$$

$$K_N = \text{gain of notch stage} \, , \qquad (11)$$

$$2\zeta_{Nz}\omega_{Nz} = 0 \, , \qquad (12)$$

$$2\zeta_{Np}\omega_{Np} = 0.48\omega_r \Rightarrow \zeta_{Np} = 0.375 \, , \quad (13)$$

and the subscript z denotes zero.

Figure 1 is a plot of the ideal matched-filter response and the approximating filter response out to 20 kHz for a 2-kbps data rate, showing good correspondence

$$\text{NumSNR} := \int_{-10^6}^{10^6} \sqrt{F(f)} \cdot \sqrt{F(f)} \, df \, ,$$

where Num SNR=1.

$$\text{DenSNR} := \int_{-10^6}^{10^6} F(f) \, df \, ,$$

where Den SNR=1.

$$\text{SNR} := 10 \cdot \log\left(\frac{\text{NumSNR}}{\text{DenSNR}}\right)$$

SNR=0 for the matched filter.
Now, implement a brick-wall filter for comparison.
H(f)=if (f>4000, 1×10⁻⁸, F(0)), HdB(f)=10×log (H(f)), and HdB(0)

=−36.021 (**Figure B**).
Now, calculate the SNR for the brick-wall filter,

$$\text{NumB} := \int_{-10^6}^{10^6} \sqrt{F(f)} \cdot \sqrt{H(f)} \, df \, ,$$

where NumB=2.301.

$$\text{DenB} := \int_{-10^6}^{10^6} H(f) \, df \, ,$$

where DenB=252.008.

$$\text{SNR} := 10 \cdot \log\left(\frac{\text{NumB}}{\text{DenB}}\right) \, ,$$

where SNR=−20.377.
The brick-wall filter response degrades SNR by 20.4 dB over the ideal matched filter with a corner at 4000 Hz.
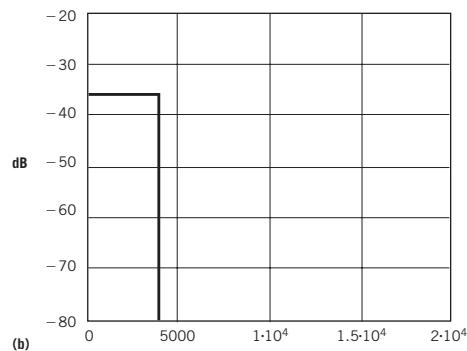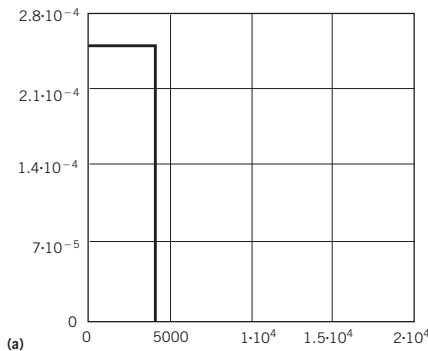
Figure B

The frequency response of the brick-wall filter appears as a power ratio (a) and in decibels (b).

between ideal and approximating out to the first side lobe. **Figure 2** shows a plot of the deviation of the approximating filter from the ideal filter in the main lobe, containing 92% of the energy. The deviation is well within 1 dB almost to the notch frequency. Near the notch frequency, the magnitude is so small that the contribution of these errors to the total is negligible.

To evaluate the SNR performance of the approximating filter, you calculate the energy ratio of both filters assuming that you are at the optimum sampling point; that is, $\tau=0$ (**Reference 8**). For the ideal matched filter, the ratio is

$$\gamma = \frac{\int_{-\infty}^{\infty} \sqrt{H_m(f)}\sqrt{H_m(f)}df}{\int_{-\infty}^{\infty} H_m(f)df} . \quad (14)$$

You take the square root of $H_m(f)$ because $H_m(f)$ was defined to be the magnitude squared in **Equation 1**.

For the approximating filter, the ratio is

$$\tilde{\gamma} = \frac{\int_{-\infty}^{\infty} \sqrt{H_m(f)}\tilde{H}_m(f)df}{\int_{-\infty}^{\infty} \tilde{H}_m^2(f)df} . \quad (15)$$

Because you normalized the energy of the matched filter in **Equation 1**, $\gamma=0$ dB. For the approximating filter, and the value represents the actual SNR degradation. For the parameter values in **equations 5** to **13**, you numerically calculated the approximating filter's SNR degradation to be $\tilde{\gamma}<0$ dB. (see **sidebar** "Matched-filter analysis" for a Mathcad sheet that calculates SNRs of matched filters).

Figure 3 shows a 2-kbps design implemented using the previous design equations. It features 1%-tolerance resistors and 10%-tolerance capacitors. The choice of a good quad op amp is important to ensure good design performance. This design uses the Motorola MC33284, which gives low offset voltages and high precision. The dc gain of the filter is 0.75. Measured results are excellent; notch frequency is 4074 Hz (with less than 2% error), and rejection is −56 dB.

Applications often call for selectable rates due to the variety of signal rates in

Figure 3
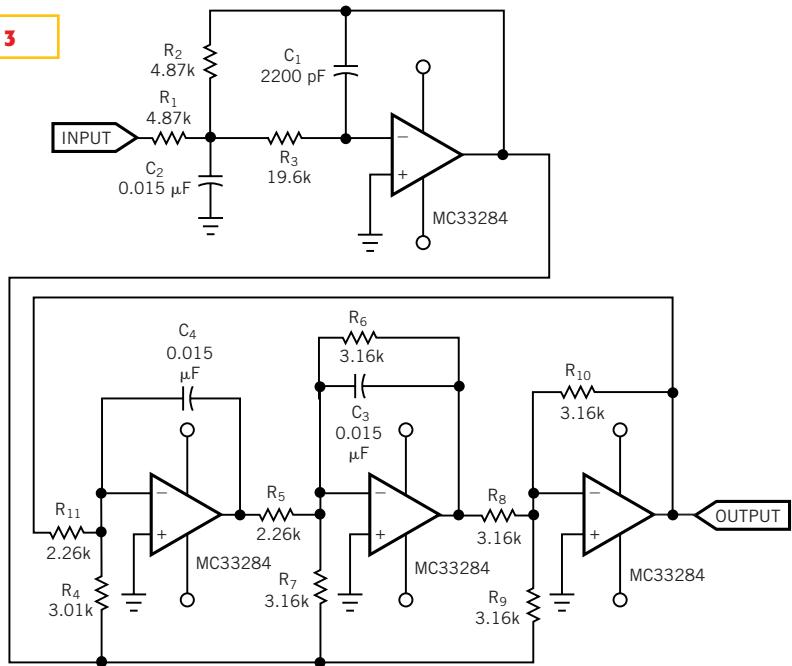
This 2-kbps design features 1%-tolerance resistors and 10%-tolerance capacitors.

Figure 4
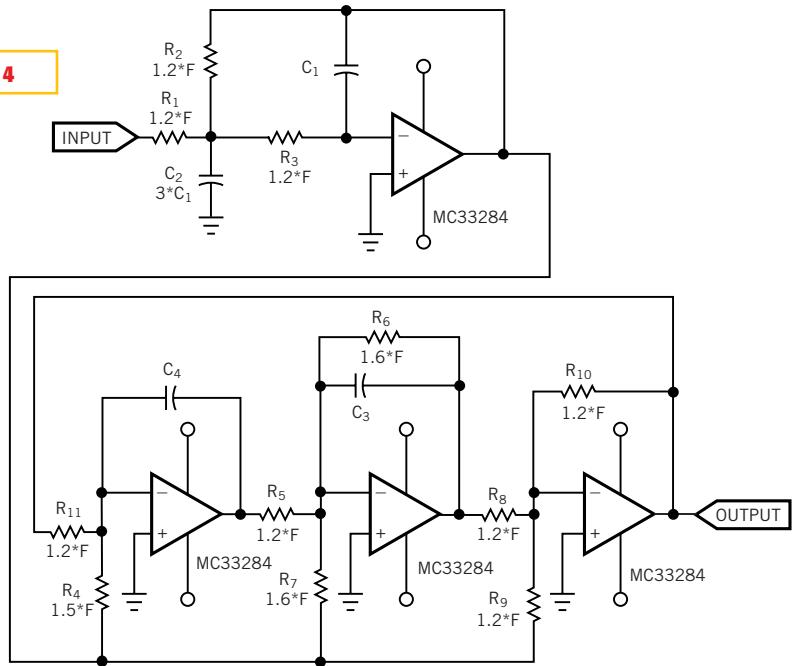
NOTE: F=FACTOR

You can change Figure 3's capacitance values for the desired rates and leave the magnitude response of the filter intact.

a system. Solutions for this type of problem abound, including banks of dedicated-rate filters and switched-in networks for the different rates. If you modify the design equations to include data rate as a parameter, you can have one design cover different data rates. By considering the design's different component values, you can also catalog all data rates you can achieve with a basic design.

MODIFYING THE DESIGN EQUATIONS

Standard component values of a prescribed tolerance are fixed and offer finite resolution. If you need a nonstandard value for a design, you often select the closest value of a prescribed tolerance. This compromise unnecessarily distorts the filter response. The approach here modifies filter-design equations to reflect standard component values and ratios. In this way, the design has no inherent bias, and the circuit performance derives mainly from the component values' distribution and potential drift, each of which you can readily analyze and control through statistical process.

To implement exact standard values in the design, it's beneficial to fix the values of the components that are most numerous—in this case, the resistors—leaving variable those components that are few in number—in this case, the capacitors. In addition, you can modify the design equations to fix the resistor values. Note

that obtaining exact standard values depends mainly upon allowable component-value tolerance ratios. If in the modification, a given design parameter is acceptable if it is within 1% of the exact value.

First, consider the notch filter, with $Q_{Np}=1.333$. Among 5%-tolerance resistors, you can set $R_{11}=1.2\times$factor and $R_6=1.6\times$factor, where factor is the base standard value of the design. Also, $R_4=1.254\times R_{11}=1.5\times$factor. You now have a good starting point with $R_4=1.5\times$factor, $R_6=R_7=1.6\times$factor, and $R_5=R_8=R_9=R_{10}=R_{11}=R=1.2\times$factor. Finally, you need to calculate the capacitance, determined from

$$C_3 = C_4 = \frac{0.133}{2f_r \times \text{factor}} \quad . \quad (16)$$

The **sidebar** "Notch, multiple-amplifier, biquad design" shows the modified equations.

As the **sidebar** "Lowpass, single-amplifier, biquad design" shows, the modified equations are slightly difficult to derive because of the implicit design requirement. To make $R_3=R_2$, K must be equal to F^2. By selecting $K=0.3333$ and $F=0.8625-0.2839=0.5785$, the ratio $K/F^2=0.9958$, or approximately 1 (less than 1%). The ratio $K=0.3333$ is fairly convenient for 10%-tolerance capacitors.

Because you have set $R_1=R_2=R_3$, you can use the same resistor value as the most numerous value in the notch stage; that is, you can set $R_1=R_2=R_3=1.2\times$factor. From this equation, you calculate the capacitances as

$$C_2 = \frac{0.108}{2f_r \times \text{factor}} , \quad (17)$$

$$C_1 = 3C_2 . \quad (18)$$

You can now review the effect of the modified design on the 2-kbps, low-rate requirement. Assuming that factor=1 kΩ, then $R_4=1.5$ kΩ, $R_6=R_7=1.6$ kΩ, and $R_1=R_2=R_3=R_5=R_8=R_9=R_{10}=R_{11}=R=1.2$ kΩ. You can readily determine the capacitances of $C_1, C_2, C_3,$ and C_4 to be approximately 0.082 (1.2% from actual), 0.027, 0.033, and 0.033 μF, respectively.

The previous process lets you design an active-RC matched filter with exact standard component values that are within 1% of the ideal. Because the capacitors were the least numerous of the components, you fix the resistor values and modify the capacitance based on the required rate. You next concentrate on determining standard capacitance values for the selectable-rate feature.

Equations 16 and **17** yield the capacitance values that depend on the rate, and **Equation 18** is fixed. If you rewrite these equations in terms of the rate, you obtain

TABLE 1—STANDARD-VALUE RATIOS FOR 5% TOLERANCE

	10	11	12	13	15	16	18	20	22	24	27	30	33	36	39	43	47	51	56	62	68	75	82	91
10	1	1.1	1.2	1.3	1.5	1.6	1.8	2	2.2	2.4	2.7	3	3.3	3.6	3.9	4.3	4.7	5.1	5.6	6.2	6.8	7.5	8.2	9.1
11	0.91	1	1.09	1.18	1.36	1.45	1.64	1.82	2	2.18	2.45	2.73	3	3.27	3.55	3.91	4.27	4.64	5.09	5.64	6.18	6.82	7.45	8.27
12	0.83	0.92	1	1.08	1.25	1.33	1.5	1.67	1.83	2	2.25	2.5	2.75	3	3.25	3.58	3.92	4.25	4.67	5.17	5.67	6.25	6.83	7.58
13	0.77	0.85	0.92	1	1.15	1.23	1.38	1.54	1.69	1.85	2.08	2.31	2.54	2.77	3	3.31	3.62	3.92	4.31	4.77	5.23	5.77	6.31	7
15	0.67	0.73	0.8	0.87	1	1.07	1.2	1.33	1.47	1.6	1.8	2	2.2	2.4	2.6	2.87	3.13	3.4	3.73	4.13	4.53	5	5.47	6.07
16	0.63	0.69	0.75	0.81	0.94	1	1.13	1.25	1.38	1.5	1.69	1.88	2.06	2.25	2.44	2.69	2.94	3.19	3.5	3.88	4.25	4.69	5.13	5.69
18	0.56	0.61	0.67	0.72	0.83	0.89	1	1.11	1.22	1.33	1.5	1.67	1.83	2	2.17	2.39	2.61	2.83	3.11	3.44	3.78	4.17	4.56	5.06
20	0.5	0.55	0.6	0.65	0.75	0.8	0.9	1	1.1	1.2	1.35	1.5	1.65	1.8	1.95	2.15	2.35	2.55	2.8	3.1	3.4	3.75	4.1	4.55
22	0.45	0.5	0.55	0.59	0.68	0.73	0.82	0.91	1	1.09	1.23	1.36	1.5	1.64	1.77	1.95	2.14	2.32	2.55	2.82	3.09	3.41	3.73	4.14
24	0.42	0.46	0.5	0.54	0.63	0.67	0.75	0.83	0.92	1	1.13	1.25	1.38	1.5	1.63	1.79	1.96	2.13	2.33	2.58	2.83	3.13	3.42	3.79
27	0.37	0.41	0.44	0.48	0.56	0.59	0.67	0.74	0.81	0.89	1	1.11	1.22	1.33	1.44	1.59	1.74	1.89	2.07	2.3	2.52	2.78	3.04	3.37
30	0.33	0.37	0.4	0.43	0.5	0.53	0.6	0.67	0.73	0.8	0.9	1	1.1	1.2	1.3	1.43	1.57	1.7	1.87	2.07	2.27	2.5	2.73	3.03
33	0.3	0.33	0.36	0.39	0.45	0.48	0.55	0.61	0.67	0.73	0.82	0.91	1	1.09	1.18	1.3	1.42	1.55	1.7	1.88	2.06	2.27	2.48	2.76
36	0.28	0.31	0.33	0.36	0.42	0.44	0.5	0.56	0.61	0.67	0.75	0.83	0.92	1	1.08	1.19	1.31	1.42	1.56	1.72	1.89	2.08	2.28	2.53
39	0.26	0.28	0.31	0.33	0.38	0.41	0.46	0.51	0.56	0.62	0.69	0.77	0.85	0.92	1	1.1	1.21	1.31	1.44	1.59	1.74	1.92	2.1	2.33
43	0.23	0.26	0.28	0.3	0.35	0.37	0.42	0.47	0.51	0.56	0.63	0.7	0.77	0.84	0.91	1	1.09	1.19	1.3	1.44	1.58	1.74	1.91	2.12
47	0.21	0.23	0.26	0.28	0.32	0.34	0.38	0.43	0.47	0.51	0.57	0.64	0.7	0.77	0.83	0.91	1	1.09	1.19	1.32	1.45	1.6	1.74	1.94
51	0.2	0.22	0.24	0.25	0.29	0.31	0.35	0.39	0.43	0.47	0.53	0.59	0.65	0.71	0.76	0.84	0.92	1	1.1	1.22	1.33	1.47	1.61	1.78
56	0.18	0.2	0.21	0.23	0.27	0.29	0.32	0.36	0.39	0.43	0.48	0.54	0.59	0.64	0.7	0.77	0.84	0.91	1	1.11	1.21	1.34	1.46	1.63
62	0.16	0.18	0.19	0.21	0.24	0.26	0.29	0.32	0.35	0.39	0.44	0.48	0.53	0.58	0.63	0.69	0.76	0.82	0.9	1	1.1	1.21	1.32	1.47
68	0.15	0.16	0.18	0.19	0.22	0.24	0.26	0.29	0.32	0.35	0.4	0.44	0.49	0.53	0.57	0.63	0.69	0.75	0.82	0.91	1	1.1	1.21	1.34
75	0.13	0.15	0.16	0.17	0.2	0.21	0.24	0.27	0.29	0.32	0.36	0.4	0.44	0.48	0.52	0.57	0.63	0.68	0.75	0.83	0.91	1	1.09	1.21
82	0.12	0.13	0.15	0.16	0.18	0.2	0.22	0.24	0.27	0.29	0.33	0.37	0.4	0.44	0.48	0.52	0.57	0.62	0.68	0.76	0.83	0.91	1	1.11
91	0.11	0.12	0.13	0.14	0.16	0.18	0.2	0.22	0.24	0.26	0.3	0.33	0.36	0.4	0.43	0.47	0.52	0.56	0.62	0.68	0.75	0.82	0.9	1

$$f_r = \frac{0.133}{2 \times \text{factor} \times (C_3 = C_4)}$$

$$= \frac{0.108}{2 \times \text{factor} \times C_2} , \qquad (19)$$

which in turn implies

$$C_3 = C_4 = 1.2C_2 \quad \text{within } 1\% . \qquad (20)$$

Equation 20 is independent of the rate, meaning that as long as you maintain the corresponding capacitance ratios, you can change capacitance values for the desired rates and leave the magnitude response of the filter intact. **Figure 4** shows the resulting design.

Of course, you can select other resistor-to-capacitor ratios for different rates, and it is of interest to select ratios that provide the greatest choice of standard component values. For example, for 5%-tolerance values and calculating the forward ratio, the most common integer ratio is 2-to-1, corresponding to 22-to-11, 24-to-12, 30-to15, 36-to-18, and 20-to-10. The next most common integer ratio is 3-to1, corresponding to 30-to-10, 33-to-11, 36-to-12, and 39-to-13. The most common ratio is 1.5-to-1, corresponding to 15-to-10, 18-to-12, 24-to-16, 27-to-18, 30-to-20, 33-to-22, and 36-to-24. Other common ratios are 1.2-to-1 and 1.1-to-1. By expanding the ratio tolerances to ±1%, a greater range of component ratios becomes available. For example, if you allow the 1.1-to-1 ratio to vary from 1.089-to-1 to 1.111-to-1, then you can have as many as 16 component ratios: 11-to-10, 12-to-11, 20-to-18, 22-to-20, 24-to-22, 30-to-27, 33-to-30, 36-to-33, 43-to-39, 47-to-43, 56-to-51, 62-to-56, 68-to-62, 75-to-68, 82-to-75, and 91-to-82. **Table 1** provides a complete span of ratios for 5% standard component values.

This article presents an approach for designing RC-active filters with exact standard component values to minimize response distortion. The basic idea is to modify the design equations and force the resistor values to ideal values while calculating capacitors using common standard value ratios. You can apply the technique to a matched-filter design with less-than-0.2-dB deviation from ideal. To incorporate a variable-rate capability, you further modify the design equations to reflect rate as a parameter. Simulation and test results show that the design has excellent response.

The method took into account not just exact standard-component values, but also parts count: You use the most common element value to limit the number of standard values necessary for the design. In the matched-filter example, the most common value was 1.2×factor (eight resistors), 1.6×factor (two resistors), and 1.5×factor (one resistor). You can apply the same approach to other biquad topologies, including Sallen-Key filters.□

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