CHAPTER 1

Direct Current Circuits

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1.1 INTRODUCTION

The wealth of electrical circuits currently present in laboratory settings and commercial equipment can be grouped into a few basic types. Electrical circuits contain a power source to supply electrical charges to the circuit and circuit elements that affect the energy of the charges in the circuit. If the circuit element is *passive*, then the charge loses energy in the element. If the circuit element is *active* then the charge may gain energy in the circuit element. A circuit is either *passive* or *active*, where active circuits have at least one active circuit element.

Measurements in electrical circuits focus on charges, their motion and their energy. If the motion (current) and energy (voltage) in a circuit remain constant in time the circuit is a *direct current* (dc) circuit. Many circuits of interest will have currents and voltages that vary in time and are alternating current (ac) circuits. However, there are many basic principles of electrical circuits that are present in dc circuits and we will explore those principles in this chapter.

Electrical circuit measurements can be made in different systems of units, and in this book the measurements will be in mks (meter kilogram second) units. Scientific notation accompanies the units so that one can write, for instance, 10^{-2} volts, or 10^{-2} V using the abbreviation for volts. Units in the mks system have standard prefixes and these are important since in electronics the prefix is preferred to scientific notation. Using the prefix one should write 10 millivolts or 10 mV instead of 10^{-2} V. Prefixes for various powers of ten are given in Table 1.1.

TABLE 1.1. Powers-of-Ten Prefixes

Prefix	*Symbol*	*Meaning*	*Example*
giga	G	10^{9}	1 gigahertz = 1 GHz = 10^{9} hertz
mega	M	10^{6}	1 megohm = 1 MΩ = 10^{6} ohms
kilo	k or K	10^{3}	1 kilobyte = 1 KB = 10^{3} bytes
milli	m	10^{-3}	1 millivolt = 1 mV = 10^{-3} volts
micro	μ or u	10^{-6}	1 microampere = 1 μA = 10^{-6} amperes
nano	n	10^{-9}	1 nanosecond = 1 ns = 10^{-9} seconds
pico	p or uu	10^{-12}	1 picofarad = 1 pF = 10^{-12} farads

Some prefix symbols appear in more than one way in electronics. This is primarily due to printing requirements on components. Greek characters are often not available and the character u will replace μ on capacitors. Since pico is the same as micro-micro the capacitors might be marked in uuF instead of pF for picofarads. Capitals occasionally replace small letters on components as well. We will use capital letter prefixes for positive powers of ten, and small letter prefixes for negative powers of ten.

1.2 ELECTRIC CHARGE AND CURRENT

There are two kinds of *electric charge*, positive and negative. They are so named because they add and subtract like positive and negative numbers. All atoms contain charge; the usual picture of an atom is a small (10^{-15} m diameter) positively charged nucleus around which negatively charged electrons move in orbits of the order 10^{-10} m diameter. Charge is measured in *coulombs* (C), and the charge on one electron is -1.6×10^{-19} coulombs. Although electric charges (positive and negative) in electric circuit problems are integral multiples of the charge of the electron, in most electric circuit problems the discrete nature of electric charge may be neglected, and charge may be considered to be a continuous variable.

One of the most fundamental conservation laws of physics says that *in any closed system the total net amount of electric charge is conserved* or, in other words, is constant in time. For example in a semiconductor if one electron is removed from a neutral atom then the atom minus the one electron (the ion) has a net electric charge of +1.6 x 10^{-19} coulombs.

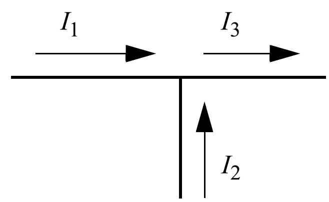
The flow of electric charge, either positive or negative, is called the *current*; that is, the current I at a given point in an electric circuit is defined as the time rate of change of the amount of electric charge Q flowing past that point.

$$I \equiv \frac{dQ}{dt} \tag{1.1}$$

Current is measured in *amperes* (amps or A); one ampere of current is the flow of one coulomb of charge per second, which is is 6.25 x 10^{18} electrons per second. One ampere is a lot of current for a benchtop circuit. More common units of current in circuits are the milliampere (mA), which is 10^{-3} A, and the microampere (μA), which is 10^{-6} A.

The direction of the current is taken by convention to be the direction of the flow of *positive* charge. If electrons are flowing from right to left in a wire, then this electron current is electrically equivalent to positive charges flowing from left to right; so, we say the current is to the right. Since current has both magnitude and direction it is a vector quantity; however, in electical circuits the wires and other components effectively restrict the direction to two choices.

We can compare electric current in a wire to water flow in a pipe, the water molecules being analogous to the electrons in the wire. A flow of water in kilograms per second is analogous to a flow of electric charge in coulombs per second or amperes. Ignoring the individual electrons in the current is similar to ignoring the individual molecules in water flow.



$$I_3 = I_1 + I_2$$

FIGURE 1.1. Conservation of current at a point.

From the law of conservation of charge we know that all the charge that goes into a point in a circuit must come out as shown in Fig. 1.1. This can be restated as the current law: *The sum of all currents at a point is zero.*

1.3 ELECTRIC POTENTIAL AND VOLTAGE

Between any two charges there is a force, called the *Coulomb force*. The Coulomb force F depends on the amount of each of the two charges Q_1 and Q_2, the distance between the two charges r and the *permittivity of free space* ε_0. The quantity $1/4\pi\varepsilon_0$ is approximately 9 x 10^9 Nm^2/C^2.

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r^2} \quad \textbf{(1.2)}$$

The direction of the force is along the line between the charges. The force is repulsive if the two charges have the same sign and attractive if the two charges have opposite sign. The *electric field* E measures the force per unit charge and is given in N/C.

$$E \equiv \frac{F}{Q} \quad \textbf{(1.3)}$$

A charge that moves in an electric field may do work. Electrical work has units of energy and is defined as the path integral of the charge as it moves under the influence of the Coulomb force.

$$W = \int_a^b F\,dl = \frac{Q_1 Q_2}{4\pi\varepsilon_0}\int_a^b \frac{dl}{r^2} \qquad \textbf{(1.4)}$$

In (1.4) W is the work and dl is the component of the path parallel to the direction of the force. The Coulomb force is conservative and the work done is independent of the path taken by a charge. If the starting point and ending point are the same the work done must be the same as if the charge had never moved and had performed no work. This is conservation of electrical energy and can be restated: *The work done by an electric charge along a closed loop is zero*.

The *voltage* (*tension* in some parts of the world), or electric *potential*, V at a point in a circuit is defined as the electrical potential energy W of a positive charge Q at that point divided by the magnitude of the charge; that is

$$V \equiv \frac{W}{Q} \qquad \textbf{(1.5)}$$

The unit of voltage is the *volt* (V); one volt is defined as one joule per coulomb (J/C). Combining this definition with the definition for the electric field (1.3) one can express the electric field in units of V/m, using L for the length between the two measured voltage points.

$$E = \frac{V}{L} \qquad \textbf{(1.6)}$$

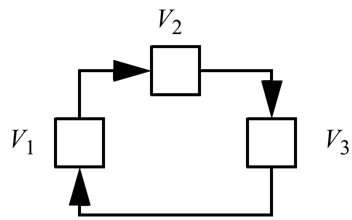
The positive charges will tend to move from points of higher voltage toward points of lower voltage; for example, positive charge will move from a point with a voltage of +15 V toward a point with a voltage of +12 V. Similarly, negative charges will tend to move from points of lower voltage toward points of high voltage; for example, negative charge (electrons) will move from a point of voltage −12 V to a point of voltage −7 V.

In (1.4) we defined electrical work relative to a starting point in the path. Similarly, voltage is measured relative to some other specified reference point in the circuit, where we say the energy of all charges is zero. This reference point is usually called ground. A good ground provides a continuous path of metal to the earth such as a cold-water pipe or sometimes just the metal chassis or box enclosing the circuit. The key is that the circuit ground has a constant potential or voltage simply because it is so large and is a reasonably good conductor. Any charge taken from or added to the earth through a circuit's ground wire will not appreciably change the total charge on the earth or the earth's voltage. The symbols used for ground notation are shown in Fig. 1.2.

FIGURE 1.2. Symbols used for ground in circuits.

The voltage at a given point has no absolute meaning but only indicates the potential energy per unit charge relative to ground. In terms of the analogy between current and water flow, the voltage is analogous to the water pressure, because water tends to flow from points of higher pressure to points of lower pressure. It is sometimes useful to think of the voltage as causing or forcing the flow of current, just as one thinks of the water pressure as causing or forcing the flow of water. In this view, voltage is a cause, and current is an effect.

Now we can talk about a circuit and the current and voltage in the circuit. An electrical circuit consists of one or more circuit elements connected together. Current flows through each element and a voltage change is present across each element. The voltage represents the change in electrical potential through a circuit element and is equivalent to the work done by an electric charge flowing through the element. When a voltage is measured at a point in the circuit it is assumed to be measured with respect to ground. The absolute voltage at a point does not matter for the behavoir of the circuit, only the change in voltage across a circuit element. We can restate the conservation of electrical energy as the voltage law: *The sum of voltages around a closed loop is zero*.



$$V_1 + V_2 + V_3 = 0$$

FIGURE 1.3. Conservation of voltage around a loop.

Power is defined as the time rate of doing work W or the time rate of expending energy.

$$P \equiv \frac{dW}{dt} \tag{1.7}$$

The unit of power is the *watt*; 1 watt = 1 joule/second (J/s). Combining equations 1.1, 1.5 and 1.6 gives an expression for the power expended in an electrical circuit element carrying a current I and with a voltage change V:

$$P = VI \tag{1.8}$$

1.4 CONDUCTORS

Our definition of a circuit element has a current that flows through the element, a voltage difference across the element, and two wires on either end of the element. This type of circuit element is called a *two-terminal device*.

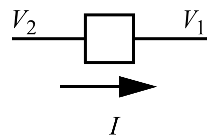


FIGURE 1.4. Current and voltage through a two-terminal device.

If a current I flows through any two-terminal device, then the current into the circuit element must exactly equal the current leaving the element, from the conservation of charge law (see Fig. 1.4). V_2 is the voltage at the terminal where the current enters the circuit element; V_1 is the voltage where the current leaves. For a passive circuit element the charge loses energy and V_2 must be larger than V_1.

The static or dc *resistance* (usually referred to simply as the resistance) of the circuit element is defined as the voltage difference, V_2 - V_1, between the terminals divided by the current I.

$$R \equiv \frac{V_2 - V_1}{I} \tag{1.9}$$

Strictly speaking, this definition applies only to a circuit element that converts electrical energy into heat, but this situation occurs in the overwhelming majority of cases in electronic circuitry. Resistance is measured in *ohm*s (Ω); one ohm is defined as one volt per ampere (V/A).

Conductivity

Real two-terminal devices are not undefined boxes, but are three-dimensional objects with real material properties. We will consider a simple two-terminal device like a wire that is cylindrical with a cross-sectional area A and length L as in Fig. 1.5.

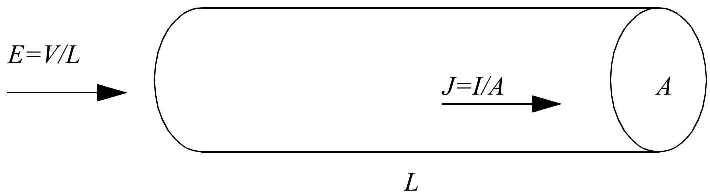


FIGURE 1.5. Flow of current through a wire.

The device has a voltage difference V which results in an electric field E (1.6). This causes a current to flow in the device like pressure causes water to flow through a pipe. The current density J is defined as the current per unit area: $J = I/A$. If we use the analogy of water flowing through a pipe then a narrow garden hose and wide fire hose may both have the same current, but the narrow hose will have a greater current density due to the smaller cross-sectional area.

Experiments show that up to some maximum voltage the current density is proportional to the electric field. The *conductivity* σ is defined as the ratio of the current density to the electric field.

$$\sigma \equiv \frac{J}{E} \tag{1.10}$$

Conductivity can depend on the material, field, temperature, pressure, or many other physical properties, but conductivity does not depend on the size and shape of the device.

To approximate the conductivity in a metal, we assume an electron is accelerating freely for time τ, due to a force $F = eE$, then it collides with an atom and starts over. The acceleration of the electron is:

$$a = \frac{F}{m} = \frac{eE}{m} \tag{1.11}$$

With constant acceleration, the electron's velocity, or drift speed, is:

$$v = \frac{eE\tau}{m} \tag{1.12}$$

And, if the number of electrons in the conductor is n, then the current density is:

$$J = nev = \frac{ne^2\tau}{m}E \tag{1.13}$$

The conductivity is the ratio of the current density to the electric field (1.10). In our approximation the conductivity depends only on the number of electrons and mean time between collisions. These terms are all constants of a particular metal, so the conductivity is a constant as well.

$$\sigma = \frac{ne^2\tau}{m} \tag{1.14}$$

Conductivity has units of current density divided by the electric field or $(A/m^2)/(V/m) = 1/\Omega\cdot m$. The conductivity multiplied by the length of the conductor gives the conductance and is measured in Ω^{-1}. This is the reciprocal of resistance. One Ω^{-1} is called a *siemen* (S) or *mho* (“ohm” spelled backward).

Resistivity

Resistivity is the inverse of conductivity: $\rho = 1/\sigma$ and is measured in units of $\Omega\cdot$m. The resistivity expresses the difficulty an electron has in moving through the material due to the collisions it experiences with the atoms. Resistivity values for various materials are given in Table 1.2.

TABLE 1.2. Resistivity Vaules of Various Materials

Material	*Resistivity at 20ºC (Ω·m)*
Conductors	
Silver	1.5×10^{-8}
Copper	1.7×10^{-8}
Aluminum	2.8×10^{-8}
Tungsten	5.5×10^{-8}
Carbon	3.5×10^{-5}
Semiconductors	
Germanium	4.3×10^{-1}
Silicon	2.6×10^{3}
Insulators	
Glass (typical)	1×10^{12}
Mica	9×10^{13}
Quartz (fused)	5×10^{16}

A *conductor* is a substance or material with "low" resistivity. Most metals are good conductors; copper or aluminum wire is usually used in electronic circuits to carry current. An *insulator* is a substance or material with "high" resistivity that is used to prevent current flow. Most plastics, rubber, air, mica, and quartz are good insulators; wire is usually covered with a plastic sheath insulator to confine the current flow to the wire. Note that germanium and silicon have resistivities that are much greater than those of metals but less than those of insulators. Hence they are often called *semiconductors*.

Resistance measures the integrated resistivity per cross-sectional area over a length of conductor. A uniform piece of conducting material of length L and cross sectional area A has a resistance of R:

$$R = \rho\frac{L}{A} \qquad \textbf{(1.15)}$$

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where ρ is the resistivity of the material. The diameter and resistance per unit length of selected copper wires are given in Table 1.3. In a practical electronic circuit, for example, a No. 22 wire 4 in. (0.1 m) long would have a resistance of only 0.006 Ω.

TABLE 1.3. Resistance of Various Sizes of Copper Wire

AWG wire size	*Diameter (in.)*	*Resistance per 1000 ft (Ω)*
24	0.0201	28.4
22	0.0254	18.0
20	0.0320	11.3
18	0.0403	7.2
16	0.0508	4.5

Ohm's Law

As in our approximation earlier in this section, for many kinds of circuit elements it is empirically true that the conductivity, and hence the resistance, of the element is constant if the temperature and composition of the element are fixed. This is true over an extremely large range of voltages and currents. Changing the voltage difference between the two terminals by any factor changes the current by exactly the same factor; that is, doubling the voltage difference $V = V_2 - V_1$ exactly doubles the current I.

Ohm's law is simply the statement that the resistance R is constant. It can be written in three ways:

$$R = \frac{V}{I} \qquad V = IR \qquad I = \frac{V}{R} \tag{1.16}$$

These three forms of Ohm's law can be thought of in the following terms. $R = V/I$ means that if there is a voltage difference across a circuit element through which a current I is flowing, then the circuit element must have a resistance V/I. $V = IR$ means that if a current I is forced through a resistance R, then a voltage difference IR will be developed between the two ends of the resistance. $I = V/R$ means that if there is a voltage difference across a resistance, then a current must be flowing through the resistance.

Perhaps the most important thing to remember about Ohm's law is that it is only the difference in voltage across a resistor which causes current to flow. Thus a 5 KΩ resistor with one end at 35 V and the other end at 25 V will pass a current of 2 mA. A 5 KΩ resistor with one end at 1078 V and the other end at 1068 V will also pass a current of 2 mA because the voltage difference is also 10 V.

1.5 RESISTORS

Any two-terminal circuit element of constant resistance is shown in circuit diagrams as a zig-zag line (see Fig. 1.6) and is called a *resistor*.

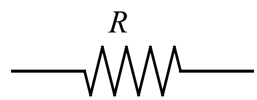

FIGURE 1.6. Schematic symbol for a resistor.

Resistors are described by their resistance, precision, and power rating. The resistance of resistors is measured in ohms (Ω) in the range 0.01 to 10^{12} Ω. Scientific notation is generally avoided for resistors less than 10^{12} Ω. The preferred units for each range of resistance are 1-999 Ω, 1-999 KΩ, 1-999 MΩ, 1-999 GΩ. Resistors are made with accuracies from 0.01% to 20% and come with power ratings from 1/8 to 1000 W.

Resistors can be made from any conductor and a number of types are common. The most common is made from a carbon composite, where the specific composition is used to vary the resistivity of the material. Carbon resistors are inexpensive and available in a wide range of resistances, but are not very precise. Resistors are also made from metal wires and films where the length and area control the resistance (1.15). Metal-based resistors are used for precision applications such as standard values. The types of resistors are addressed more fully in Appendix A.

A three-digit number code is commonly used to describe the resistance printed on an individual resistor, instead of the actual resistance. The first two digits of the code should be read as a number, and the third digit is read as an exponent for a power of 10. For example, a resistor printed with the number 471 is 47×10^1 Ω = 470 Ω; a resistor printed with the number 103 is 10×10^3 Ω = 10 KΩ. Note that this second case is not 10^3 Ω, which would be 1 KΩ instead of the correct value 10 KΩ.

The two digit numbers in the resistor code do not appear in all 100 possible values. Because many common resistors have precision no better than 5%, 10% or 20%, the standard two digit numbers go up in steps roughly double the precision - 10%, 20% or 50%. These are shown in Table 1.4, where the 20% precision steps use only six values, the 10% steps use 12 values, and the 5% steps use all 24 values in the table.

TABLE 1.4. Standard two digit numbers for resistor values.

20%, 10%, 5%	5%	10%, 5%	5%
10	11	12	13
15	16	18	20
22	24	27	30
33	36	39	43
47	51	56	62
68	75	82	91

We should keep in mind the precision of the components in a circuit when selecting a resistor value. If a circuits calls for a resistance of 205 Ω, but the components in the circuit are only accurate to within 10%, a 220 Ω resistor could be used because 205 Ω is not available and 220 Ω is within 10% of the required value. Don't try to get too precise through a combination of different resistors or with expensive components when the rest of the circuit doesn't warrant the precision.

For the common carbon resistors in cylindrical packages the resistance of a resistor usually is given by a color code instead of a number. The code is the same as if a three-digit number was used, with each color representing a number: black=0, brown=1, red=2, orange=3, yellow=4, green=5, blue=6, violet=7, grey=8, white=9. If you have difficulty remembering the color code, note that the colors follow the colors of the visible spectrum, starting with red for the number two and going through violet for the number seven. In the color code, the first digit of the resistance is given by the color band closest to an end of the resistor.

Many color-coded resistors have more than three bands. The fourth color band gives the precision, also called the tolerance, of the resistor. Silver means 10% tolerance, gold means 5% tolerance, and no fourth colored band means 20% tolerance. A 2 KΩ 10% resistor will have a resistance somewhere between 1.8 and 2.2 KΩ.

Certain high-reliability resistors are tested for failure rates under conditions of maximum power and voltage, and the results are expressed in percentage failure per thousand hours. The fifth colored band represents the failure rate according to the following scheme: brown 1%, red 0.1%, orange 0.01%, and yellow 0.001%. A table of the resistor color code is included in Appendix A.

Variable resistors are also available (Fig 1.7). They come in many sizes and styles, and are usually adjusted by manually turning a shaft. One special type of variable resistor is the potentiometer or "pot." Pots have three terminals: one at each end of the resistor and one for the variable position of the tap. The total resistance R_T between the two end terminals A and B is always constant and equals the resistance value of the pot. The resistance R_1 between A and the tap and the resistance R_2 between B and the tap vary as the shaft is turned. Notice that if the shaft is turned fully toward terminal A, the tap is electrically connected to terminal A and $R_2 = R_T$, $R_1 = 0$.

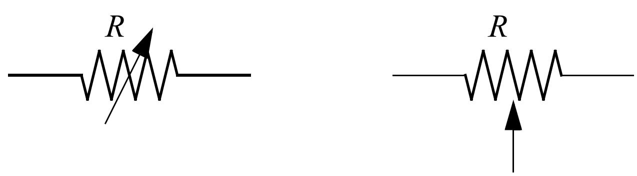

FIGURE 1.7. Schematic symbols for (a) variable resistors and (b) potentiometers. *R* measures the maximum resistance.

A dc current I flowing through a resistor R develops a power dependent on the voltage V across the resistor. Recalling that voltage is electrical potential energy per unit charge, we see that a charge has less electrical potential energy when it leaves a resistor than when it enters because of the decrease in voltage, or voltage "drop" across the resistor. From (1.8), $P = IV$. But, from Ohm's law, $V = IR$; the power can also be expressed as $P = I^2R$. And, again from Ohm's law, I = V/R, so another way of expressing the power is $P = V^2/R$. These three expressions for the power are equivalent and apply only to direct current flowing through a resistor. In short the power developed by the resistor is I^2R or VI or V^2/R.

The power developed through a resistor shows up as heating of the resistor. In other words, the loss of electrical potential energy (due to the IR voltage drop) of the charge flowing through the resistor is converted into random thermal motion of the molecules in the resistor. The kinetic energy of the flowing charges remains approximately constant everywhere in the circuit. Electrical potential energy is converted into heat energy in any dc circuit element across which there is a voltage drop and through which current flows.

If too many watts of power are converted into heat in a resistor or in any circuit element, the resistor may “burn up,” in which case the resistor turns brown or black and may actually fragment, thus breaking the electrical circuit. In other words the resistor is open or has an infinite resistance. Or, if the resistor is heated too much, its resistance value may increase tremendously, thus changing the operation of the circuit drastically. For these reasons resistors are rated by the manufacturer according to how much power they can safely dissipate without being damaged. Resistors are commonly available with wattage ratings of 1/8 W, 1/4 W, 1/2 W, 1 W and 2 W. Special purpose resistors are available with higher wattage ratings. The larger the resistor is physically, the more power it can safely dissipate as heat.

Note that the resistance does not depend on the actual physical size. The physical size determines the power rating; for example, a 1/2 W, 2.2 KΩ resistor is the same size as a 1/2W, 470 KΩ resistor. Circuit designers usually choose a power rating of at least three or four times the expected power. For example, if a 1.5 KΩ resistor is to carry 20 mA of direct current, then the power dissipated as heat in the resistor will be $(1.5\ \text{K}\Omega)(20\ \text{mA})^2 = 0.6\ \text{W}$.

In an actual circuit a 1.5 KΩ 2 W resistor would be used, perhaps even a 5 W resistor. If a large wattage is developed in a certain part of a circuit, care should be taken to provide an adequate vertical flow path for air around the hot element so that the heat can be carried away by the resulting convection air currents. A resistor dissipating a large amount of power should never be placed in a closed chassis. Heat is an enemy of transistors and other circuit elements.

1.6 TEMPERATURE DEPENDENCE

Resistivity varies for different materials; it depends also on the temperature, typically increasing with increasing temperature in most metals. For carbon resistors used in most electronic circuits, resistivity increases by approximately 0.5% to 1% for a 10ºC temperature increase. For most metallic conductors, the resistivity increases slowly with increasing temperature. At high temperatures the thermal motion of the atoms increases; hence the average distance moved by a free electron between collisons decreases, producing a slightly lower drift speed. The resistance of a metallic conductor depends upon temperature according to the relation

$$R_T = R_0(1 + \alpha\Delta T) = R_0(1 + \alpha T - \alpha T_0) \qquad \textbf{(1.17)}$$

where R_0 is the resistance at some reference temperature T_0, $\Delta T = T - T_0$ and is the temperature rise in ºC and α is the linear temperature coefficient of resistance in $(^\circ C)^{-1}$ (Table 1.5). α is typically $0.005(^\circ C)^{-1}$ for most metals or about 0.5%/ºC. For example, if a piece of copper has a resistance of 100 Ω at 10ºC, at 300ºC it will have a resistance of (100 Ω)[1 + (0.0039/ºC)(300ºC-10ºC)] = 213 Ω.

TABLE 1.5. Linear Temperature Coefficients for Various Materials.

Material	$\alpha(^\circ C)^{-1}$
Carbon	–0.0005
Silver	+0.0038
Copper	+0.0039
Aluminum	+0.0039
Platinum	+0.0039
Tungsten	+0.0045
Iron	+0.0050
Nickel	+0.0067

The temperature dependence of conductors is used to measure temperature in electronic devices. Two types of such devices are the *resistance thermometer* and the *thermistor*.

The resistance of pure platinum is often used as a standard of temperature over a wide temperature range, from -190ºC to over 600ºC; the device is called a resistance thermometer. A constant known current is forced through a pure piece of platinum wire, and the voltage across the platinum wire is measured. This voltage drop is linearly proportional to the resistance of the platinum wire, which in turn depends upon the temperature, according to the equation (1.17).

A thermistor is a special two-terminal device designed to have a strong function of temperature. The resistance, R_T, of a thermistor is given by

$$R_T = R_0 e^{A(1/T - 1/T_0)} \tag{1.18}$$

where A is a constant in kelvins (K) whose value depends upon the particular thermistor, R_0 is the resistance at temperature T_0 (K), T is the temperature of the thermistor (K), and $e = 2.718$ (the basis of the natural logarithms). The constant A depends slightly on temperature. Using (1.18) over a 50ºC range with A constant would typically produce a 3ºC error. For greater accuracy the thermistor should be calibrated experimentally over the temperature range expected.

The resistance change for a thermistor is typically ten times the resistance change for copper for the same temperature change. The thermistor resistance decreases with increasing temperature, which is like carbon, but opposite to the temperature dependence for most metals.

Commercial thermistors are usually made from sintered mixtures of Mn_2O_3 and NiO or platinum alloys and are often encapsulated in metal or a thin glass bead with two wire leads. They are available in a wide range of resistance values. The thermistor resistance at 25ºC can range from 30 Ω to 20 MΩ for various types, and the ratio of the resistance at 25ºC to resistance at 125ºC may range from 10:1 to 100:1.

The thermistor takes a certain time to come to equilibrium if its surrounding temperature is changed. The thermistor time constant is defined as the time required for the thermistor resistance to change by 63%, with 100% being the total change in resistance for an infinite time. For example, consider a thermistor with a 10 KΩ resistance at 100ºC, and a 110 KΩ resistance at 30ºC, and a time constant of 100 ms. If the thermistor is initially in equilibrium at 100ºC and is suddenly immersed in a 30ºC environment, then its resistance will increase to 110 KΩ + 0.63(110 KΩ - 10 KΩ) = 73 KΩ in the first 100 ms. Bare thermistors with no glass covering are the fastest; they are available with time constants as short as 4 ms in water and 100 ms in air. Larger thermistors encapsulated in glass would, of course, have longer time constants, perhaps 200 ms in air and 5 s in water.

Finally it should be pointed out that the self-heating of the thermistor due to the current I flowing through it should be as small as possible. The manufacturer will specify a self-heating (I^2R_T) power per degree Celsius error produced. This might be 0.5 mW/ºC in air and 2.5 mW/ºC in water for a typical thermistor. For such a thermistor in water, the internal self-heating should be kept much less than 2.5 mW for the temperature error to be much less than 1ºC.

1.7 POWER SOURCES

A *power source* or *supply* is a two-terminal circuit element that supplies the voltage and current to other elements in a circuit. In a circuit with a power source the remaining elements of the circuit are called the *load*. In a dc circuit a power source has a positive terminal and a negative terminal, with the positive end acting as a source of positive voltage or current. In general, current flows from the positive terminal through a circuit and into the negative terminal. The fact that there is a more positive and more negative end of a battery defines the *polarity*.

It is often convenient to use the concept of an ideal power source to describe various circuits. An *ideal voltage source* is one that supplies a constant voltage to the rest of the circuit regardless of the current drawn by the other elements in the circuit. An ideal voltage source has *zero internal resistance*. The circuit symbol for an ideal voltage source is a circle as shown in Fig. 1.8(a); a voltage V is always present between points A and B. A specific type of dc voltage source is the *battery*, shown in Fig 1.8(b). In many cases the symbol for a battery, with its alternating long and short lines, is used for any dc voltage source. The positive "+" terminal of a battery is not always marked, but always coinicides with the end with a long line.

An *ideal current source* is one that supplies a constant current regardless of the voltage across the load. An ideal current source has an i*nfinite internal resistance* so that changes in the load resistance will not affect the current supplied by the source. The circuit symbol for an ideal current source is shown in Fig. 1.8(c); a current I always flows through the circle representing the ideal current source regardless of the rest of the circuit.

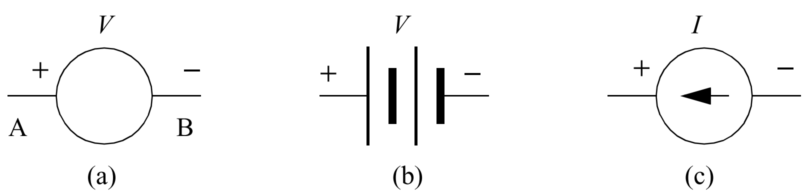

FIGURE 1.8. Symbols for (a) voltage source, (b) battery and (c) current source.

Real power supplies cannot supply an arbitrary amount of voltage or current. A real voltage source is limited in the amount of current it can supply before the voltage fails to meet the specified value. Similarly a real current supply is limited by the amount of voltage it can supply. A real supply behaves like it has some resistance inside, but for most purposes we can treat real supplies as ideal supplies.

A real voltage source or battery can be represented by an ideal battery or voltage source V_{bb} in series with a resistance r, as shown in Fig. 1.9. Two circuit elements are in *series* when all the current passing through one element also passes through the other element. The terminal voltage of a real battery falls as we draw more and more current from it because there is a voltage drop across the battery's internal resistance. The amount of the voltage drop comes directly from Ohm's law and is equal to Ir.

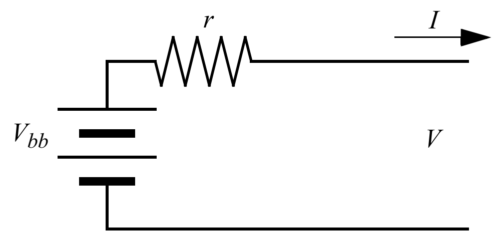

FIGURE 1.9. Real voltage source showing a series resistance.

For example a 6 V battery with a 1 MΩ internal resistance will essentially deliver 6 uA to a 1 KΩ, a 10 KΩ, or a 100 KΩ resistor connected across it. The current remains constant so long as the load resistance is small compared to the internal resistance. Such a source can be approximated by a battery with a series resistance r much larger than any resistances in the load connected across the battery. Then the battery will always supply a current of V_{bb}/r regardless of any changes in the load resistance.

A real current source can be represented by an ideal current source in parallel with a resistance r, as shown in Fig. 1.10. Two circuit elements are in *parallel* when both elements share the same voltage drop across their terminals. The current supplied by a real current source falls if the load is large since it will demand a voltage drop across the load, but this voltage drop will send current through the large internal resistance r. The current taken up by the internal resistance is equal to V/r.

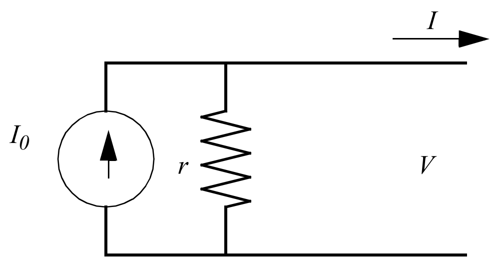

FIGURE 1.10. Real current source showing a parallel resistance.

Batteries

A battery is a power supply that converts a chemical reaction into electrical energy. Batteries may use a variety of chemicals to generate the electrical energy. One of the most common is the alkaline battery commonly used in household and electronic equipment. This battery runs on a mixture of manganese dioxide and carbon as a cathode or negative source and powdered zinc for the anode or positive source. A solution of potassium hydroxide in water forms an electrolyte that caries charges between the anode and cathode. Other common batteries use only carbon and zinc, camera and watch batteries may use lithium ions as their chemical components.

Batteries are rated by their size, voltage, lifetime, and power density. Many battery sizes are standardized by size with names like AA and D for use in electrical instruments. There are also many custom battery sizes for particular manufacturers, such as for cellular phones and handheld computers.

Typical battery voltages range from just over 1 V to 12 V. This voltage rating is the same as V_{bb} in Fig 1.9 and indicates the difference in voltage that the battery will maintain between its two terminals. As a battery uses its chemicals it loses the ability to maintain its rated voltage, and is said to go bad. At this point its internal resistance r increases sharply. For good batteries this internal resistance can vary from 0.03 Ω in a 12 V automobile battery, to 13 Ω in a standard 9 V battery.

A battery's lifetime is measured ampere-hours (Ah), and indicates the total current times the time that battery can supply its intended voltage before the chemical reaction has weakened and the voltage falls off. For instance, a battery rated at 3000 mAh can supply a continuous current of 10 mA for 300 hours, or 100 µA as a backup for 30,000 hours or about 3 1/2 years.

Power density measures the lifetime of the battery compared to the weight of chemical in the battery. Power density only depends on the type of chemical used to create the electrical energy. Higher power densities, such as in lithium batteries, permit smaller battery size.

An important class of batteries are rechargeable. These batteries also use chemical reactions, but are designed to allow the reactions to be reversed. This allows the battery to be recharged by putting the battery in parallel with another power supply which restores the lifetime of the battery. Examples of rechargeable batteries include nickel-cadmium (NiCad) and nickel metal hydride (NiMH) as well as the lead-acid battery used in cars. One disadvatage to a rechargeable battery is that it continues to lose power even while no current is drawn from it.

For a brief summary of different types of batteries, see Appendix A.

1.8 KIRCHOFF'S LAWS AND NETWORK ANALYSIS

A battery with a voltage V and a resistor with resistance R can be connected by wires to form a simple circuit as shown in Fig 1.11. Notice again that the straight lines drawn in the circuit diagram represent wires with zero resistance, and therefore no change in voltage along those wires; the voltage at the positive battery terminal is exactly the same as the voltage at the top of the resistor. When wires connect both ends of a component to the other components in the circuit the circuits is said to be *closed*. If there are wires or components that are unconnected as in Fig. 1.9 the circuit is said to be *open*. A special case of a closed circuit is a circuit closed a zero resistance wire, which is called a *short* circuit.

Also notice that we may ground any one point in a circuit as shown in Fig. 1.11.Whether or not there is a ground the voltage difference between the battery terminals is V, and the current in the circuit is the same: $I = V/R$. The ground connection merely sets the zero voltage level; no current flows into or out of the ground connection. In some circuits more than one ground wire is shown. In this case, a virtual wire is assumed to connect all ground points in a circuit so that the ground connections can be used to close a circuit and current may flow through that virtual wire.

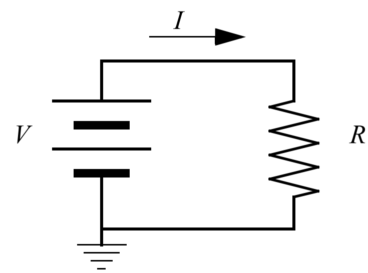

FIGURE 1.11. Simple circuit with a battery and resistor.

Kirchoff's Laws

The two basic laws of electricity that are most useful in analyzing circuits are Kirchoff's laws of current and voltage:

Kirchoff Current Law (KCL) *: The sum of currents entering a junction is equal to the sum of currents leaving the junction.*

Kirchoff Voltage Law (KVL) *: The sum of voltage increases around a closed loop of a circuit is equal to the sum of voltage drops around the loop.*

The current law merely says that no electric charge is being created or destroyed at the junction in question; that is the total current entering equals the total current leaving. The voltage law says that there is no net gain in electrical potential energy for any charge making a trip around any closed loop. For example the energy the charge gains passing though a battery must all be lost as heat in the resistor in the rest of the loop.

To solve for the currents and voltages in a circuit or network, using Kirchoff's laws, we must first identify and label all the voltage sources and voltage drops in the circuit. We then draw an arrow for a current direction in each branch of the circuit and define a current symbol such as I for that current. It is useful to draw the current arrows in the direction the current actually flows in the circuit; if this is done, the numerical value obtained for the current at the end of the calculation will turn out to be positive. However, if we guessed wrong as to the current direction, the numerical value obtained for the current will be negative but of the same magnitude as if we guessed the current direction correctly. It should also be emphasized that once the current directions are chosen, the polarities of the voltage drops are fixed.

Network analysis is the method of calculating voltages and currents in a circuit. There are basically three methods we will discuss, and only experience will enable you to choose the easiest method for a given situation. In the following paragraphs, a *branch* is simply one path or wire through which one current can flow, a *loop* is a closed path in the circuit (an electron can flow around any loop and return to its starting point without leaving the actual circuit), and a *junction* or *node* is a point in a circuit where three or more wires come in contact together. In all three methods we must have a clear circuit diagram with all the voltages and currents clearly defined.

The Branch Method

In this method we define the current in each of n branches of the circuit. Then we can write a KCL equation at each junction and a KVL equation for each closed loop in the circuit. However, a little thought will show that such a procedure applied to each junction and each closed loop will produce a number of nonindependent equations; that is, we might obtain five equations in three unknowns. Obviously, we wish to obtain n independent equations to solve for the n unknown currents.

It can be shown that if there are k junctions in the circuit, there are only $k-1$ independent KCL equations. For example if there are four junctions, there are three independent equations. It can also be shown that there are only $n-(k-1)$ independent KVL equations. If there are three branches and two junctions there are only two independent loops. Totaling the number of independent equations (KCL and KVL) is $(k-1)+n-(k-1)=n$. We can solve these n equations for the n unknown currents. The only constraint in selecting independent equations is that each current must appear in at least one KCL junction equation and in at least one KVL loop equation.

As an example of the branch method, let us solve for the currents and voltages in the circuit of Fig 1.12. In this problem all the voltages and resistances are known. First we define the three currents in the three branches and draw the arrows to mark the current directions. Note that current I_1 flows through V_1, R_1, and R_4. The ground connection is ignored for the purposes of the branch since it is not needed to connect to another ground. The other two currents each flow through a single resistor.

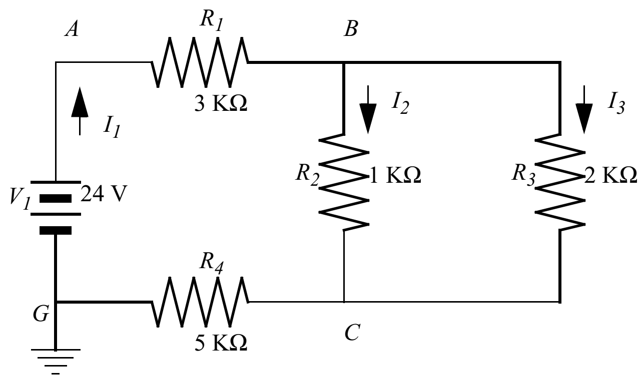

FIGURE 1.12. Circuit problem using branch currents.

For this circuit there are two junctions, B and C, so $k = 2$, so there is $k - 1 = 1$ independent KCL equation. The KCL says that the *total* current entering a junction equals the *total* current leaving the junction. At junction B we have

$$I_1 = I_2 + I_3 \tag{1.19}$$

This equation uses all three branch currents I_1, I_2, and I_3 so we can use this equation for our analysis.

There are $n = 3$ branch currents, and $k = 2$ junctions, so there are $n - (k - 1) =$ 2 independent KVL equations. The KVL says that the sum of voltage *increases* equals the sum of voltage *drops* around one loop. We can obtain one KVL equation by starting at G, which is ground (0 V), and going around the circuit clockwise through R_2, getting

$$V_1 = R_1 I_1 + R_2 I_2 + R_4 I_1 \tag{1.20}$$

Another KVL equation can be found by starting at G and going around clockwise through R_3, getting

$$V_1 = R_1 I_1 + R_3 I_3 + R_4 I_1 \tag{1.21}$$

Since all three currents appear in one or the other of these two KVL equations they can be used for our analysis.

Substituting (1.19) for I_1 in (1.20) and in (1.21) immediately gives us two equations in the two unknowns I2 and I3.

$$V_1 = R_1(I_2 + I_3) + R_2 I_2 + R_4(I_2 + I_3) = (R_1 + R_2 + R_4)I_2 + (R_1 + R_4)I_3 \quad \textbf{(1.22)}$$

$$V_1 = R_1(I_2 + I_3) + R_3 I_3 + R_4(I_2 + I_3) = (R_1 + R_4)I_2 + (R_1 + R_3 + R_4)I_3 \quad \textbf{(1.23)}$$

And now (1.22) can be solved for I_3 and substituted into (1.23).

$$V_1 = (R_1 + R_4)I_2 + (R_1 + R_3 + R_4)\left(\frac{V_1 - (R_1 + R_2 + R_4)I_2}{R_1 + R_4}\right) \quad \textbf{(1.24)}$$

This equation can finally be solved for I_2, then used to find I_3 from (1.23) and I_1 from (1.19).

$$I_2 = V_1\left(\frac{R_3}{R_1R_2 + R_1R_3 + R_2R_3 + R_2R_4 + R_3R_4}\right) \quad \textbf{(1.25)}$$

$$I_3 = \frac{V_1 - (R_1 + R_4)I_2}{R_1 + R_3 + R_4} = V_1\left(\frac{R_2}{R_1R_2 + R_1R_3 + R_2R_3 + R_2R_4 + R_3R_4}\right) \quad \textbf{(1.26)}$$

$$I_1 = V_1\left(\frac{R_2 + R_3}{R_1R_2 + R_1R_3 + R_2R_3 + R_2R_4 + R_3R_4}\right) \quad \textbf{(1.27)}$$

Note that no specific values have been placed into the equations up to this point. This greatly helps avoid errors in arithmetic. Finally we can insert the given values for resistors and batteries in the circuit, and find that $I_1 = 2.77$ mA, $I_2 = 1.85$ mA and $I_3 = 0.92$ mA.

We can calculate the voltages at points B and C by using Ohm's law and the values for the branch currents. Starting from ground at 0 V, $V_C = +I_1R_4 = (2.77$ mA)(5 KΩ) = 13.85 V. Moving from G to A to B, $V_B = V_1 - I_1R_1 = 24$ V - (2.77 mA)(3 KΩ) = 15.7 V.

The Loop Current Method

In this method one loop current is drawn for each closed loop of the circuit, as shown in Fig. 1.13. Note that this is the same circuit we used in the example for the branch method. It is convenient to draw all the loop currents going clockwise. Also, the currents in the loop method are not necessarily the same as the currents in the branch method.

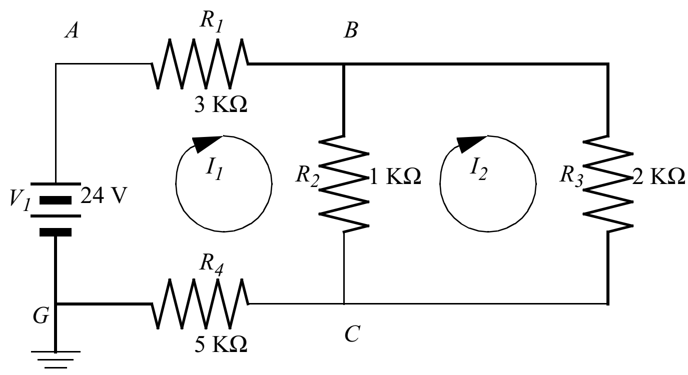

FIGURE 1.13. Circuit problem using loop currents.

Notice that a loop current, by definition, always flows through a junction, so the KCL is automatically satisfied. Unlike the branch method, we write only the KVL equations, and we need write only one KVL equation for each loop. We will always have an equal number of KVL equations and loop currents, so we can solve for the loop currents.

In writing the KVL equations, notice that we must always use the total current flowing through a resistance to calculate the voltage drop across that resistance. For this example there are two currents flowing through R_2 so the voltage drop from B to C is $(I_1 - I_2)R_2$. It is very important to keep track of the direction of the current when calculating the voltage changes.

The two KVL equations are

$$V_1 = R_1 I_1 + R_2(I_1 - I_2) + R_4 I_1 \quad \textbf{(1.28)}$$

$$0 = R_2(I_2 - I_1) + R_3 I_2 \tag{1.29}$$

In the loop used for (1.29) there was no voltage source, and the direction of the loop through R_2 was opposite to the direction of the loop through that resistor in (1.28). Solving (1.29) for I_2 and substituting that value into (1.28),

$$V_1 = R_1 I_1 + R_2\left(I_1 - \frac{R_2}{R_2 + R_3} I_1\right) + R_4 I_1 = \left(R_1 + \frac{R_2 R_3}{R_2 + R_3} + R_4\right) I_1 \tag{1.30}$$

Finally we find for the two loop currents

$$I_1 = \frac{V_1}{R_1 + \dfrac{R_2 R_3}{R_2 + R_3} + R_4} = V_1\left(\frac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3 + R_2 R_4 + R_3 R_4}\right) \tag{1.31}$$

$$I_2 = \left(\frac{R_2}{R_2 + R_3}\right) I_1 = V_1\left(\frac{R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3 + R_2 R_4 + R_3 R_4}\right) \tag{1.32}$$

These are the same results found in the branch method, where I_1 and I_2 in this loop method are respectively the same as I_1 and I_3 in the branch method.

The Nodal Method

In this method the *voltages* are the unknowns, and the KCL is written for each junction in terms of voltages and resistances. For this method, the number of equations equals the number of independent junctions (k - 1), where k is the total number of junctions. Currents are drawn with directions as in the branch method.

Again we'll use the circuit of Fig. 1.13 for our example. There are two junctions: B and C. We therefore obtain one independent KCL equation in terms of the voltages. Let V_B and V_C be the voltages at junctions B and C, respectively. The KCL at B is $I_1 + I_2 = I_3$, or, in terms of voltages,

$$\frac{V_1 - V_B}{R_1} = \frac{V_B - V_C}{R_2} + \frac{V_B - V_C}{R_3} \tag{1.33}$$

The voltage $V_C = I_1 R_4 = (V_1 - V_B) R_4 / R_1$. So (1.33) can be rewritten to eliminate V_C.

$$\frac{V_1 - V_B}{R_1} = \frac{V_B}{R_2} - \frac{(V_1 - V_B)R_4}{R_1 R_2} + \frac{V_B}{R_3} - \frac{(V_1 - V_B)R_4}{R_1 R_3} \qquad \textbf{(1.34)}$$

This equation can be solved for V_{B},

$$V_B = V_1\left(\frac{R_2R_3 + R_3R_4 + R_2R_4}{R_1R_3 + R_3R_4 + R_1R_2 + R_2R_4 + R_2R_3}\right) \qquad \textbf{(1.35)}$$

And finally we can return to I_1,

$$I_1 = \frac{V_1 - V_B}{R_1} = V_1\left(\frac{R_2 + R_3}{R_1R_3 + R_3R_4 + R_1R_2 + R_2R_4 + R_2R_3}\right) \qquad \textbf{(1.36)}$$

Which matches the result from the branch method.

Notice that we solved for the currents by using Ohm’s law. The algebra is often simplified by using the conductance G instead of resistance: $G_1 = 1/R_1$, etc. The nodal method is useful when the total number of junctions is less than the number of loops. Then there will be fewer nodal equation than loop equations.

1.9 RESISTOR NETWORKS

Many problems in network analysis can be simplified before applying a particular method to solve the problem. In many cases a connected set of resistors, power sources, and other components can be replaced by a single equivalent component.

If two resistors are connected in series, they are connected end-to-end so that the same current flows through each of them, as in Fig. 1.14(a). The effective resistance of the two resistors can be found from Ohm’s law.

$$R_{eff} = \frac{V_A - V_C}{I} = \frac{V_A - V_B}{I} + \frac{V_B - V_C}{I} = R_1 + R_2 \qquad \textbf{(1.37)}$$

The total effective resistance is simply the sum of the two individual resistances. Thus a 1 KΩ resistor and a 3 KΩ resistor in series act like a single 4 KΩ resistor. This rule can be extended to N resistors in series, in which case the total effective resistance is equal to the sum of all the N individual resistances.

$$R_{total} = R_1 + R_2 + \ldots + R_N \qquad \textbf{(1.38)}$$

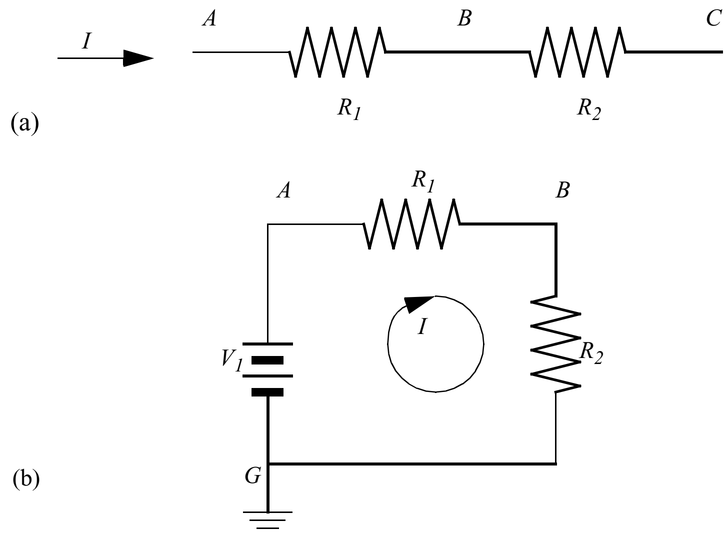

FIGURE 1.14. Two resistors in series.

Consider the same problem using a one loop application of Kirchoff's law in Fig. 1.14(b). If there is an equivalent resistor, R_{eff}, then the current I through the circuit will be the same whether the loop has the two resistors, R_1 and R_2 or a single resistor, R_{eff}. The loop equation for two resistors is

$$V_1 = R_1 I + R_2 I = (R_1 + R_2) I \qquad \textbf{(1.39)}$$

And for the single equivalent resistor

$$V_1 = R_{eff} I \qquad \textbf{(1.40)}$$

Combining these two equations again gives the result for a series resistor (1.37).

If two resistors are connected in parallel or side by side as in Fig. 1.15(a) the same voltage appears across each one. The effective resistance is given by splitting the current into two parts:

$$R_{eff} = \frac{V_A - V_B}{I} = \frac{V_A - V_B}{\frac{V_A - V_B}{R_1} + \frac{V_A - V_B}{R_2}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2} \tag{1.41}$$

If we have N resistors connected in parallel, then

$$R_{eff} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}} \tag{1.42}$$

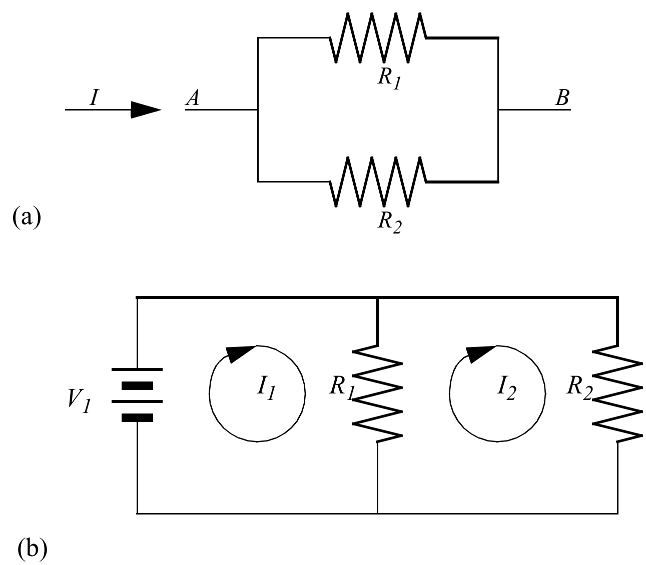

FIGURE 1.15. Two resistors in parallel.

Now consider the circuit using a two loop application of Kirchoff's law in Fig. 1.15(b). If there is an equivalent resistor, R_{eff}, then the current I_1 through the circuit will be the same whether the loop has the two resistors, R_1 and R_2 or a single resistor, R_{eff}. The loop equations are

$$V_1 = R_1 I_1 - R_1 I_2 \tag{1.43}$$

$$0 = R_1 I_2 - R_1 I_1 + R_2 I_2 \tag{1.44}$$

And for the single equivalent resistor

$$V_1 = R_{eff} I_1 \tag{1.45}$$

Combining these three equations gives the result for a parallel resistor (1.42).

The use of series and parallel equivalent resistors can be extended to an arbitrary network. In Fig. 1.16(a) four resistors are arranged with some in parallel and some in serial configurations. To reduce this look for one pair that makes either a serial or parallel connection. Here R_3 and R_4 are in series and can be replaced by $R_{34} = R_3 + R_4$ as shown in Fig. 1.16(b).

In Fig. 1.16(b) we now see resistors R_2 and R_{34} in parallel. Their parallel resistance, R_{234}, is

$$R_{234} = \frac{R_2 R_{34}}{R_2 + R_{34}} = \frac{R_2(R_3 + R_4)}{R_2 + R_3 + R_4} \tag{1.46}$$

That substitution is made in Fig. 1.16(c). The final substitution is to replace R_1 and R_{234} with their equivalent.

$$R_{eq} = R_1 + \frac{R_2(R_3 + R_4)}{R_2 + R_3 + R_4} \tag{1.47}$$

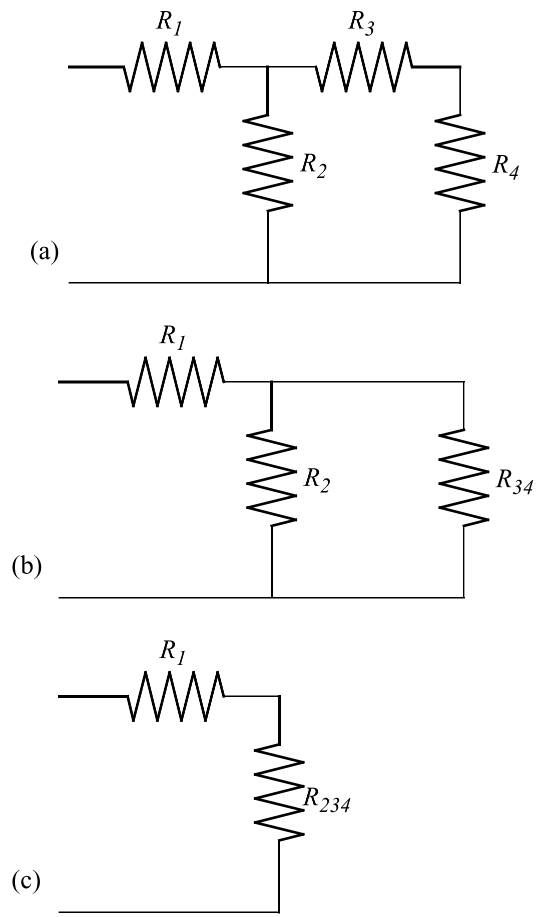

FIGURE 1.16. Simplification of a resistor network.

Voltage Dividers

Let's look at the voltage at point B in Fig. 1.14(b). We can find it from (1.39) and use Ohm's law from ground at point G.

$$V_B = R_2 I = \left(\frac{R_2}{R_1 + R_2}\right) V_1 \tag{1.48}$$

The value of V_B is always a fraction of V_1 that depends on the values of the resistors. This type of circuit where the resistors create a voltage that is a fraction of the initial voltage is called a *voltage divider*. This a very useful combination to spot in a circuit, because the voltage divider is perhaps the most common circuit combination!

For example, we often wish to reduce a power supply voltage to a smaller value. In Fig. 1.17 we have a 20 V battery and we would like to supply –4 V to a load resistor R_L. If R_L is unconnected, forming an open circuit, $I = V_1 / (R_1 + R_2) =$ 1 mA, and the voltage at A is set by the voltage divider $V_A = -V_1 R_2 / (R_1 + R_2) =$ –4 V. Note that the battery has its positive terminal grounded and A is negative with respect to G because the current flows from G towards A.

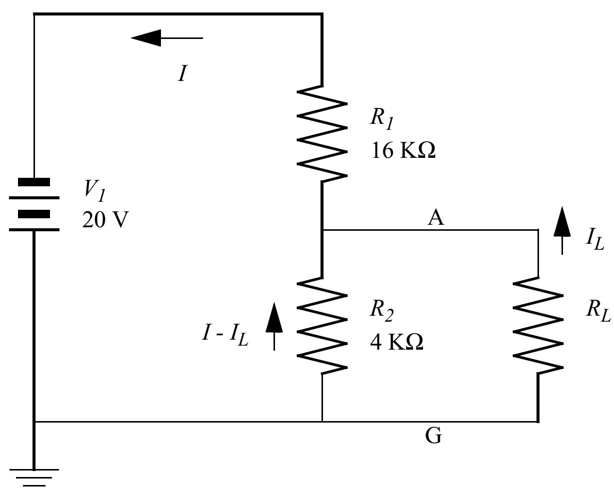

FIGURE 1.17. Voltage divider.

As a real circuit application of a voltage divider, there are two things to keep in mind. First, keep the total current drawn from the battery low to ensure reasonably long battery life. Second, realize that any current drawn from terminal may greatly affect the voltage at A. In Fig 1.17 the load R_L is in parallel with R_2, so V_A is not a simple voltage divider and is given by

$$V_A = \left[\frac{(R_2 \parallel R_L)}{R_1 + (R_2 \parallel R_L)}\right] V_1 = \left[\frac{\dfrac{R_2 R_L}{R_2 + R_L}}{R_1 + \dfrac{R_2 R_L}{R_2 + R_L}}\right] V_1 \qquad \textbf{(1.49)}$$

For example, if $R_L = 1$ KΩ, then by inserting the values, the parallel resistance of R_2 and R_L is 0.8 KΩ and the voltage at the load is $V_A = -0.95$ V. This is substantially less than the designed value of –4 V, and we say that the 1 KΩ resistor has *loaded* the voltage divider.

Similarly, any change in R_L will cause V_A to vary. This variation can be minimized by making R_2 much less than R_L. This is equivalent to making the current I through the divider much larger than the current I_L through the load. If this is true in the divider any change in R_L will have minimal effect on the voltage V_A. Such a divider is said to be *stiff*. Note that a stiff divider can use a lot of current compared to the load and may be wasteful if the battery has limited power or lifetime.

1.10 EQUIVALENT CIRCUITS

In the loop method of network analysis we saw that two loop currents could be treated independently and then summed to get the correct current in a branch. This is an example of the superposition theorem which applies to any linear circuit. A *linear circuit* is one in which all the KVL equations are mathematically linear in the currents and the voltage. This requires that the resistance and voltage of the circuit elements are independent of the current, and there are no terms in the equation containing products or quotients like $V_1 V_2$ or V_1/V_2. All dc circuits containing only batteries and resistances are linear. In general, all circuits are linear when the voltages and currents are relatively small; they become nonlinear for larger currents and voltages.

For any linear circuit containing more than one voltage source, the *superposition theorem* states that the total current in any part of the circuit equals the algebraic sum of the currents produced by each source separately. To calculate the current due to any one particular source, replace all the other voltage sources by short circuits and all other current sources by open circuits.

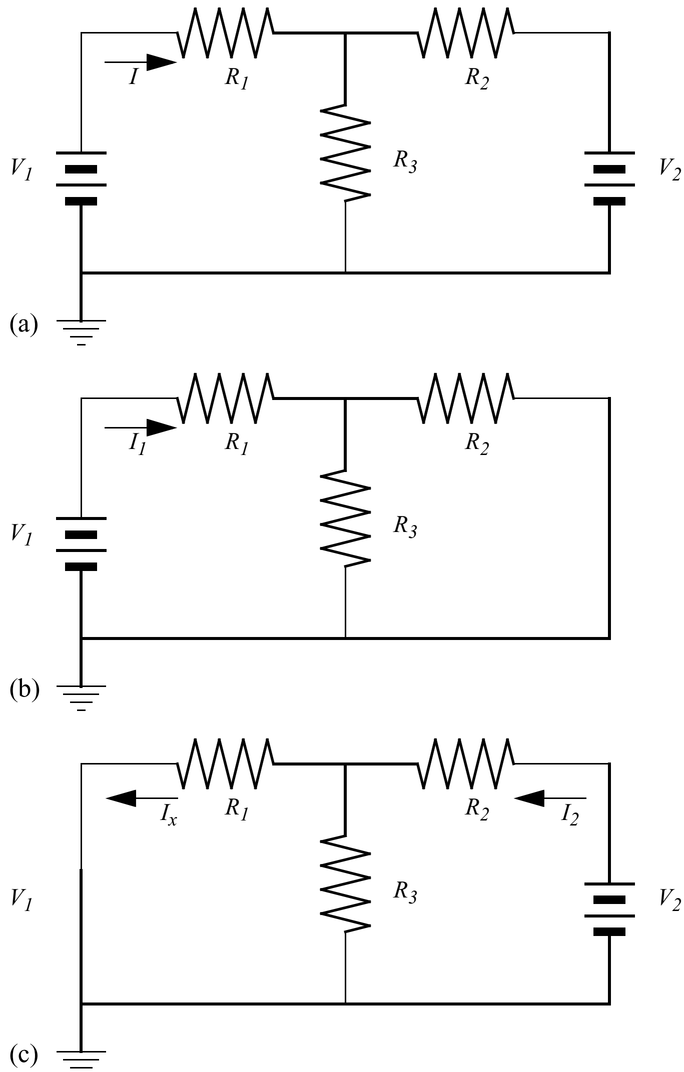

FIGURE 1.18. Superposition theorem problem.

In the circuit of Fig 1.18(a), let us use the superposition theorem to calculate the current I through R_1. The current through R_1 due to V_1 alone is obtained by shorting out V_2, as shown in Fig. 1.18(b), and

$$I_1 = \frac{V_1}{R_1 + (R_2 \parallel R_3)} \tag{1.50}$$

The current through R_2 due to V_2 alone is obtained by shorting out V_1, as shown in Fig. 1.18(c):

$$I_2 = \frac{V_2}{R_2 + (R_1 \parallel R_3)} \tag{1.51}$$

and the current I_x through R_1 is then

$$I_x = \left(\frac{V_2}{R_2 + (R_1 \parallel R_3)}\right)\left(\frac{R_3}{R_1 + R_3}\right) \tag{1.52}$$

By the superposition theorem the current through R_1 is just $I = I_1 - I_x$ or

$$I = \frac{V_1}{R_1 + (R_2 \parallel R_3)} - \left(\frac{V_2}{R_2 + (R_1 \parallel R_3)}\right)\left(\frac{R_3}{R_1 + R_3}\right) \tag{1.53}$$

This result could be obtained from a Kirchoff analysis with considerably more effort.

V-I Curves

One the requirements of the superposition theorem is that the circuit elements be linear. It is very useful to look at the current through a circuit element or connected group of circuit elements compared to the voltage through the same circuit elements. Typically the voltage is plotted on the *x*-axis of the graph and the current is plotted on the *y*-axis. Linear devices and circuits will appear as straight lines on these voltage-current (V-I) graphs.

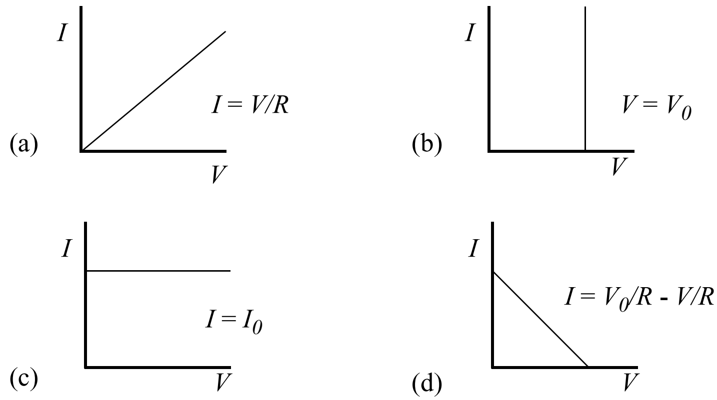

FIGURE 1.19. Voltage-current curves: (a) resistor, (b) voltage source, (c) current source, (d) battery and resistor in series.

A resistor, or any device that strictly obeys Ohm's law as in Fig. 1.19(a) will appear on a V-I graph as a straight line with a slope of $1/R = I/V$. An ideal voltage source produces the same voltage regardless of the current drawn and the V-I graph will be a vertical line as shown in Fig. 1.19(b) with an x-intercept at the nominal voltage V_0. An ideal current source produces the same current regardless of the voltage across its terminals and the V-I graph will be a horizontal line as shown in Fig. 1.19(c) with a y-intercept at the nominal current I_0.

Consider the voltage-current relationship for a battery and resistor in series such as in Fig. 1.9 from Section 1.7. This is the same relationship that is found with a real battery with internal resistance. If there is no load, then the circuit is open and there is no current and the voltage between the open terminals would be V_0. Once a load is placed on the circuit there will be a voltage drop due to the resistor r equal to Ir. The expression for voltage as a function of current is

$$V = V_0 - Ir \tag{1.54}$$

or expressed as current as a function of voltage

$$I = \frac{V_0}{r} - \frac{V}{r} \tag{1.55}$$

This can be graphed as a straight line with y-intercept V_0/r and slope $-1/r$ as shown in Fig. 1.19(d). Graphically we see that circuits with resistors and power sources are linear – their V-I curves are straight lines.

In a formal sense a battery has a negative resistance because the battery gives energy to the charge, making V_1 greater than V_2; but this occurs because chemical energy is converted into electrical energy in a battery. Some circuit elements are said to have negative resistance because the slope is negative, but this is not the real resistance. This rate of change of voltage with respect to current is defined as *dynamic*, or ac, resistance. A circuit for which the V-I curve is not a straight line has non-linear resistance.

Thevenin's Theorem

Thevenin's theorem is useful for analyzing many circuits. The theorem states that any combination of batteries and resistances with two terminals is electrically equivalent to an ideal battery of voltage e in series with one resistance r, as shown in Fig 1.20. This a direct result of the superposition theorem, and the same requirements for that theorem apply to Thevenin's theorem.

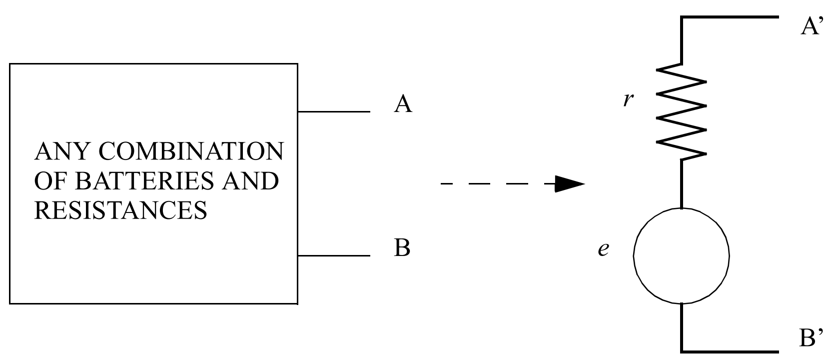

FIGURE 1.20. Thevenin equivalence circuit.

Note that an ideal battery has a consistent voltage regardless of the rest of the circuit, a resistor's value does not depend on current, and all the Kirchoff voltage loop equations for linear circuit elements are linear. Thus for two points A and B in the circuit, the current I flowing out of A (to the "outside world") must be linearly related to the voltage difference V_{AB}. Such a linear relationship, of course, yields straight-line plot of V_{AB} versus I of the form

$$V_{AB} = e - Ir \tag{1.56}$$

But (1.56) is precisely the equation for $V_{A'B'}$ of the Thevenin equivalent circuit of Fig 1.20 that we saw in (1.54).

We can now look at both (1.56) and the V-I graph in Fig 1.19(d) and see that the Thevenin equivalent voltage e is the open circuit voltage V_{AB} measured when no current flows into or out of A and B. It also follows that the Thevenin resistance r is given by

$$r = \frac{e - V_{AB}}{I} \tag{1.57}$$

Once we obtain the linear relationship between V_{AB} and I, both e and r are easily determined. Any method that determines the graph in Fig. 1.19(d) will uniquely determine e and r. For the equivalent circuit, terminals A' and B' will act electrically exactly like terminals A and B of the actual circuit.

Theoretically speaking, e is the open circuit voltage measured when R_L is infinite. If the internal voltages and resistances are known, this can be determined through Kirchoff's laws or the network analysis techniques of Section 1.8. The equivalent resistance r is also the open circuit voltage V_{AB} divided by the short-circuit current I_{sc}, which is the current flowing from terminals A and B when they are shorted together. A useful way to calculate r is to consider the same circuit, but short out all batteries - replace them with wires. The resistance r is then the total net resistance between output terminals A and B. However, this method of shorting out the batteries to determine r is appropriate only for constant voltage sources such as batteries.

In practice, we can use measuring instruments to determine the voltage-current relationship for a circuit, even if we cannot calculate the behavior by knowing the individual component values. It is easy and safe to measure the open-circuit voltage to determine e. However, it is often disastrous (producing sparks, smoke, vile odors, destruction of the circuit, and embarrassment) actually to short terminals A and B to determine r. A better way to determine r experimentally is to measure the voltage V_{AB} between the two circuit terminals for a known load resistance R_L connected between A and B. The r is given again by (1.57).

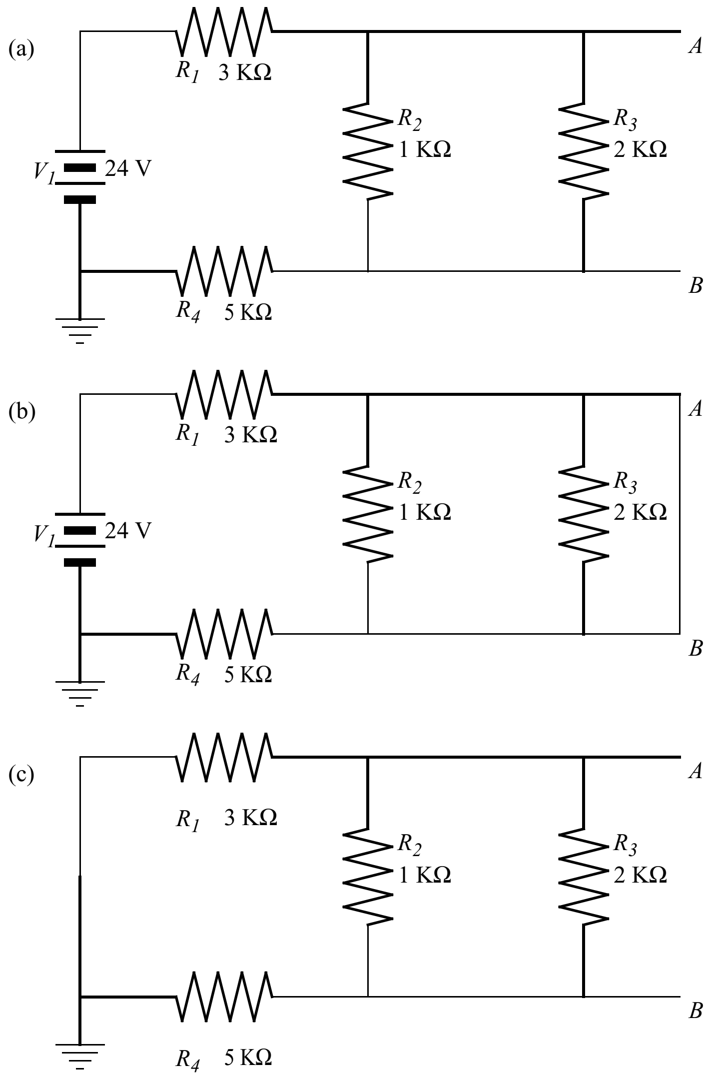

FIGURE 1.21. Thevenin equivalent circuit problem.

Let us consider the Thevenin equivalent circuit for the circuit of Fig. 1.21(a) which was also the circuit that appeared in Section 1.8. We found there that the current through the 2 KΩ resistor was 0.92 mA, so the open circuit voltage V_{AB} = (0.92 mA)(2 KΩ) = 1.84 V. The short circuit voltage is found by connecting a wire from A to B and then finding the current through that wire. For Fig 1.21(b) that becomes a one loop circuit with V_1, R_1, and R_4 in series. The current $I_{sc} = V_1/(R_1 + R_4)$ = 3 mA. By division r = (1.84 V)/(3 mA) = 610 Ω. If instead we short out the battery V_1 as in Fig. 1.21(c) then calculate the equivalent resistance we get 610 Ω just as we did by finding the short circuit current.

Norton’s Theorem

Norton’s theorem simply states that any combination of batteries and resistors with two terminals is electrically equivalent to an ideal current source in parallel with a resistance, as shown in Fig 1.22. Remember from Section 1.7, any real current source can be represented by an ideal current source i in parallel with a resistance r, so this is an example of Norton’s theorem. The equivalent current is just the short circuit current I_{sc} found in the same way as we found it for the Thevenin equivalent circuit. The equivalent resistance is the same as the equivalent resistance of the Thevenin equivalent circuit and can be found the same way by finding the open circuit voltage V_{AB}, then $r = V_{AB}/I_{sc}$.

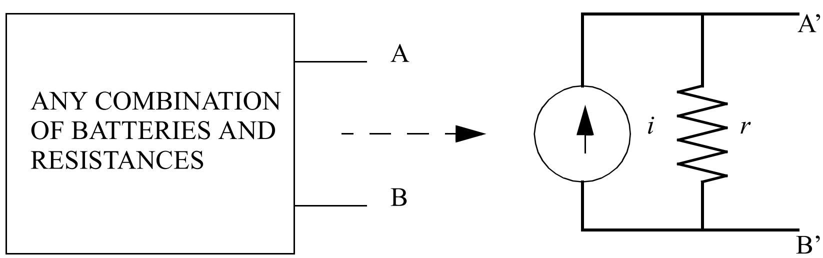

FIGURE 1.22. Norton equivalence circuit.

We emphasize that the Thevenin equivalent is just as good as the Norton equivalent, and vice versa. Each is electrically equivalent to the original circuit being analyzed and to each other. Which equivalent circuit you use is a matter of taste and convenience, although we will see that there is a tendency to use the current equivalent circuit for certain types of circuits based on industry usage.